CONTROLLING LINEAR FLEXIBLE SYSTEMS DYNAMICS BY USING ACTIVE CONTROL WITH OPTIMAL CONTROL THEORY

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Keywords: *optimal input signals, optimal trajectory, performance index, linear regulator, nonzero set point tracking*

Abstract

Controlling flexible systems dynamics is always a challenge in developing a control system. Maneuvers done by a flexible system will induce unwanted vibrations residue, which usually disturb the system performances. Thus in order to control the dynamics of the system; active control systems were developed by using optimal control theory. These controllers generate optimal input signals, which drive the systems to follow optimal trajectories that minimize the performance indexes and satisfy certain criterions. Linear regulator and nonzero set point tracking were the criterions used in this research. The applications of these controllers on the flexible systems improve the flexible systems dynamics.

1 General Introduction

1.1 Background

On very flexible systems, vibrations have become a more challenging problem because unwanted vibrations can easily induce these kinds of system. Vibrations are usually unwanted because, at certain levels, they cause disturbances on system performances, e.g. changes on satellites attitude. They can even cause structural failures on big structures, such as bridges, aircraft wings, tall buildings, solar panels etc. Therefore, in order to control these unwanted vibrations, methods need to be developed.

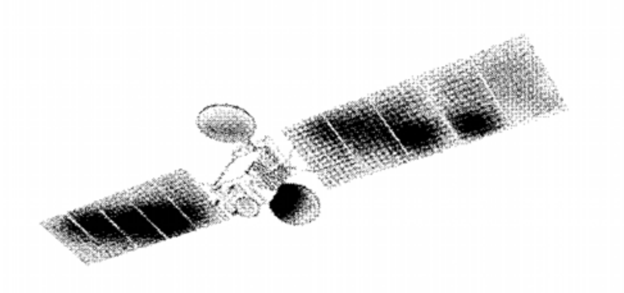


Fig. 1. Satellite with Flexible Solar Panels

Some methods that are used to control the vibrations on flexible systems, such as adding system stiffness, adding dampers into the system and using input shaping method, have been developed. In this research, active vibrations control that uses optimal control theory were developed. These controllers generate optimal input signals that drive the flexible systems to follow optimal trajectories that minimize the performance indexes and satisfy certain criterion.

1.2 Used assumptions

In this research, the flexible systems were modeled as 2-Degree of Freedom (DOF) and 4-DOF linear time-invariant spring-mass systems with zero damping characteristic ($\zeta = 0$). The effects of friction on the systems were considered negligible and external force was given to only one of the masses at the time. All state variables were assumed could be used as feedback, therefore the systems were considered observable. Linear regulator and nonzero set point tracking were the criterions used in this research. These two criterions were used to develop two different controllers separately.

1.3 General procedures

In doing this research, the linear time-invariant flexible systems equations of motion were first derived. Then, the normal mode of the flexible systems were analyze. State spaces of the systems were then formed based on the equations of motion. The controllability and the observability of the systems were tested first before the implementation of the optimal control theory. All simulations were done by using MATLAB® and SIMULINK®. The simulations results were then analyzed.

2 Experiment

2.1 Equations of motion

This experiment used the following physical models to derive the mathematical model of the linear time-invariant flexible systems in the form of equations of motion.

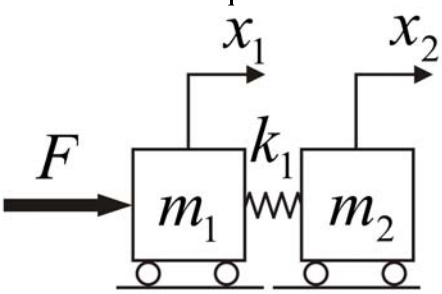


Fig. 2. 2-DOF Spring-Mass System

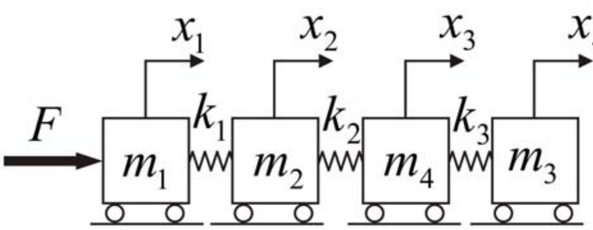


Fig. 3. 4-DOF Spring-Mass System

By using Newton's second law of motion and free body diagram, the following general form of equations of motion for 2-DOF and 4-DOF are obtained.

$$[M]\{\ddot{X}\}+[K]\{X\}=\{F\}_i \qquad (1)$$

Where subscript *i* define onto which mass the external force was being applied. Several equations of motion were obtained.

