NEW POSSIBILITIES OF SIGNIFICANT IMPROVEMENT OF AEROSPACE LAUNCHER EFFICIENCY BY RIGOROUS OPTIMIZATION OF ATMOSPHERIC FLIGHT

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Abstract

The possibility of a significant increase of a payload mass injected into an orbit using the qualitatively new type of optimal atmospheric ascent trajectories is investigated. The Pontryagin maximum principle is rigorously applied to the optimization taking into account aerodynamic load limits and dependence of the vehicle structure mass on the aerodynamic loading conditions.

Several types of extremals satisfying necessary conditions of the maximum principle but featuring qualitatively different control law structures are demonstrated. The qualitative distinction is caused mainly by the effect of the aerodynamic lift even if it is small as compared with the weight. It is shown the new optimal ascent trajectories may add 15÷20% to the payload mass inserted into orbit.

Nomenclature

Symbols:

C_D	aerodynamic drag coefficient
C_{D0}	zero-lift drag coefficient
C_L	aerodynamic lift coefficient
F_0	reference cross section area
F_w	outer wing area
h	altitude
L/D	lift-to-drag ratio
$(L/D)_{max}$	$= \max\limits_{\alpha}(L/D)$
M	Mach number
m	vehicle mass
n	thrust-to-weight ratio
q	dynamic pressure
$\mathbf{T}$	thrust vector
t	time
$\mathbf{V}$	velocity vector
α	angle of attack
γ	path angle
θ	pitch angle

Subscripts:

$(\,)_f$	at the final point
$(\,)_i$	at the initial point
$(\,)_{max}$	maximum value
$(\,)_{min}$	minimum value
$(\,)_{opt}$	optimal value

Superscripts:

$(\,)^*$	at the bifurcation point

1 Introduction

The maximization problem of payload mass injection into an orbit has a long history. It has well-known theoretical approximate solutions, which are employed in practice. Qualitatively, they are characterized with respect to the role of aerodynamic forces as follows. In the theory: aerodynamic forces are small and act shortly after launcher start and therefore they can be neglected [1, 2]. In practice: the nominal control program of a launcher has to minimize aerodynamic drag losses of the characteristic velocity and therefore realizes the so-called gravitational turn (with zero angle of attack) in dense atmospheric layers; at the later stages of flight the control program uses the quasioptimal solution with the pitch angle being linear in time. We refer to such solutions as “traditional” ones.

At the same time, in view of aerodynamic forces the application of the rigorous optimization on the basis of the Pontryagin maximum principle made it possible to reveal existence of two qualitatively different types of optimal solutions [3]. The first type corresponds to the traditional solutions mentioned above and provides the global optimum for launchers with a low lift-to-drag ratio. Another, the qualitatively new type of optimal solutions, is realized for launchers with improved lifting capabilities.

In this paper the emphasis is on the 2nd type of the optimal solutions, which is conditionally named as "*Aerodynamic*". The fundamental difference between new and traditional control laws becomes apparent in the fact that the functions of sensitivity of the maximum payload mass to the launcher parameters can differ in orders of value and signs.

The non-traditional ("*Aerodynamic*") optimal pitch program has an explicit tendency to oscillations into the atmosphere. It is worth noting that existence of the optimal oscillatory solution for aerospace vehicles was well studied for the functional determined by the flight range. In our case, the functional is the injected payload mass, and the range is not included in any boundary condition. The new type of optimal trajectories is caused by the influence of the aerodynamic lift even if it is small compared to the weight [3]. This is why these trajectories can be found under the rigorous optimization using rather complete models of the atmospheric motion only in comparison with classical consideration [1, 2].

2 Trajectory optimization

The launcher mass centre motion is considered in the coordinate system fixed to the start point:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x},\mathbf{u},t)\,, \mathbf{f} = \{\mathbf{V},\, \mathbf{T}/m + \mathbf{A}/m + \mathbf{g} + \boldsymbol{\Omega},\, -\mu\}^{\mathrm{T}}\,, \quad (1)$$

where $\mathbf{x}=\{\mathbf{r},\, \mathbf{V},\, m\}^{\mathrm{T}}$ is the state vector, $\mathbf{r}$ is the radius-vector, $\mathbf{u}$ is the control vector, $\mathbf{A}$ is the vector of aerodynamic forces, $\mathbf{g}$ is the gravitational acceleration vector, $\boldsymbol{\Omega}$ is the acceleration vector due to coordinate system noninertiality, μ is the mass flow rate.

The vector of aerodynamic forces can be written in the form [3]:

$$\mathbf{A} = qF_0\Big(C_L^{\alpha}\mathbf{e}_{\tau} - \big(D_0 + \big(C_L^{\alpha} + D_{\alpha}\big)(\mathbf{e}_{\tau},\mathbf{e}_{\mathrm{v}})\big)\mathbf{e}_{\mathrm{v}}\Big), \quad (2)$$

where $\mathbf{e}_{\tau}$ is the unit vector directed along the vehicle's longitudinal axis, $\mathbf{e}_{\mathrm{v}}$ is the velocity unit vector. The following form of aerodynamic coefficients is used [3]:

$$C_L = C_L^{\alpha}\,\sin\alpha,\quad C_D = D_0 + D_{\alpha}\cos\alpha\,, \quad (3)$$

that is in accordance with the square aerodynamic polar at a small angle of attack.

