EXPERIMENTAL AND THEORETICAL SYSTEM IDENTIFICATION OF FLEXIBLE STRUCTURES WITH PIEZOELECTRIC ACTUATORS

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Abstract

Two methodologies of extracting dynamic models of flexible structures with piezoelectric transducers (i.e. smart structures) are presented in this paper. However, these techniques are general and valid for system identification of any dynamic system. As for the first method, i.e. theoretical system identification, the Finite Element Method (FEM) is employed to model the dynamic system and to obtain the Frequency Response Function (FRF). Then the μ synthesis technique is used to match a transfer function to the FRF plot. A test bed was developed to investigate the second method, i.e. experimental system identification. A state space model produces the best fit to the experimental data. Results demonstrated good agreement between the two methods. The outcome of this study is an essential part of the control design and implementation for smart structures.

1 Introduction

Conventionally system identification, i.e. system ID, refers to identification of the transfer function of a dynamic system from experimental data. This is essential for complex structures when derivation of a mathematical model of the system is difficult or impossible. However, as described in this paper for complex structures the transfer function of the system can also be obtained from a Finite Element (FE) model. This so called theoretical system ID is much easier and cheaper than experimental system ID particularly in the development of an integrated flexible structure with numerous sensors and actuator placement possibilities, such as in smart structures.

Several experimental system ID techniques have already been investigated in the literature. Genetic algorithms have been employed to identify systems through their input-output behavior. This technique does not depend on the deterministic or stochastic nature of the systems [1]. Recursive identification techniques have been used to obtain the transfer function of a system in real time [2]. The development and application of a fast transversal filter was also presented. A method to obtain an unique optimized system ID model has been investigated [3]. It was shown that the identified stiffness matrix was always unique if the identified matrix was unique as well.

In flexible structures with integrated sensors and actuators, such as piezoelectric transducers (i.e. smart structures), due to the complexity of the system it is difficult to predict the dynamic response by simulation. Thus, numerous studies have been performed on the experimental system identification of smart structures. System identification and state estimation have been used to achieve self-maintenance of a self-sensing piezoelectric cantilever structure [4]. Using a simple experiment, a model of the system has been identified by the subspace state space identification method. A Multi-Input Multi-Output (MIMO) model using the Hankel singular value decomposition was identified from the experimental data of a curved plate with two piezoelectric actuators [5]. The model was fitted simultaneously across all of the transfer functions in the MIMO system. Neural networks have been used for an experimental system identification of smart structures with piezoelectric sensors and actuators [6]. Auto Regressive Moving Average (ARMA) has been implemented for a real-time system identification [7]. A hardware demonstration of a smart structure using this algorithm for real-time health monitoring has been presented in that study. Finite Impulse Response (FIR) and State Space Model techniques have been applied for experimental system identification of a smart isolation mount in an active vibration control system [8]. In another study a frequency domain curve fit technique was employed to identify the transfer function of a system [9]. In that study an experimentally identified transfer function was used to synthesize a MIMO active smart antenna.

Due to the integration of actuators and sensors with the host structure, it is usually very cumbersome, if not impossible, to develop a mathematical model for a complex smart structure. Thus, in many cases, FEM is used to predict the structural response. For example an electromechanical coupling effect of piezoelectric materials was employed to establish a FEM model of a flexible plate with piezoelectric sensors and actuators [10]. In the present study, NASTRAN was employed as the FE solver with the thermal load analogy used to model piezoelectric actuators.

FE codes usually provide the structural response to a specific loading. However, in this study, a procedure of extracting a mathematical representation of the system (i.e. theoretical transfer function) from a FE model is described and results are compared to the experimental system ID. As an application, an Aluminum flexible fin with PZT (Lead Zirconate Titanium) piezoceramic actuators was considered as the target structure. Results of this research will be used in the system identification and control of a full-scale model under a joint US/Australia/Canada project [11] for active suppression of vertical tail buffeting vibrations.