2.2 Normal mode analysis

The linear time-invariant flexible systems used in this research have the following configurations. The 2-DOF system has a configuration as the following.

m_1 (kg)	m_2 (kg)	k (N/m)
2	3	3

There are two configurations for 4-DOF system. The followings are the first and second configuration respectively.

m_1 (kg)	m_2 (kg)	m_3 (kg)	m_4 (kg)	k_1 (N/m)	k_2 (N/m)	k_3 (N/m)
1	1.2	1.5	1	2	1.8	2.2
1	1.2	1.5	1	3	2.7	3.3

With the help of MATLAB®, by using the equation of motion with zero external force, the normal mode of the linear time-invariant flexible systems were analyzed. The for 2-DOF system, the eigen values are as the following.

λ_1	λ_2
2.5	0

Hence, the natural frequencies are as the following.

ω_1 (Hz)	ω_2
0.2516	0

The normal modes are as the following.

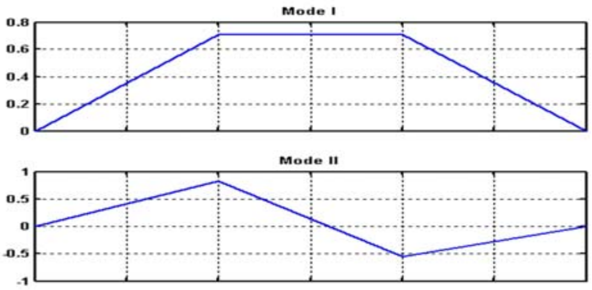


Fig. 4. 2-DOF System Normal Modes

While for the 4-DOF systems, for the first and second configuration the following eigen values, respectively, are obtained

λ_1	λ_2	λ_3	λ_4
5.3031	3.6667	0	1.0635

7.9547	5.5	0	1.5953

Therefore, the following natural frequencies for both configurations, respectively, are obtained.

ω_1	ω_2	ω_3	ω_4
0.3665	0.3048	0	0.1641
0.4489	0.3733	0	0.201

Consequently, the following normal modes are obtained for system with the first configuration.

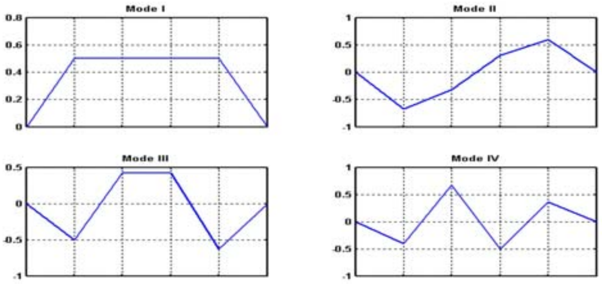


Fig. 5. 4-DOF System, First Configuration, Normal Modes

4-DOF system with the second configuration has similar normal modes.

2.3 State space

In the control system application, it is more convenient to use state space. Hence, the linear time-invariant flexible systems equations of motion were then transform into state spaces. The general form of the systems state spaces was then obtained.

$$\begin{Bmatrix} \dot{X} \\ \ddot{X} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix} \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \{F\}_i \quad (2)$$

Where subscript i define onto which mass the external force was being applied. Several state spaces were obtained.

2.4 Optimal control theory application

Before the application of the optimal control theory, the linear time-invariant flexible systems controllability and observability have to be tested first. The controllability of the systems can be tested by using the controllability Matrix.

$$S = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} \quad (3)$$

Where n is the number of the state variables. For a system to be completely controllable, it is necessary and sufficient that the controllability matrix has a rank of n. By using MATLAB®, it could be seen that all flexible systems controllability matrixes have a rank of n.

While for the observability, because all the state variables were assumed could be used as feedback, the systems were assumed observable.

Since the controllability and observability condition of the linear time-invariant flexible systems had been tested, the optimal control theory could then be applied onto the systems accordingly. There were two criterions used in this research. They were linear regulator and nonzero set point tracking.

The linear regulator controllers were developed by finding a solution for the following equations.

$$u^*(t) = Gx(t) \quad (4)$$

$$J = \int_{t_0}^{t_f} \left[x^T(t)Qx(t) + u^T Ru(t) \right] dt \quad (5)$$

With

$$G = -R^{-1}B^T P \quad (6)$$

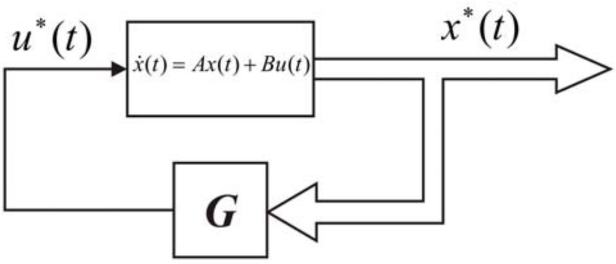


Fig. 6. Flexible System with Linear Regulator Controller Block Diagram

The nonzero set point tracking controllers were developed by finding a solution for the following equations.