The thrust is assumed to be directed along the launcher longitudinal axis determined by the unit vector $\mathbf{e}_{\tau}$:

$$\mathbf{T} = T\mathbf{e}_{\tau}\,,\; T_{\min} \le T \le T_{\max}. \quad (4)$$

At the initial moment t_i the vehicle position and velocity value are fixed, but the velocity direction can be free:

$$\mathbf{r}(t_i)=\mathbf{r}_i,\; \mathrm{v}(t_i)=\mathrm{v}_i,\; m(t_i)=m_i. \quad (5)$$

The task is to find the admissible control

$$\mathbf{u} = \{\mathbf{e}_{\tau}, T\}^{\mathrm{T}} \in \mathbf{U} \quad (6)$$

to transfer the vehicle from the initial point to a specified Earth orbit with the minimum mass consumption that corresponds to maximization of the final vehicle mass:

$$\Phi \equiv m_f \Rightarrow \max_{\mathbf{u}\in\mathbf{U}}\,. \quad (7)$$

Solving problems with the Pontryagin maximum principle [4], the optimal control is found from the condition:

$$\{\mathbf{e}_{\tau}, T\}_{opt} = \operatorname{argmax}\mathcal{H}\,, \quad (8)$$

where

$$\mathcal{H} = \boldsymbol{\Psi}^{\mathrm{T}}\mathbf{f} \quad (9)$$

is the Hamiltonian of the system (1). The adjoint vector $\boldsymbol{\Psi}$ satisfies the equation

$$\dot{\boldsymbol{\Psi}} = -\left(\frac{\partial\mathcal{H}}{\partial\mathbf{x}}\right)^{\mathrm{T}} \quad (10)$$

and the transversality conditions. Thus, the reference optimization problem reduces to a multipoint boundary-value problem (BVP) for equations (1), (10).

The numerical solution is found using the ASTER package [5]. It realizes the practically regular procedure of the BVP solution due to application of the modified Newton method, parameter continuation method and local extremal selection [3].

According to the adjoint system property the coefficients $\frac{\partial\Phi}{\partial\mathbf{p}}$ of a sensitivity of the functional Φ to variations of a vector parameter $\boldsymbol{p}$ can be obtained with a high accuracy simultaneously with the optimal trajectory integration avoiding noticeable additional computations:

$$\frac{\partial \Phi}{\partial \mathbf{p}} = \int_{t_i}^{t_f} \frac{\partial \mathscr{H}\left(\mathbf{x}_{opt}, \mathbf{u}_{opt}, \mathbf{p}\right)}{\partial \mathbf{p}} dt \,. \qquad (11)$$

where $\mathbf{x}_{opt}$ and $\mathbf{u}_{opt}$ are taken at the nominal optimal trajectory.

3 Extremal types

The strategy of the optimal trajectory control is determined by the correlation of the thrust **(T)**, aerodynamic **(A)** and gravitational **(G)** forces (Fig. 1). If the thrust dominates, the optimal control law is qualitatively in conformity with the traditional one [1, 2] obtained for uniform gravitational field under an assumption of the negligibility of aerodynamic forces. However, if $|\mathbf{T}| >> |\mathbf{A}|$ и $|\mathbf{G}| >> |\mathbf{A}|$, but

$$|\mathbf{T+G}| \sim |\mathbf{A}|, \qquad (12)$$

the effect of aerodynamic forces can change the structure of the optimal control law and generate multiplicity of extremals.

The classification of possible extremals is given in [3]:

***B*-type (“*Ballistic*”) extremals:**

- the optimal pitch angle programs are quasi-linear to correspond the well-known “traditional” solutions [1, 2];
- the aerodynamic forces influence weakly on the optimal control law structure;
- the optimal start is nearly vertical;
- the atmosphere is "perceived" only as a medium with some drag;
- typical to be used in guidance algorithms for current space ballistic launchers;
- provide the global optimum at low lift-to-drag ratios.

A-type (“Aerodynamic”) extremals:

- the optimal pitch angle program during the atmospheric flight has a pronounced oscillatory nature;
- an inclined and quasihorizontal start is optimal;
- the atmosphere is mainly perceived as a medium that produces a lift; the optimal trajectories pass into regions with higher dynamic pressures as compared with the *B*-type extremals;
- provide the global optimum at high lift-to-drag ratios.

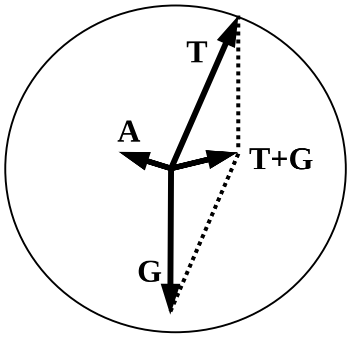

Fig. 1. The scheme of forces acting on a launcher.