2 Physical Model of the Smart Structure

The physical model considered in this study was a simple scaled model of the vertical tail fin of a F/A-18 fighter jet, which approximately replicated the first two natural frequencies of the full-scale vertical fin. This model included a flexible Aluminum fin, with a thickness of 1 mm, fixed at the base. A total of 24 Piezoceramic actuators[1] were bonded onto both sides (12 on each side) of the Aluminum plate. An accelerometer was used to monitor the dynamic response of the fin tip. The actual system is shown in Fig. 1 and the schematic of the smart fin showing the placement of the actuators and sensors is presented in Fig. 2. Material properties of the structural components (including the Aluminum fin and piezoelectric patches) are given in Table 1.

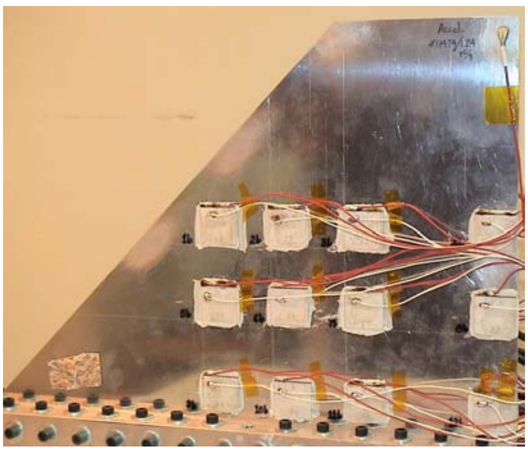

Figure 1. Flexible fin with piezoelectric actuators

[1] BM500 PZT from SensorTechnology Ltd.

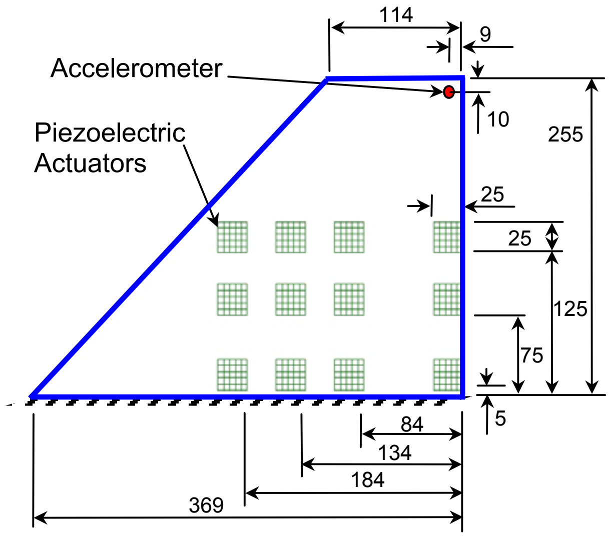

Figure 2. Schematic of smart fin (all dimensions in mm)

Table 1. Material properties

Property	Aluminum 2024-T3	PZT BM500
Density [Kg/m^3]	2796	7650
Elastic Module [MPa]	73.0	64.5
Thermal Expansion [µm.°C]	23.2	-
Thickness [mm]	1.02	0.50
Charge Constant, d_{31}[pC/N]	-	175

3 FE Modeling

Finite Element Modeling (FEM) was used to model the integrated smart fin. A frequency response analysis was performed to get the FRF of the dynamic system. Afterward, the MATLAB µ synthesis toolbox was used to derive the transfer function which identified the model of the dynamic system.

3.1 Piezoactuator Modeling

Although there are a few FEM software packages which provide an electromechanical modeling of piezoelements, for simplicity a thermal analogy was employed in this study to simulate the piezoelectric effect in the finite element model. This is a valid analogy in the linear range (low applied voltage and low bandwidth) and for thin piezoelements.