$$u^*(t) = -Gx(t) + H_c^{-1}(0)z_0 \quad (7)$$

$$J = \int_{t_0}^{t_f} \left[x'^T(t)Qx'(t) + u'^T Ru'(t) \right] dt \quad (8)$$

With

$$G = -R^{-1}B^{T}P \qquad (9)$$

$$H_c(0) = E\left(-\bar{A}\right)^{-1}B \qquad (10)$$

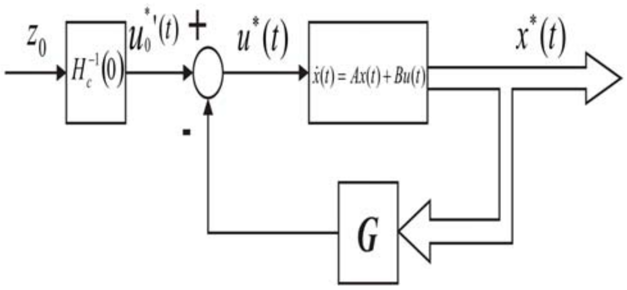


Fig. 7. Flexible System with Nonzero Set Point Tracking Controller Block Diagram

Both, linear regulator and nonzero set point tracking solution were obtained by solving Riccati's equations.

$$0 = Q - PBR^{-1}B^{T}P + PA + A^{T}P \qquad (11)$$

MATLAB® was used to solve Riccati's equations. Q and R are respectively state and input weighting matrixes. Several weighting matrixes were used in this research. They were $Q = [I]_{n\times n}$, $Q = 10[I]_{n\times n}$, $Q = 100[I]_{n\times n}$, $R = [0.01]$, $R = [1]$ and $R = [10]$.

For 2-DOF system, the solutions for linear regulator criterion were obtained by varying Q over a constant $R = [1]$ and also by varying R over a constant $Q = [I]_{n\times n}$, while the external force was applied to only one of the masses. The same processes were repeated for the other mass. The nonzero set point tracking criterion solutions were obtained in a similar way.

For 4-DOF systems, for both configurations, the solutions for linear regulator criterion were obtained by varying Q over a constant $R = [1]$, while the external force was applied to only one of the masses. The same processes were repeated for the other mass. The nonzero set point tracking criterion solutions were obtained in a similar way. For the systems with linear regulator criterion, the external force was considered as a disturbance.

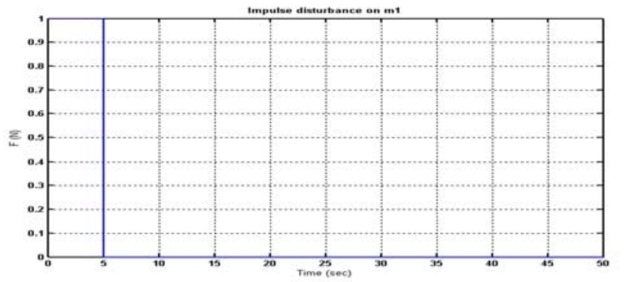


Fig. 8. Impulse Disturbance for Linear Regulator (F vs. time)

SIMULINK® was then used to simulate the linear time-invariant flexible systems dynamics, which had been equipped with controllers that were developed by using the optimal control theory. The following simulation blocks, respectively, were used to represent the 4-DOF linear time-invariant flexible systems, which had been equipped with the control systems that were developed by using linear regulator and nonzero set point tracking criterion. The set points for 2-DOF systems were 4 m, 6 m and 8 m. For the 4-DOF systems, the set points for the systems with the first configuration were 4 m, 6 m and 20 m. While systems with the second configuration had the following set points, 2.5 m, 4 m and 13 m.

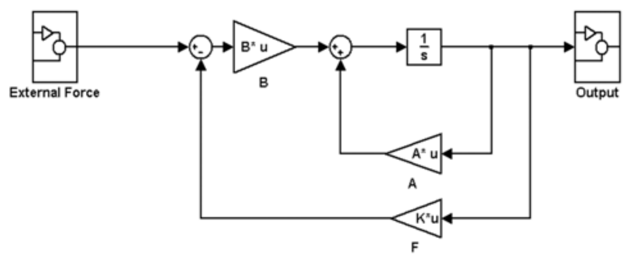


Fig. 9. Simulation Block of the 4-DOF Systems with Linear Regulator Controller

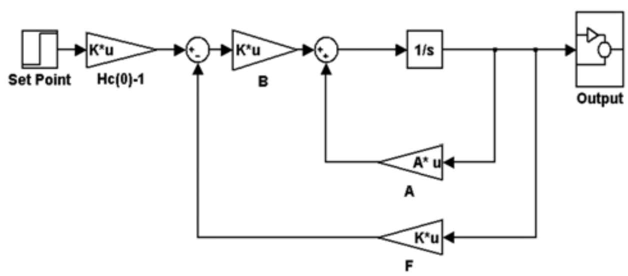


Fig. 10. Simulation Block of the 4-DOF Systems with Nonzero Set Point Tracking Controller

3 Results and Discussions

3.1 2-DOF Linear time-invariant flexible systems

3.1.1 Linear regulator

3.1.1.1 Variation of Q *over a constant* $R = [1]$

For 2-DOF systems with external force being applied onto the first mass, the following results were obtained.