M-type (“interMediate”) extremals:

– do not provide a global optimum.

The availability of two basic types of global extremals can be explained by following physical reasons.

To minimize aerodynamic losses of the characteristic velocity, it is necessary to reduce the angle of attack and to increase the initial path angle. On the contrary, to minimize gravitational losses, it is necessary to use the aerodynamic lift and to decrease the initial path angle. Thus, to minimize the sum of aerodynamic and gravitational losses, the optimal control has to implement a compromise between two indicated trends. The result depends on the correlation of aerodynamic lift and drag of the launcher. A measure of the correlation is the maximum lift-to-drag ratio $(L/D)_{\max}$. Therefore $(L/D)_{\max}$ is possible to be considered as the important generalized parameter determining the optimal control structure.

The condition (12) is fulfilled, as a rule, on the initial flight segment. For current ballistic launchers with a ground start the initial trust-to-weight ratio is moderately greater than 1. The initial thrust of air launch transportation systems can be less than weight.

Thus, the qualitative effect of aerodynamic forces on the optimal control structure is expectable both for advanced aerospace vehicles, and for typical "rocket" layouts of launchers.

Let us analyze physical reasons of mass efficiency increase of a launcher at the optimal use of the aerodynamic lift in details.

Firstly, we will point out typical doubts:

1. The nonzero angle of attack results in growth of the aerodynamic drag and, hence, aerodynamic losses of the characteristic velocity.
2. The aerodynamic lift is perpendicular to the velocity vector and does not make the work. Thus, ostensibly, it does not influence an energy balance at the ascent.

The paradox is settled as follows:

1. The increase of the aerodynamic drag at a small angle of attack is a value of a higher order as compared to the lift.
2. The lift allows to unload the weight and, hence, to drop gravitational losses of the characteristic velocity that dominate the aerodynamic losses.
3. The nonzero angles of attack at the ascent corresponds to the classical optimal control law [1, 2] where if the aerodynamic lift is ignored. In fact, in a general case, the optimal thrust vector in a gravitational field is not collinear with velocity.

Later these arguments back up on typical examples (specific data are defined in Appendix A).

In Fig. 2 the optimal angle of attack program for a launcher with zero effective lift are shown. The optimal program even in this "ideal" case is not trivial as for gravity turn.

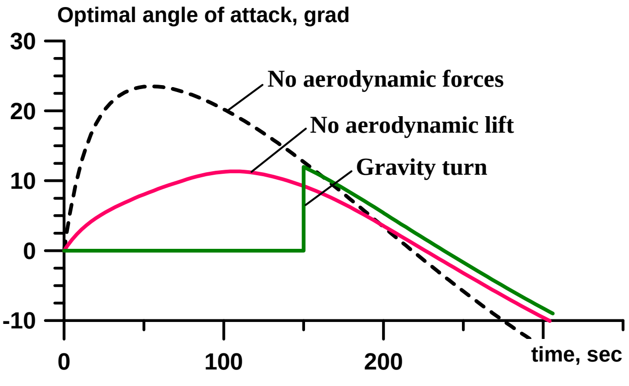

Fig. 2. The optimal time-programs of angle of attack of the launcher with zero effective lift.

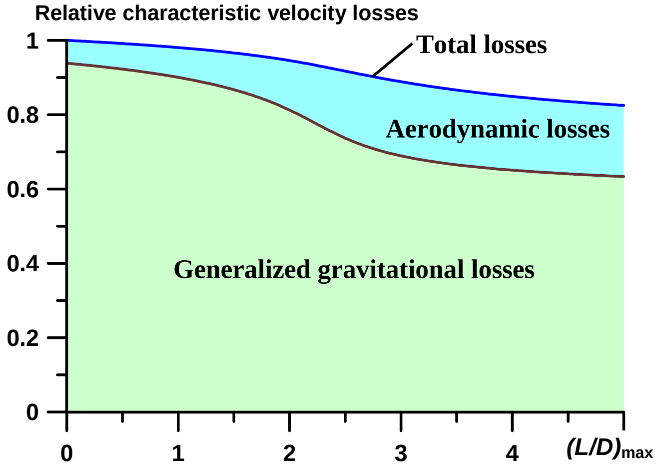

Fig. 3. Redistribution of characteristic velocity losses due to the maximum lift-to-drag ratio variation, $n_i = 1.4$.

