ROBUST CONTROL OF THE AIRCRAFT-ON-GROUND LATERAL MOTION

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Keywords: *Aircraft-on-Ground ; Nonlinear Control ; Feedback Linearization ; Sliding Mode Control*

Abstract

This paper aims to present the study of a yaw rate control of the aircraft-on-ground. Such a control law can ease ground handling and turn it safer. It helps to stabilize the aircraft trajectory and guarantees a predictable behavior.

Because of the high non-linearities of the model, the control design methodology is based on feedback linearizing. This technique use supposes that the reference model perfectly corresponds to the real system. This assumption is not verified (high tire/ground friction coefficient uncertainty) and under degraded conditions, the response time can significantly increase while static errors may occur.

Therefore, this communication proposes a control law adaptation based on sliding mode control, to improve robustness with respect to such uncertainties.

1 Introduction

The sustained air transportation growth during the last decades has led to the congestion of many airports. It appears that one way to improve airside traffic conditions at these airports, is to perform more accurate and faster aircraft movements while increasing the safety level. All these points have recently enlarged the concern with the amelioration of airport use and safety and have promoted new studies. Some of them are involved in ground traffic control projects such as A-SMGCS (Advanced Surface Movement Guidance and Control Systems), while others are more specifically concerned with new on board systems devoted to ground navigation and guidance.

The problem considered in this paper aims to improve the aircraft ground handling qualities. Nowadays, the lateral motion of commercial aircraft is achieved by means of an open loop steering control. One possible way of amelioration consists in adding an "aircraft loop" to this direct control. This philosophy corresponds to what has been done on flight control channels with the "fly by wire" concept : the pilots do not directly command the actuator deflection but command aircraft "attitude" parameters (e.g. roll rate and vertical acceleration). A first adaptation of this principle for ground handling has been studied by F. Villaumé to improve runway axis tracking during landing, take-off and rejected take-off [1]. This adaptation is based on a yaw rate control of the aircraft. Such a control objective can allow to ease on-ground control and turn it safer. It helps to stabilize the aircraft trajectory through a reduction of the number of corrections needed to ensure straight line roll and tends to guarantee an homogeneous aircraft behavior whatever the weather and the ground conditions (dry, wet, icy, …).

The study mainly concentrates on taxiing. At such speeds, aircraft presents highly non-linear behavior. The aerodynamic forces become negligible compared with the tire/ground friction. During short turns, the sideslip angles often reach high values. It implies that linear models of the natural aircraft behavior are not representative during such maneuvers.

Therefore, non-linear control techniques can be advantageously used to cope with this non-linear dynamics.

The proposed approach is based on feedback linearizing control. This technique aims to design a non-linear controller that compensates the system non-linearities and constraints its outputs to follow a linear reference behavior.

As a previous study shows [2], for normal taxiing conditions, the feedback linearizing control of yaw rate produces very satisfactory outcomes. However, when the tire/ground friction coefficient is degraded, the response time increases and static errors occur.

This communication aims to present the feedback linearizing yaw rate control methodology and to show how robustness of the resulting control law can be improved using a non-linear robust control technique based on sliding mode control. At first, it briefly describes the equations of aircraft-on-ground dynamics, including tire-ground friction and steering system effect. Then, a second part recalls the feedback linearizing control theory and displays its application to the aircraft-on-ground yaw rate control. A third part deals with the sliding mode control theory and describes how this technique can be used to adapt the feedback linearizing control law.

2 Aircraft On-Ground Dynamics

Before studying the control law, let us briefly describe the equations of the aircraft on-ground dynamics. Different types of models can be considered, from a 12 degree of freedom non-linear model to a 2 degree of freedom linear one [3], considering or not actuators dynamics.

The aim of this section is to present the basic equations that lead to these models. This presentation allows to put forward the global model architecture.