The induced strain in a piezoelement due to an applied voltage of ΔV is given by:

$$\varepsilon = \frac{d_{31}}{t} \Delta V \qquad (1)$$

where d_{31} is the piezoelectric charge constant. t and ΔV represent piezoelement thickness and applied voltage respectively. For a thermoelastic material, the temperature change of ΔT causes a strain of

$$\varepsilon = \alpha \times \Delta T \qquad (2)$$

where α is the thermal coefficient. Comparing linear Equations 1 and 2 one can simply make an analogy by substituting d_{31}/t with α. Thus, in the finite element model, piezoelectric elements can be modeled as thermoelastic elements. In this paper SOLID elements, HEX20 (20 nodes) were used to model the piezoelectric actuators in PATRAN as shown in Fig. 3. Thermal loads were applied to these elements to simulate the applied voltage to the piezoactuators.

3.2 Integrated Structure Modeling

The substructure Aluminum fin was modeled using SHELL elements, QUAD8 (8 nodes) in PATRAN. The FE model of the integrated smart fin is presented in Figure 4. NASTRAN was used as the solver in this study. A modal analysis of the Aluminum fin alone was performed to obtain the first 5 dynamic modes and the natural frequencies are listed in Table 2.

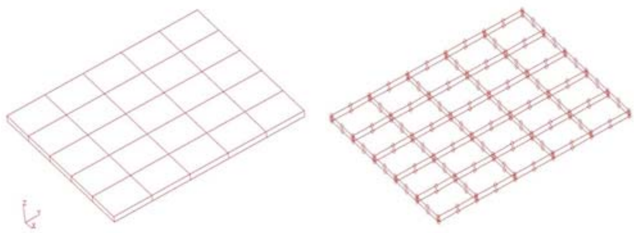
Figure 3. Solid HEX20 elements used for piezoelectric actuator modeling

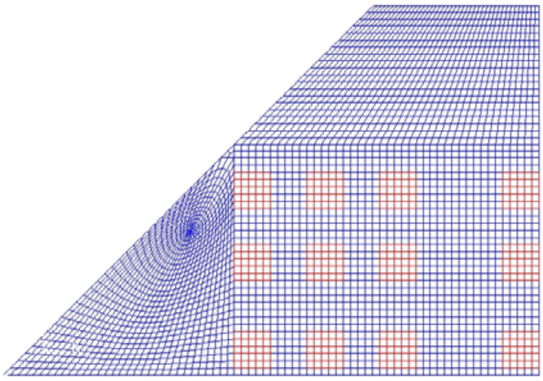

Figure 4. FE model of the smart fin alone

Table 2. Natural frequencies of the flexible fin

Mode	Experiment [Hz]	FEM [Hz]	Error [%]
1	15.7	16.6	+5.7
2	55.1	51.8	-6.4
3	82.2	88.5	+7.2
4	117.0	125.5	+6.8
5	184.0	194.4	+5.2

3.3 Experimental Verification of the FE Model

In order to verify the simulation results and the accuracy of the FE model, the actual fin was tested on an electrodynamic shaker. Initially, a sine sweep of 5-200 Hz was conducted on the fin alone (before bonding piezoactuators) to obtain the natural frequencies of the fin. The LMS TEST.Lab vibration control system was used to run the test. A real time FFT analysis by LMS provided the frequency spectrum of the tip acceleration which in turn identified the natural frequencies of the smart fin. Results of the FE modal analysis and experimental sine sweep are compared in Table 2. The results match closely. However, the FEM predicted higher frequencies except for the second mode. This might be due to the assumption of perfect boundary conditions at the cantilever end of the fin in the FE model. Later, the smart fin was tested on the shaker. Natural frequencies of the integrated flexible fin with piezoelectric actuators obtained from the experiment are compared with the FE modal analysis results in Table 3. Modal damping ratios are also extracted from the experimental data and are listed in Table 3. These modal damping ratios were later used to incorporate damping into the FE model to obtain the FRF.