The necessary reserve of the characteristic velocity V_Σ is presentable as the sum of the given orbital velocity V_{orb}, the generalized gravitational losses ΔV_g and losses ΔV_a on the aerodynamic drag:

$$V_\Sigma = V_{orb} + \Delta V_g + \Delta V_a, \text{ where} \qquad (13)$$

$$V_\Sigma = \int_{t_i}^{t_f} \frac{T}{m} dt, \qquad \Delta V_a = \int_{t_i}^{t_f} \frac{C_D q F_0}{m} dt,$$

$$\Delta V_g = \int_{t_i}^{t_f} \frac{T}{m}(1-\cos\alpha)\,dt + \int_{t_i}^{t_f} g \sin\gamma\, dt.$$

A typical diagram of the optimal redistribution of the characteristic velocity losses due to the maximum lift-to-drag ratio $(L/D)_{\max}$ variation of a launcher with the initial thrust-to-weight ratio $n_i = 1.4$ is shown in Fig. 3.

The conversion of the optimal pitch program with $(L/D)_{\max}$ growth is shown in Fig. 4.

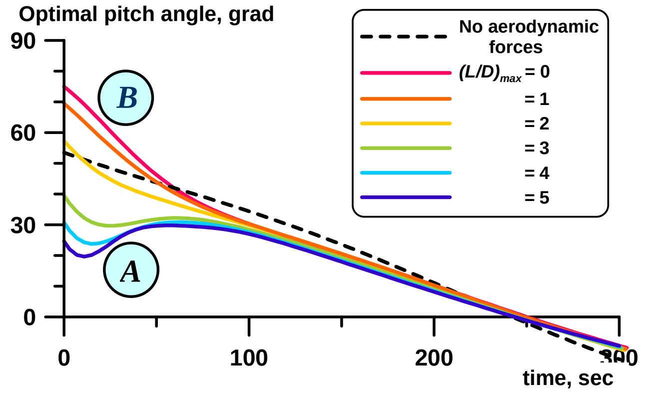

Fig. 4. Optimal time-programs of the pitch angle of the launcher with several lift-to-drag ratios, $n_i = 1.4$.

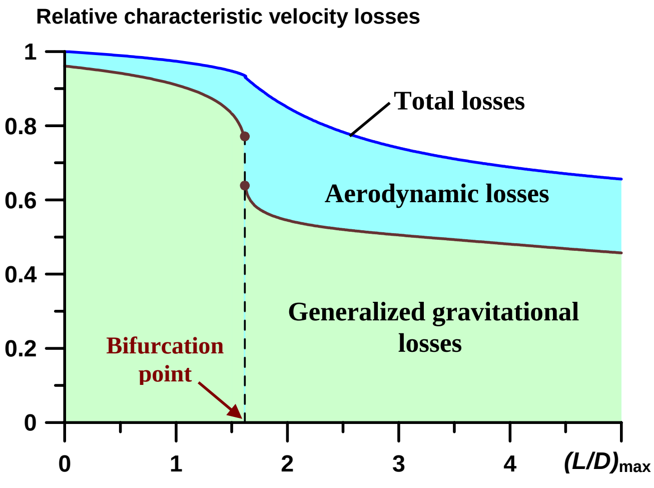

Fig. 5. Redistribution of characteristic velocity losses due to the maximum lift-to-drag ratio variation, $ni = 1.04$.

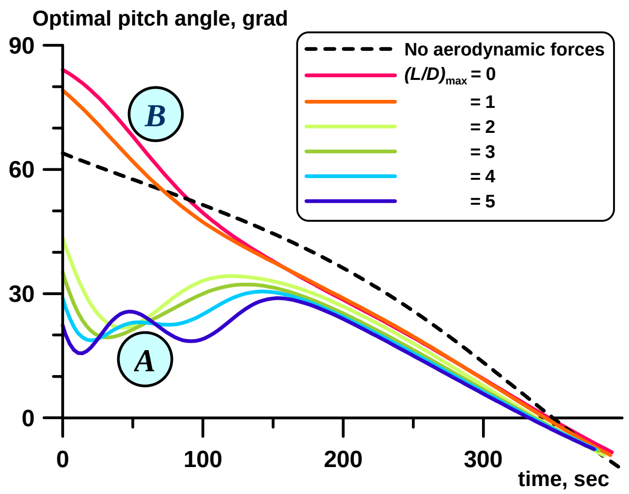

Fig. 6. Optimal time-programs of the pitch angle of the launcher with several lift-to-drag ratios, $n_i = 1.04$.

For comparison, the "ideal" program which would be optimal in the Newtonian gravitational field ignoring aerodynamic forces is shown here by a dot line.

4 Bifurcation of extremals

The presence of two qualitatively different extremal types causes also a probability of the fact that the change of the optimum control law at a variation of launcher parameter will have bifurcation character. At the problem parameters described in Appendix A, such change of optimal solutions at $(L/D)_{\max}$ variation takes place at a small initial thrust-to-weight ratio.

The relations of the characteristic velocity losses to the maximum lift-to-drag ratio of the launcher and $n_i = 1.04$ are shown in Fig. 5.

As soon as $(L/D)_{\max}$ increases over the bifurcational value the change of the optimal control structure from ***B***-type on ***A***-type (Fig. 6) happens ,that results in a discontinuous change of aerodynamic and gravitational losses (Fig. 5).