	Optimal Gain (G)			
	x_1	x_2	v_1	v_2
$Q=[I]$	1.2191	0.1951	2.4241	1.77
$Q=10*[I]$	5.089	-0.6169	5.5096	3.9898
$Q=100*[I]$	18.351	-4.2089	13.1683	12.5312

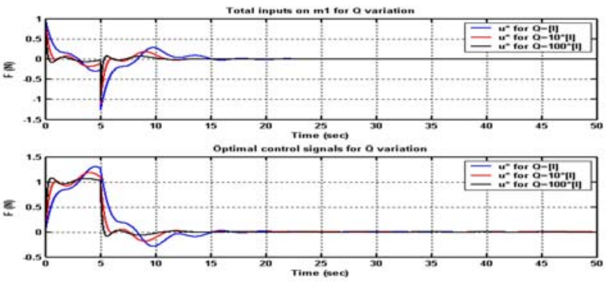


Fig. 11. Total inputs on m_1 and optimal control signal, for Q variations (F vs. time)

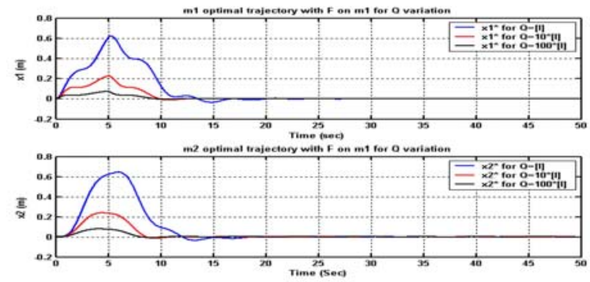


Fig. 12. m_1 and m_2 optimal trajectories with F on m_1 for Q variations (x vs. time)

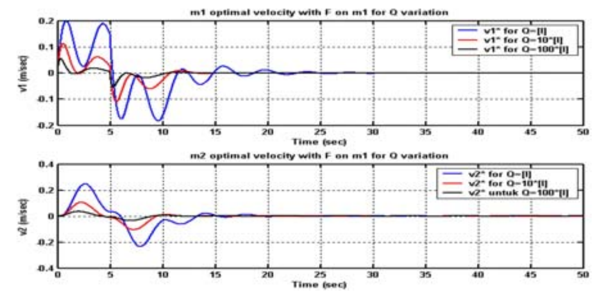


Fig. 13. m_1 and m_2 optimal velocities with F on m_1 for Q variations (v vs. time)

	Peak Response (m)		Settling Time (sec)	
	x_1	x_2	x_1	x_2
$Q=[I]$	0.626	0.645	20	18
$Q=10*[I]$	0.228	0.24	12.5	11
$Q=100*[I]$	0.074	0.079	15	13.5

Simulations showed that bigger Q caused the systems to reached their destinations faster and, consequently, bigger inputs were needed.

2-DOF systems with external force being applied onto the second mass gave similar simulation results.

3.1.1.2 Variation of R *over a constant* $Q = [I]_{n \times n}$

For 2-DOF systems with external force being applied onto the first mass, the following results were obtained.

	Optimal Gain (G)			
	x_1	x_2	v_1	v_2
$R=0.01$	18.351	-4.2089	13.1683	12.5312
$R=1$	1.2191	0.1951	2.4241	1.77
$R=10$	0.2943	0.1529	1.1302	1.0459

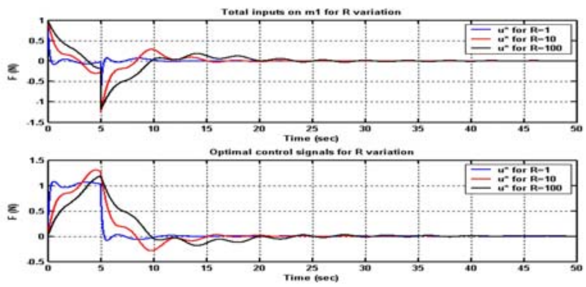


Fig. 14. Total inputs on m_1 and optimal control signal, for R variations (F vs. time)

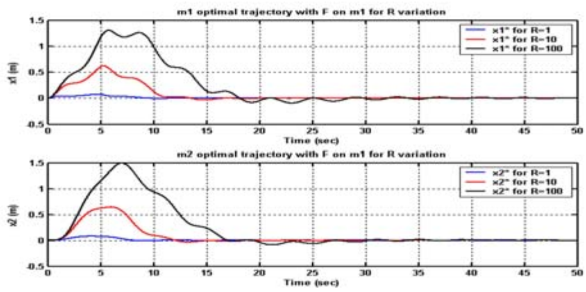


Fig. 15. m_1 and m_2 optimal trajectories with F on m_1 for R variations (x vs. time)