There are some interesting observations in comparing Tables 2 and 3. In the FEM results the natural frequencies of the integrated smart fin are lower than those of the fin alone except for the first mode. This means that adding piezoelements in the model had a great contribution to increasing the modal masses than the modal stiffness in general. In the experimental results the natural frequencies of the smart fin are higher than those of the fin alone except for the second and fifth mode. This is due to the effect of the stiffness introduced by the bonding glue and wiring, which apparently exceeds the effect of the added mass of the piezoelements. The natural frequencies of the integrated smart fin predicted by FEM were lower than the experimental frequencies mainly due to the effects of the glue and wiring of the actuators, which made the structures stiffer (e.g. higher natural frequencies).

The first three dynamic mode shapes are shown in Fig. 5. The first mode was the first bending mode of the fin, the second mode was the first torsional mode of the fin and finally the third mode was the second bending mode.

Table 3. Modal frequencies and damping ratios of the integrated flexible fin with piezoelectric actuators

Mode	Frequency [Hz]			Experimental
	Exp.	FEM	Error [%]	damping ratio
1	17.9	16.9	-5.3	0.016
2	53.8	49.0	-8.7	0.012
3	83.5	80.3	-3.7	0.025
4	126.0	120.7	-3.9	0.008
5	172.0	166.4	-3.3	0.016

4 Theoretical System Identification

System identification often means obtaining the transfer function of a dynamic system from the experimental data (a relation between output and input). However, it is possible to simulate the same technique using a FE model of the system (i.e. theoretical system

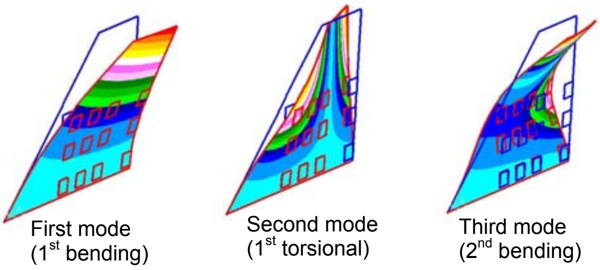

Figure 5. First 3 modes of the smart fin

identification). In this section a methodology to extract the transfer function between the accelerometer signal (output) and the piezoactuators voltage (input) of the FE model of the smart fin is presented.

A Frequency Response Analysis between 5-100 Hz with a step of 0.5 Hz was performed on the smart fin. Nodal thermal loads were applied onto the piezoactuators to simulate the input voltage. The phase and magnitude of acceleration, which represented the FRF, at the location of the accelerometer was read from the FE results. This FRF was basically a representation of the smart fin in the frequency domain. The μ synthesis Toolbox of MATLAB was then used to curve fit a transfer function to this FRF. A 6th order system was found to be a good representation of the FRF for the first three modes. The bode plot of the original FRF obtained from the FE analysis is compared with the bode plot of the 6th order transfer function in Fig. 6, and as it is seen, they match well. The μ curve fit is the final presentation of the smart fin in the frequency domain and is given as the following transfer function in the Laplace domain, Eq. 3, which concludes the theoretical system identification task. This transfer function could replace the actual system for any dynamic response simulation as well as control design and implementation.

$$G(s) = \frac{-21.6s^6 - 39.8s^5 - 1.4\times10^5 s^4 - 1.6\times10^5 s^3 - 2.0\times10^8 s^2 - 4.4\times10^7 s + 1.9\times10^{10}}{s^6 + 2.2s^5 + 9.2\times10^3 s^4 + 1.1\times10^4 s^3 + 1.8\times10^7 s^2 + 6.5\times10^6 s + 4.5\times10^9} \tag{3}$$

5 Experimental System Identification

The transfer function of a dynamic system can also be obtained experimentally which in general is called "system ID". In order to identify the dynamic model of the smart fin and also to verify the theoretical system identification model such an experiment was conducted.