Fig. 7 shows the typical dependence of the optimal initial path angle $\gamma_{i\,\mathrm{opt}}$ and relative injection mass $\bar{m}_f = m_f((L/D)_{\max})/m_f(0)$ on the maximum lift-to-drag ratio of the launcher.

The Pontryagin maximum principle is based on necessary conditions of optimality. It is visible from Fig. 7, that in the neighborhood of the bifurcation point in accordance with classification [3] three local extremals: ***A*** , ***B*** and ***M-types*** (the last does not provide the global optimum) exist simultaneously. The transition from one to another extremal type at the numerical solution of the problem are performed by the parameter continuation method [3].

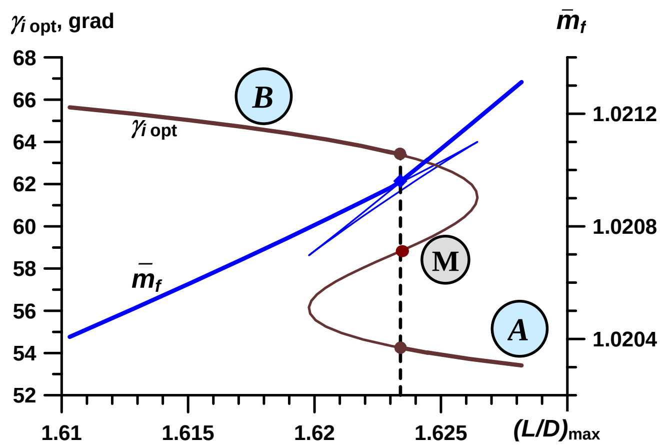

Fig. 7. Dependencies of the optimal initial path angle $\gamma_{i\,\mathrm{opt}}$ and relative injected mass $\bar{m}_f$ on $(L/D)_{\max}$ of the launcher in the vicinity of the bifurcation point.

5 Aerodynamic shape influence on the optimal ascent trajectory

In the previous sections we supposed that the dependence $C_D(\alpha)$ is fixed, $(L/D)_{\max}$ is constant with flight regime changes, and the influence of

$(L/D)_{\max}$ variations on other launcher parameters are not taken into consideration.

As indicated above, the $(L/D)_{\max}$ is one of main parameters of the aerodynamic configuration that govern the structure of the optimal control law and trajectory, but variations of other parameters, for example, C_{D0}, can have some effect on the functional and optimal control law [3]. In view of the fact that a change in the aerodynamic configuration leads to a variation of the whole set of aerodynamic characteristics, the analysis of the dependence of optimal solutions on geometric parameters of the launcher configuration is of a practical interest.

Several types of aerodynamic configurations covering most-used vehicle configurations are considered, including the follows:

1. A conical body with an elliptic cross section.
2. A cylindrical body with a delta wing.

For all the launcher types (the aerodynamic data are presented in [6]), the influence of its geometric configuration modifications, ensuring enhanced lifting capabilities, on the maximum injected mass, the optimal control laws and maximum aerodynamic loads is analyzed..

In the first case the vehicle shapes are modified by passing from circular cross sections into elliptic ones with the volume of the body remaining the same. The ratio of the launcher width a to the launcher height b is hereinafter referred to as the "contraction parameter" a/b.

The effect of the launcher cross section and the outboard wing area F_w on the injected mass and optimal injection trajectories is analyzed for the nominal conditions defined in Appendix B.

According to [6], the variations of the cross section geometry and outboard wing area cause $(L/D)_{\max}$ and C_{D0} changes of the same sign. Thus, a change in the injected mass with variations of the aerodynamic configuration depends on a compromise between two opposite trends: the increase in aerodynamic drag and growth of launcher lifting capabilities. The investigations show that the effect of variations of the aerodynamic configuration is significantly different for optimal trajectories of A-type and B-type.

Fig. 8 presents the relative injected mass $\bar{m}_f = m_f(a/b)/m_f(1)$ for different values of the contraction parameter of the conical vehicle and for two types of trajectory control programs. It is seen that the parametric analysis based on approximate “traditional” control laws qualitatively distorts the objective parameter dependence of the launcher weight efficiency.

To the left of the bifurcation point, increased contraction parameter results in reduced injected mass. The optimal trajectories for these configurations correspond to traditional launcher injection schemes (the B-type) and almost do not use the lifting capabilities. Therefore, the increase in the contraction parameter followed by the increased aerodynamic drag results in reduced mass m_f.

The increase of the contraction parameter in excess of the bifurcation value $a/b \approx 1.3$ leads to the growth of the injected mass. As noted above, the qualitative change in the contraction parameter dependence of the maximum injected mass is caused by change of the optimal control law structure. To the right of the bifurcation point, the injected mass is the maximum when the A-type control law is used.