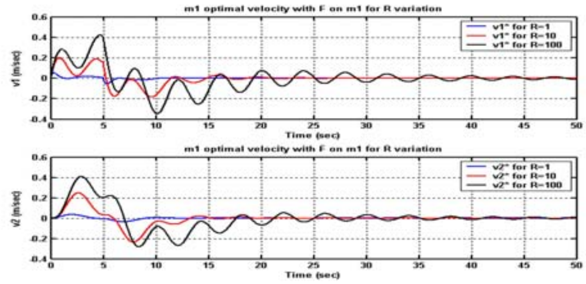


Fig. 16. m_1 and m_2 optimal velocities with F on m_1 for R variations (v vs. time)

	Peak Response (m)		Settling Time (sec)	
	x_1	x_2	x_1	x_2
R=0.01	0.074	0.08	9.5	8
R=1	0.626	0.645	20	18
R=10	1.305	1.5	32	30

Bigger R caused smaller inputs; consequently caused the systems to take more time to reached their destinations.

2-DOF systems with external force being applied onto the second mass gave similar simulation results.

3.1.2 Nonzero set point tracking

3.1.2.1 Variation of Q *over a constant* $R=[1]$

The simulations were done for several maneuver destinations, which were 4 m, 6 m and 8 m.

	Optimal Gain (*G*)				$H_c(0)^{-1}$
	x_1	x_2	v_1	v_2	z_0
Q=[I]	1.2191	0.1951	2.4241	1.77	1.4142
Q=10[I]*	5.089	-0.6169	5.5096	3.9898	4.4721
Q=100[I]*	18.351	-4.2089	13.1683	12.5312	14.1421

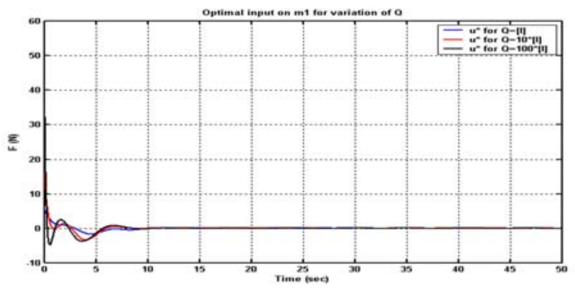


Fig. 17. Optimal input on m_1 for variations of Q (F vs. time)

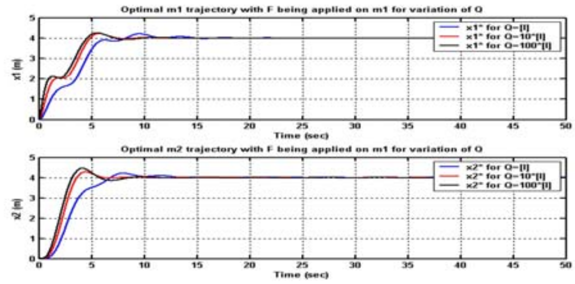


Fig. 18. m_1 and m_2 optimal trajectories with F being applied on m_1 for variation of Q (x vs. time)

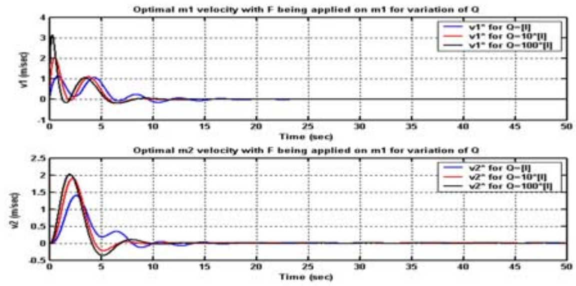


Fig. 19. m_1 and m_2 optimal velocities with F being applied on m_1 for variation of Q (v vs. time)

	Peak Response (m)		Settling Time (sec)	
	x_1	x_2	x_1	x_2
Q=[I]	4.206	4.215	15	13
Q=10[I]*	4.224	4.282	7.5	6
Q=100[I]*	4.253	4.463	10	8

Simulations showed that bigger Q caused the systems to reached their destinations faster and, consequently, bigger inputs were needed.

The simulation results for the other maneuver destinations were similar. These results were obtained from 2-DOF systems simulation with external force being applied onto the first mass. Systems with external force being applied onto the second mass gave similar simulation results.

3.1.2.2 Variation of R *over a constant* $Q=[I]_{n\times n}$

The simulations were done for several maneuver destinations, which were 4 m, 6 m and 8 m.