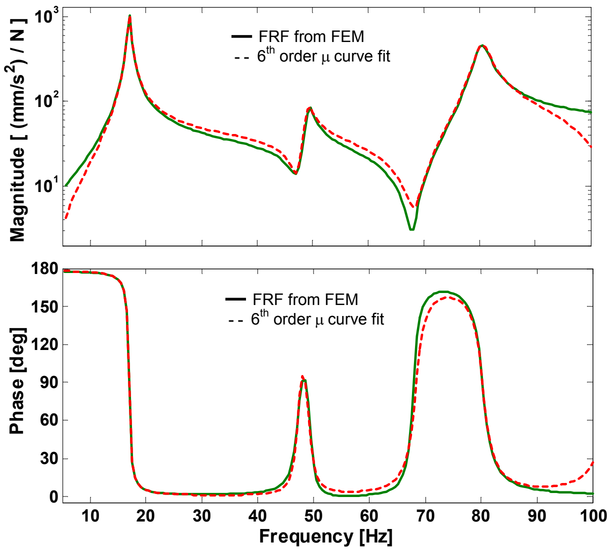

Figure 6. FRF from FEM and the 6th order μ synthesis curve fit

Figure 7. Experimental configuration

5.1 Experimental Configuration

The experimental test bed for the smart structure system identification developed at the National Research Council Canada (NRC) is shown in Fig 7. The smart fin configuration is presented in Fig. 1. The Real-Time Workshop Toolbox and xPC TargetBox of MathWorks were employed to conduct the real time data acquisition.

5.2 System identification from experimental data

The frequency range of interest was focused on 5-100 Hz which covered the first three dynamic modes. These are the major modes of the actual model of the vertical tail fin. A frequency sweep of 5-100 Hz was applied to the piezoactuators using a chirp signal of $\pm 40V$. The fin response monitored by the tip accelerometer is shown in Fig. 8 which clearly shows the first three resonances of the smart fin. Input and output signals of the smart fin have been fed into the Matlab System Identification Toolbox to define a proper dynamic model of the system. Several experimental system ID techniques such as ARMAX and the state space model were investigated to establish the most accurate dynamic model of the system from the time domain response. The state space model technique offered the best approximation of the system and a 6th order state space model was found to be a good representation of the smart fin with a correlation factor of 84.5%. The Bode plot of this model is shown in Fig. 9 which represents the transfer function between the accelerometer and piezoelectric actuators. This transfer function can also be described in state space format (A, B, C and D matrices) or as an equation in Laplace domain for control design and implementation.

6. Results Comparison

The bode plots of the theoretical and experimental system identification models of the smart fin are compared in Fig. 10. As for the theoretical model, a discretization with time step of 0.004 sec (the same sampling rate as in the experiment) has been performed to obtain the discrete model from the continuous model shown in Fig. 6. Both methods used a 6th order model to represent the smart fin. It is worthwhile to mention that there are always approximations associated with FE modeling such as perfect boundary conditions, perfect bonding, and negligible glue stiffness, mass and slip motion at the interface of the piezoelectric transducers and substructure. Consequently, it is typical to have some discrepancies between