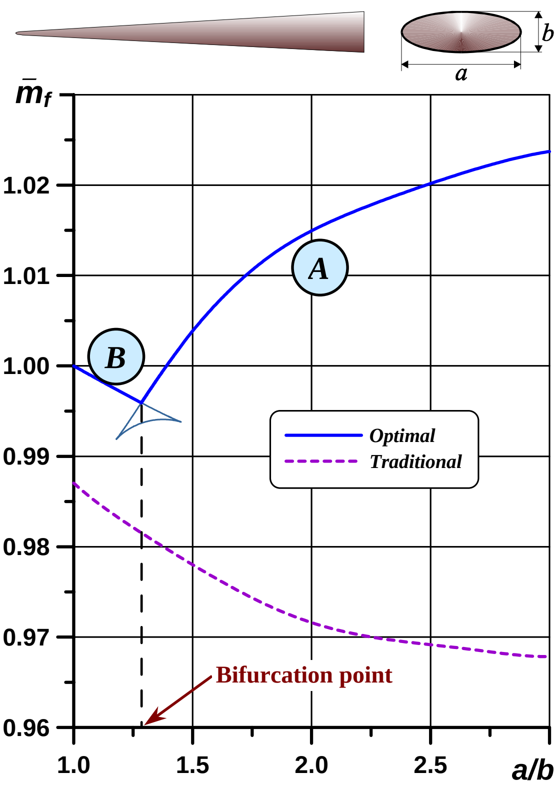

Fig.8. Relative injected mass versus contraction parameter of the conical launcher. The bold line selects the global optimum.

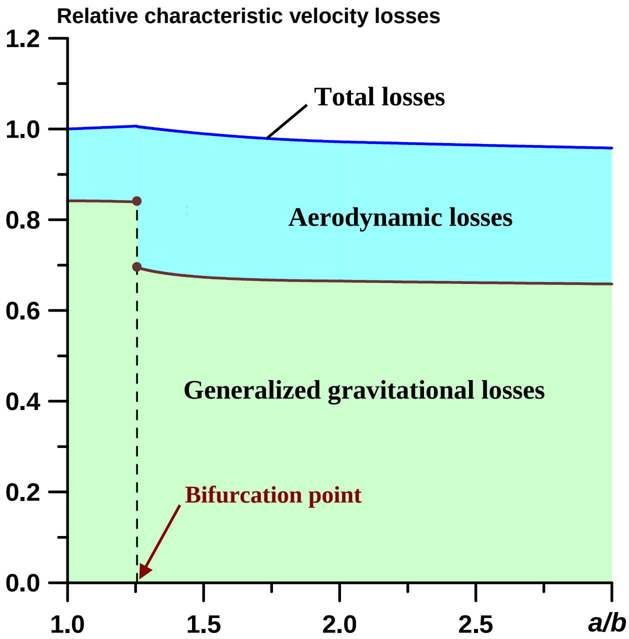

Fig. 9. Redistribution of characteristic velocity losses due to the contraction parameter variation of the conical launcher.

The comparison of the functional values on different-type trajectories shows that when optimal control laws are used the gain in the injected mass on *B*-type trajectories is 1.3% as compared with traditional control laws. When the lifting capabilities are used (on the optimal trajectories of type *A*), this gain is four times greater already at $a/b \approx 2.5$.

From Fig. 9 follows that increase of the injected mass at *a/b* higher than bifurcational value is correlated to the abrupt fall of gravitational losses of the characteristic velocity.

Changes in the relative injected mass $\bar{m}_f = m_f(\bar{F}_w)/m_f(0)$ with variations of the relative outboard wing area $\bar{F}_w = F_w/F_0$ (Fig. 10) have the same peculiarities as with variations of the contraction parameter. There exists a bifurcation value of the parameter $\bar{F}_w^*$, which separates the optimality regions for extremals of types *A* and *B*. At $\bar{F}_w < \bar{F}_w^*$ the *B*-type extremals is globally optimal, while at $\bar{F}_w > \bar{F}_w^*$ the *A*-type extremals are of this sort because they better use the lifting capabilities. It must be emphasized that in this case qualitative changes in the dependence of the maximum injected mass on the outboard wing area are demonstrated already at very small outboard wing panels which area is only several percent of the mid-section area.

Note especially the fundamental difference in sensitivity coefficients $\partial \bar{m}_f/\partial(a/b)$ characterizing the effect of the contraction parameter on the injected mass for optimal and traditional control laws. It is widely believed that approximate estimates of the injected mass are sufficient at the initial design stages when optimal variations of the configuration parameters are often based on the analysis of only partial derivatives, i.e., sensitivity coefficients. In this case, the hypothesis of their independence from the parameter to be varied is implicitly accepted. However, it is seen in Fig. 11 that the real dependence of sensitivity coefficients on the parameter, which takes account of the optimal use of lifting capabilities, is not only inquasiconstant, as opposed to traditional control laws, but experiences a bifurcation change. During the jump, not only the derivative magnitude