	Optimal Gain (*G*)				$H_c(0)^{-1}$
	x_1	x_2	v_1	v_2	z_0
R=0.01	18.351	-4.2089	13.1683	12.5312	14.1421
R=1	1.2191	0.1951	2.4241	1.77	1.4142
R=10	0.2943	0.1529	1.1302	1.0459	0.4474

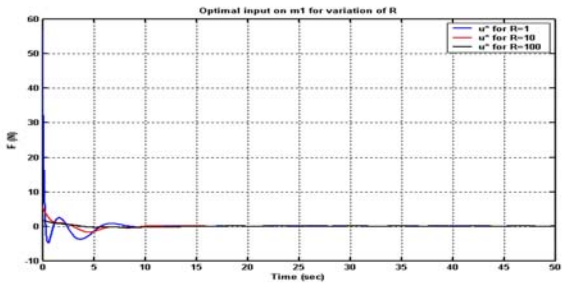


Fig. 20. Optimal input on m_1 for variation of R (F vs. time)

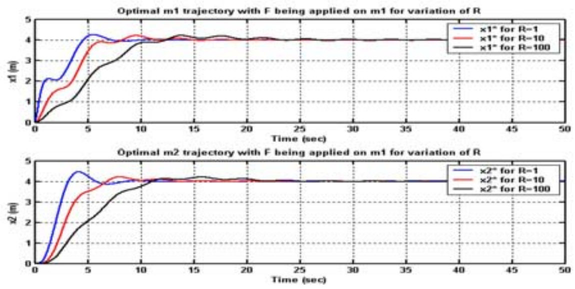


Fig. 21. m_1 and m_2 optimal trajectories with F being applied on m_1 for variation of R (x vs. time)

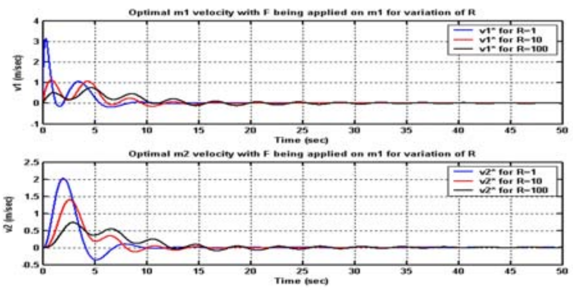


Fig. 22. m_1 and m_2 optimal velocities with F being applied on m_1 for variation of R (v vs. time)

	Peak Response (m)		Settling Time (sec)	
	x_1	x_2	x_1	x_2
R=0.01	4.253	4.463	10	8
R=1	4.206	4.215	15	13
R=10	4.211	4.206	25	21

Bigger R caused smaller inputs; consequently caused the systems to take more time to reached their destinations.

The simulation results for the other maneuver destinations were similar. These results were obtained from 2-DOF systems simulation with external force being applied onto the first mass. Systems with external force being applied onto the second mass gave similar simulation results.

3.2 4-DOF Linear time-invariant flexible systems

3.2.1 Linear regulator

The following results were obtained from 4-DOF systems, the first configuration, simulations where the external force was applied only to the first mass.

	Optimal Gain (*G*)			
	x_1	x_2	x_3	x_4
Q=[I]	2.5067	-1.3278	0.8703	-0.0492
Q=10[I]*	9.1646	-4.7401	1.4541	0.4459
Q=100[I]*	28.724	-12.378	0.5025	3.152
	Optimal Gain (*G*)			
	v_1	v_2	v_3	v_4
Q=[I]	2.4522	1.3316	1.3415	1.1709
Q=10[I]*	5.3225	5.3461	3.1512	3.662
Q=100[I]*	12.548	19.0956	9.3155	11.2876

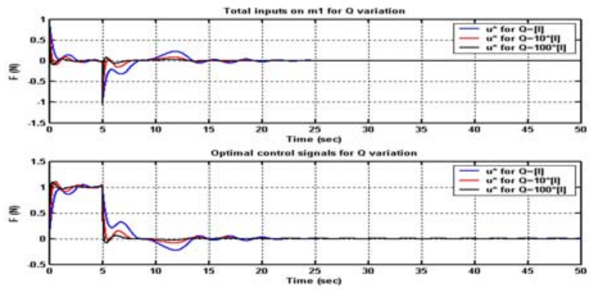


Fig. 23. Total input on m_1 and optimal control signals, for variations of Q (F vs. time)

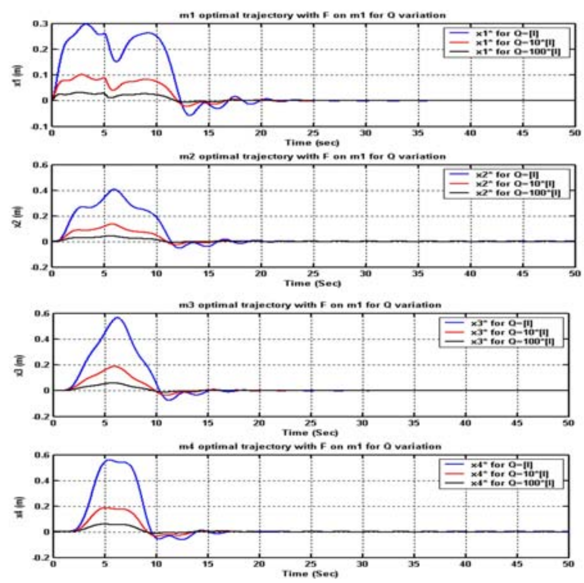


Fig. 24. m_1, m_2, m_3 and m_4 optimal trajectories with F on m_1 for Q variations (x vs. time)