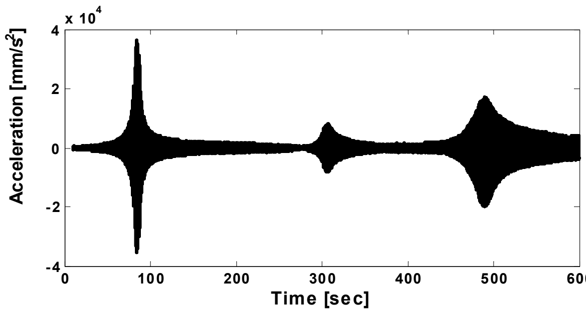

Figure 8. Fin acceleration sweep between 5-100 Hz

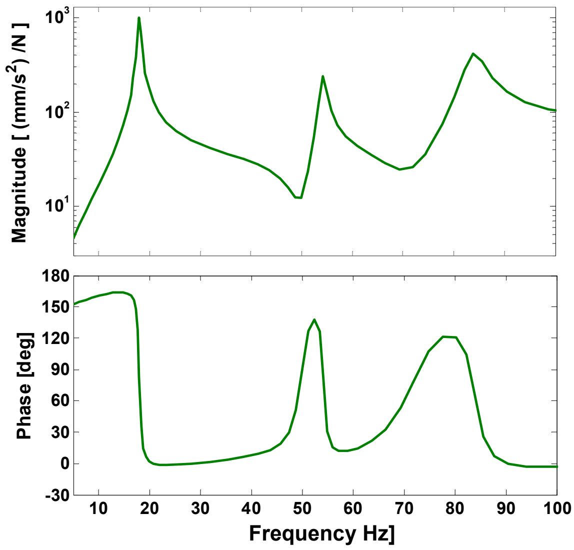

Figure 9. Bode plot of the experimental system ID model using a 6th order state space model

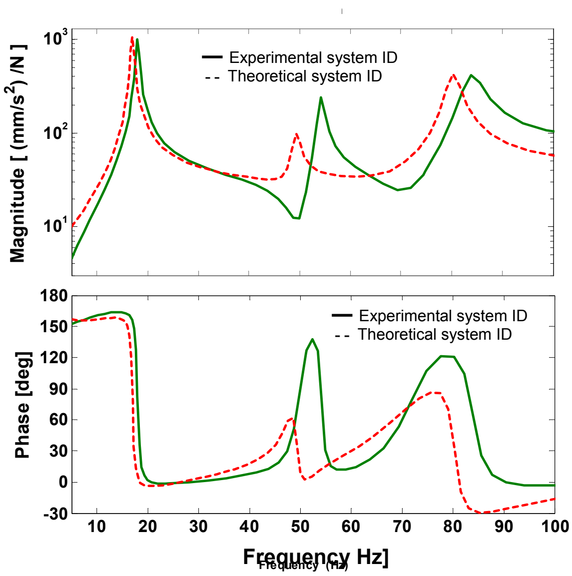

Figure 10. Comparison of the experimental and theoretical system ID models

simulation and the experimental system ID. Moreover, structural damping values and the method of incorporating these into the FE model are major challenges, which need to be investigated to improve the agreement between the simulation (µ synthesis) and experimental models. Despite these facts, it is seen that good agreement between the experimental system ID using the state space model technique and the theoretical system ID using µ synthesis has been achieved (Fig. 10). The natural frequencies (peak frequencies) match closely and the overall trend in both magnitude and phase plots are similar. The impulse response of the two models is also compared in Fig. 11 which demonstrates the accuracy of the theoretical model compared with the experimental model.

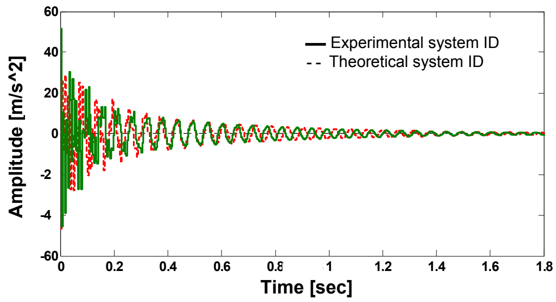

Figure 11. Impulse response of the experimental and theoretical models

7. Conclusions

Although the terminology of system ID is largely used for experimental identification of the dynamic model of an actual system, it was shown in this paper that even for complex flexible structures it is possible to obtain a representation of the system as a transfer function from the FE model, i.e. theoretical system identification. The FRF of a flexible fin with bonded piezoactuators was obtained from a frequency analysis of the FE model. As an example a 6^{th} order transfer function matching the FEM FRF was then extracted by the µ synthesis technique. In order to verify the theoretical system ID model an experimental test bed was developed at NRC on which a 6^{th} order state space model was found to be a good representation of the actual smart fin for the frequency range of 5-100 Hz. Very good agreement of peak frequencies, magnitudes and phases between theoretical and experimental system ID models was observed. Theoretical system ID is much easier and cheaper than experimental system ID particularly in the development of an integrated smart structure with numerous sensors and actuator placement possibilities. The results of this paper demonstrate the reliability and accuracy of the theoretical system ID.

Acknowledgement

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