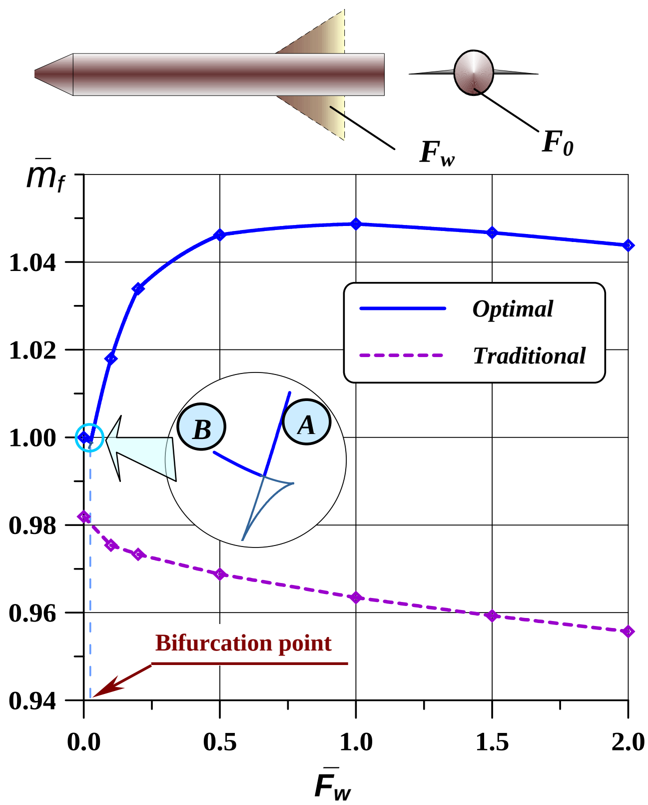

Fig. 10. The relative injected mass versus the relative outboard wing area for the winged cylindrical launcher. The bold line selects the global optimum.

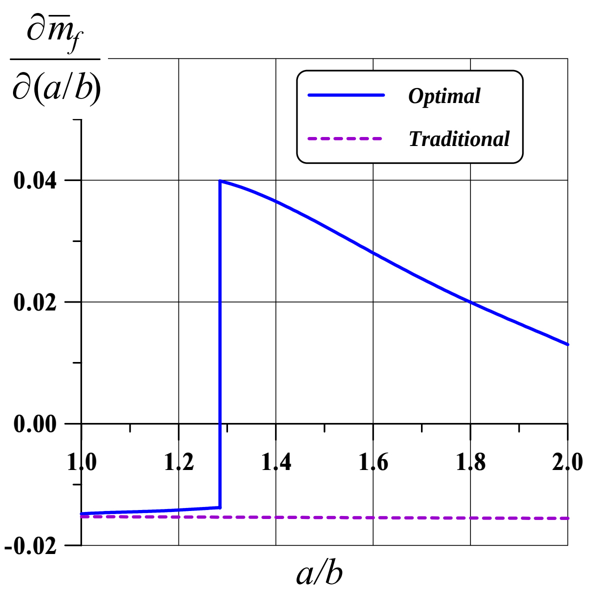

Fig. 11. The sensitivity coefficient versus the contraction parameter of the conical launcher.

changes (sometimes in orders) but also its sign (Fig. 11).

It is important to stress that in this case the bifurcation behavior is characteristic for the injected mass sensitivity coefficients not only to variations of the contraction parameter but also to almost all other configuration parameters (for example, the initial thrust-to-weight ratio, specific load on mid-section etc.). It follows from Fig. 11 that if traditional approximate control laws were used in determining the influence of the parameters on the functional, the derivative $\partial \bar{m}_f/\partial(a/b)$ would be essentially constant.

Compare now the optimal contraction parameter values for optimal and traditional control laws. Fig. 8 shows that the approximate (traditional) approach gives the only "optimal" configuration solution: $(a/b)_{\rm opt} = 1$, i.e., the circle is the best cross section shape of a conical body of the launcher. When the strict optimization procedure taking account of structural changes in optimal control laws is used, it is obtained the ellipse with a great contraction parameter is sufficiently better in the functional than the circle. Thus, the approximate approach to constructing the trajectory control laws for such launchers can violate the optimal concept of the vehicle under design. Investigations of diverse aerodynamic configurations reveal that complex analysis of the influence of vehicle parameters on the vehicle effectiveness is of great importance in designing due regard for significantly nonlinear dependence of optimal solutions on launcher parameters.

6 Multidisciplinary optimization of launcher parameters

As it has been shown above, the rigorous approach to the trajectory optimization leads to the qualitatively new optimal solution. The new optimal trajectories demand, in general, the according modification of launcher layout parameters. Therefore the multidisciplinary optimization (MDO), based on the rigorous technique [7] may recognize effective non-traditional solutions.

Fig.12. The winged-body aerodynamic model of a launcher in TsAGI's wind tunnel.