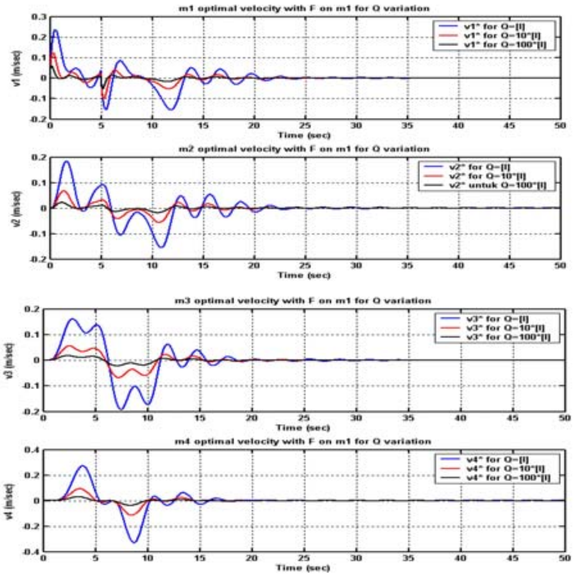


Fig. 25. m_1, m_2, m_3 and m_4 optimal velocities with F on m_1 for Q variations (v vs. time)

	Peak Response (m)			
	x_1	x_2	x_3	x_4
$Q=[I]$	0.3	0.407	0.565	0.56
$Q=10^*[I]$	0.1	0.136	0.187	0.186
$Q=100^*[I]$	0.032	0.043	0.06	0.06
	Settling Time (sec)			
	x_1	x_2	x_3	x_4
$Q=[I]$	25	23	19	18
$Q=10^*[I]$	22	20	18	17
$Q=100^*[I]$	21	20	18	17

Simulations showed that bigger Q caused the systems to reached their destinations faster and, consequently, bigger inputs were needed.

The simulations were also done for the systems with external force given to the second, third and forth mass, the results were similar. The 4-DOF systems with the second configuration also had similar simulation results.

3.2.2 Nonzero set point tracking

The following results were obtained from 4-DOF systems, the second configuration, simulations where the external force was applied only to the first mass. The maneuver destinations were 2.5 m, 4 m and 13 m.

	Optimal Gain (G)				$H_c(0)^{-1}$
	x_1	x_2	x_3	x_4	z_0
$Q=[I]$	2.5322	-1.4685	1.0377	-0.1014	2
$Q=10^*[I]$	10.103	-6.1804	2.2308	0.1716	6.3246
$Q=100^*[I]$	32.657	-17.344	1.8304	2.856	20
	Optimal Gain (G)				
	v_1	v_2	v_3	v_4	
$Q=[I]$	2.4626	1.0771	1.2458	1.0275	
$Q=10^*[I]$	5.4959	4.4315	2.6004	3.1787	
$Q=100^*[I]$	12.857	16.7328	7.1824	9.8663	

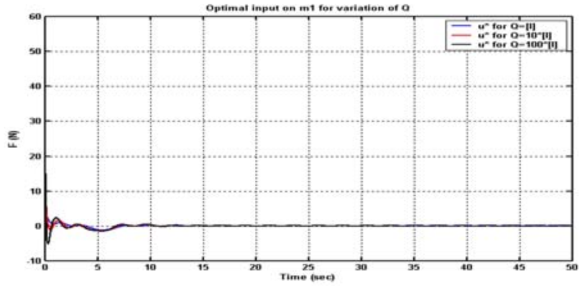


Fig. 26. Optimal input on m_1 for variations of Q (F vs. time)

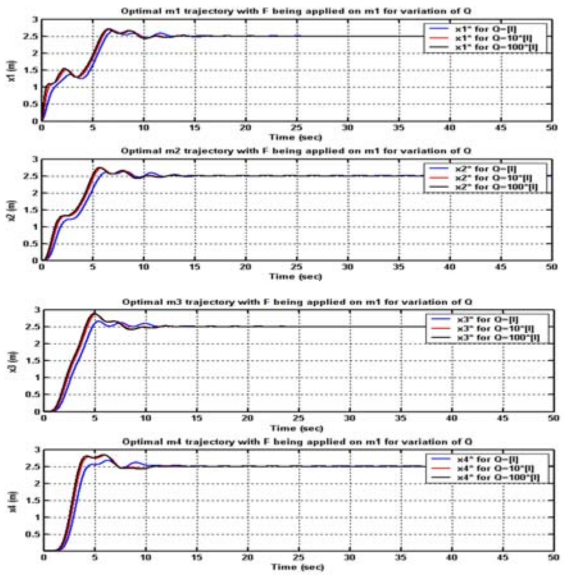
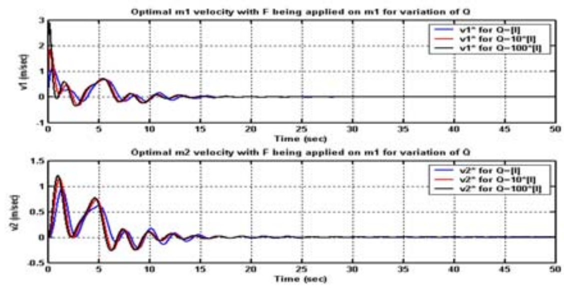


Fig. 27. m_1, m_2, m_3 and m_4 optimal trajectories with F on m_1 for Q variations (x vs. time)



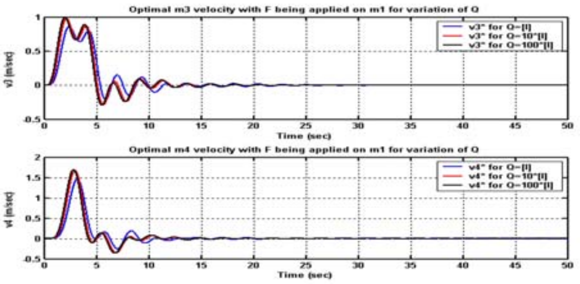


Fig. 28. . m_1, m_2, m_3 and m_4 optimal velocities with F on m_1 for Q variations (v vs. time)

	Peak Response (m)			
	x_1	x_2	x_3	x_4
$Q=[I]$	2.662	2.64	2.66	2.677
$Q=10*[I]$	2.711	2.735	2.864	2.827
$Q=100*[I]$	2.712	2.75	2.886	2.846
	Settling Time (sec)			
	x_1	x_2	x_3	x_4
$Q=[I]$	16	14	13	12
$Q=10*[I]$	13	12	11.5	11
$Q=100*[I]$	13	12	11.5	11

Simulations showed that bigger Q caused the systems to reached their destinations faster and, consequently, bigger inputs were needed.

The simulation for the other destinations gave similar results. Simulations were also done for the systems with external force given to the second, third and forth mass, the results were similar. The 4-DOF systems with the first configuration also had similar simulation results. The maneuver destinations for the systems with the first configuration were 4 m, 6 m and 20 m.

4 Conclusions

The application of control systems that were developed by using linear regulator criterion on linear flexible systems can reduce vibration residues, which are caused by force vibration, well.

Linear flexible systems can do rest-to-rest maneuver with several destinations; the destinations are used as the set points, by using the controllers that were developed by using nonzero set point tracking criterion. Vibration residues, which are caused by the rest-to-rest maneuvers, can also be controlled by using control systems that were developed by using nonzero set point tracking criterion.

Variations on states weighting matrix Q will be varying states trajectories optimization. The bigger the value given to Q the more optimal the states trajectories will; lesser time needed to reach their destinations. Variations on inputs weighting matrix R will be varying input signal, which will drive the systems to follow optimal trajectories, optimization. The bigger the value given to R, the more optimal will the input signals be; smaller inputs needed by the systems to reach their destinations.

The best control system that was obtained from this research was acquired by using $Q = 10*[I]_{n\times n}$ and $R = 1$. Inputs and trajectories that are achieved by controllers, which are developed by using this combination of weighting matrixes, are the most optimal ones; compare to inputs and trajectories achieved by using other controllers developed in this research.

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6 Nomenclatures

6.1 Notations

A	System dynamics Matrix
$\overline{A}$	Closed loop system dynamics matrix
B	Inputs matrix
E	Controlled variables matrix
F	External force
$\{F\}_i$	Force vector on *i*-mass
G	Feed back gain
$H_c(0)$	$u_0^{*}{}'(t)$ to z_0 transfer function
J	Performance Indexes
$[K]$	Stiffness matrix
k	Stiffness coefficient
$[M]$	Mass matrix
m	Mass
P	Riccati's equation solution
Q	State variables weighting matrix
R	Input weighting matrix
S	Controllability matrix
t	Time variable
$u(t)$	Input vector
$u^*(t)$	Optimal input vector
v	Velocity
$\{X\}$	Displacements vector
$\{\ddot{X}\}$	Accelerations vector
$x(t)$	State variable vector
$x^*(t)$	Optimal trajectories vector
x	State variable or displacement
z_0	Set points

6.2 Greeks

λ	Eigen value
ω	Angular frequency
ζ	Damping characteristic

6.3 Superscript

-1	Inverse of a matrix
*	Optimal or minimum
‘	Shifted variables
T	Transpose of a matrix

6.4 Subscript

0	Initial conditions
f	Final conditions
i	Define onto which mass the external force was being applied
n	Number of state variables