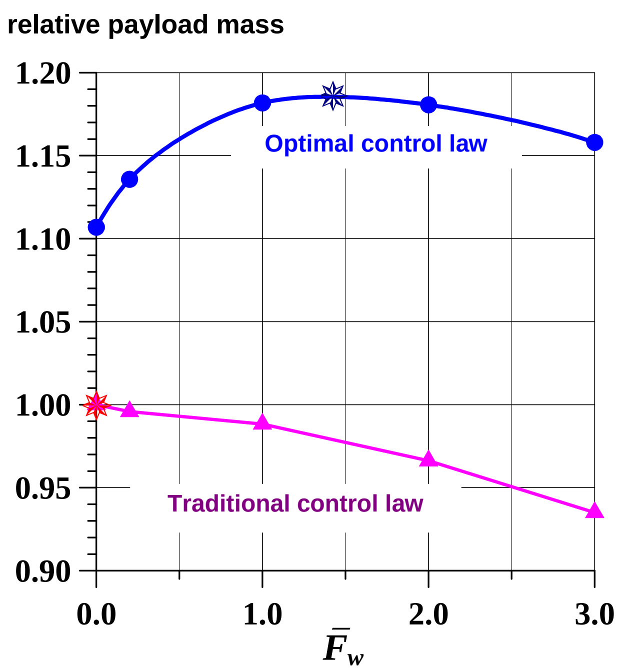

Fig. 13. Relative payload mass versus relative outer wing area $\bar{F}_w$ of the launcher with defined engines and optimal initial mass.

Results of the MDO of the outer wing area and the initial mass of launchers (the aerodynamic model of the launcher see in Fig. 12) at the defined engines are shown in Fig. 13.

There is compared two MDO approaches: with the rigorous trajectory optimization [3, 7] and with the “traditional” trajectory control program [1, 2]. The rigorous trajectory optimization provides qualitative and quantitative new solution in the frame of MDO. The developed approach let improve the inserted payload mass up to 18% in comparison with the traditional one owing to the use of qualitatively new optimal control programs and launcher load-carrying structure adapted for the new flight regimes.

Conclusion

The use of the rigorous method of the trajectory optimization based on the Pontryagin maximum principle makes it possible to significantly increase the efficiency both current and advanced launchers. Appearance of qualitatively new optimal solutions is caused by the effect of the aerodynamic lift even if it is small as compared with the weight.

References

[1] Okhotsimskii, D.E., and Eneev, T.M. Some variational problems connected with launching of artificial Earth satellites. *Uspekhi Fizicheskikh Nauk*, Vol. 63, No. 1a, 1957.

[2] Lawden, D.F. *Optimal Trajectories for Space Navigation*. Butterworths, London, 1963.

[3] Filatyev, A.S. "Paradoxes" of optimal solutions in problems of space vehicle injection and reentry. *Acta Astronautica*, vol.47, issue 1, pp 11-18, 2000.

[4] Pontryagin, L.S., Boltyansky, V.G., Gamkrelidze, R.V., and Mischenko, E.F. *The Mathematical Theory of Optimal Processes*. Interscience Publishers, New York, 1962.

[5] Filatyev, A.S., Golikov, A.A., and Yanova O.V. Optimization of aerospace system control with constraints on return trajectories of spent elements. *Proceedings of the 4th ESA International Conference on Spacecraft Guidance, Navigation and Control Systems,* 18-21 October 1999, ESTEC, Noordwijk, The Netherlands.

[6] Filatyev, A.S., Yermak, Yu.N., Golikov, A.A., and Zadonsky, S.M. New solutions of the optimal injection problem based on complex investigation of dynamics and aerodynamics. Proceedings of 22nd International Congress of Aeronautical Sciences ICAS-2000, 27 August - 1 September 2000, Harrogate, UK, ICAS-2000-4.10.5.

[7] Filatyev, A.S., Golikov, A.A., Kosykh, A.P., Nersesov, G.G., Shanygin, A.N., Voityshen, V.S., Yumashev, V.L. Multidistiplinary Optimization of Aircraft Parameters by the Local Distributed Criteria Method. Proceedings of 23nd International Congress of Aeronautical Sciences ICAS-2002, Toronto, Canada.

Appendix A

Pararameters of optimal injection problem reviewed in sections 3 and 4.

initial conditions	υ_i = 180 m/s, h_i = 12 km
final orbit	circular, h_{orb} = 275 km
ballistic coefficient	$C_{D0}F_0/m_i = 6.68 \cdot 10^{-5}$ m^2/kg
specific mass flow rate	$\mu / m_i = 3.0 \cdot 10^{-3}$ c^{-1}

Appendix B

Pararameters of optimal injection problem reviewed in section 5.

initial conditions	υ_i = 50 m/s, h_i = 200 m
final orbit	circular, h_{orb} = 275 km
initial specific mid-section load	$m_i / F_0 = 6 \cdot 10^3$ kg/m^2 (for the cylindrical boby) $= 1.6 \cdot 10^3$ kg/m^2 (for the conical body)
specific mass flow rate	$\mu / m_i = 3.83 \cdot 10^{-3}$ c^{-1}
initial thrust-to-weight ratio	$n_i = 1.1$