2.1 General equations of ground motion

The equations of motion form the core of the aircraft on ground dynamic model. They derive from basic Newton mechanics, applied to a rigid body. These differential equations can be written in a non-linear state-space format (m is the weight and I the inertia tensor) :

$$\frac{d}{dt}\begin{bmatrix} V \\ \Omega \end{bmatrix} = \begin{bmatrix} \dfrac{F}{m} - \Omega \wedge V \\ I^{-1} \cdot [\, M - \Omega \wedge (I \cdot \Omega) \,] \end{bmatrix} \qquad (1)$$

with :

$V = [Vx \;\; Vy \;\; Vz]^T$ The velocity vector at the center of gravity,

$\Omega = [p \;\; q \;\; r]^T$ The angular velocity vector about the c.g.,

$F = [Fx \;\; Fy \;\; Fz]^T$ The total external force vector,

$M = [Mp \;\; Mq \;\; Mr]^T$ The total external torque vector,

Here the body-axes components of linear and angular velocities can be regarded as the state and the body-axes components of the external forces and torques are the input variables.

The external forces and torques themselves are non-linear functions of the aircraft motion variables and of the real inputs from the control law. By combining these functions in a single matrix equation, a highly non-linear state space model can be obtained (cf. *(2)*).

$$\frac{dx}{dt} = f(x) + g(x\,,u), \qquad (2)$$

with $x = \begin{bmatrix} V \\ \Omega \end{bmatrix}$ and $u = \theta_{NW}$

where θ_{NW} is the controlled steering angle.

Many forces and torques should be considered : e.g. the gravity acceleration, the engines thrust, the aerodynamic effects and the tire-ground friction. During low speed taxi roll or maneuvers, the tire-ground friction has a major influence on the aircraft on-ground dynamics.

2.2 Tire-ground friction

At low speed, the aircraft lateral behavior mostly relies upon the sideslip of the nose and main landing gear wheels (the angle between the wheel axis and the direction of motion).
These angles are given by :

$$\beta_{MLG\,R} = \operatorname{arctg}\left(\frac{Vy - L_{MLG} \times r}{Vx - l_{MLG} \times r}\right)$$
$$\beta_{MLG\,L} = \operatorname{arctg}\left(\frac{Vy - L_{MLG} \times r}{Vx + l_{MLG} \times r}\right) \qquad (3)$$
$$\beta_{NW} = \operatorname{arctg}\left(\frac{Vy + L_{NW} \times r}{Vx}\right) - \theta_{NW} = \beta_F - \theta_{NW}$$

Where :

- L and l represent respectively the wheels distances to the center of gravity and to the aircraft axis,
- The suffixes "MLG R", "MLG L" and "NW" point out variables respectively relative to the right and left main landing gears and to the nose landing gear.

Sideslip generates lateral forces perpendicular to the plane of the wheels (the "cornering" forces). The tire behavior is highly complex (non-constant friction coefficient, lateral bending of carcass and belt, varying pressure distribution, etc). The most representative models are based on empirical functions, designed and tuned to fit measure data. Some mathematical formulations can be already found in literature. The Pacejka's Magic Formula (trigonometric formulation) [4 and 5] is a popular model in the automotive industry. The NASA, through an AGARD project, proposed a mathematical formulation applied to commercial airplanes and based on polynomial equations [6].

In this study, the cornering force model uses a simplified formulation, adapted to functional inversion [7 and 8].

$$Fy = Gy \times \beta \times \frac{\beta_{OPT}^{\;2}}{\beta_{OPT}^{\;2} + \beta^2} \qquad (4)$$

Where β_{OPT} is the optimal sideslip angle and Gy the associated cornering gain (linearization of (4)).

Such quasi-static tire models are not representative for very low speeds. In this case, it can be considered that the buckling of the tire carcass acts on the sideslip angle as a low-pass filter. To cover all the speed range, the tire flexibility should be taken into account, but at a cost of a more complex tire model.
This particular case has been studied in [9]. For simplicity reason, in this communication, such very low speeds will not be considered.

2.3 Steering system model

On most Airbus commercial aircraft, the steering system is made of a hydraulic actuator electrically controlled by a steering control unit (SCU) [10 and 11]. Beyond a certain bandwidth, the steering angle rate directly corresponds to the electric order (I) through a non-linear characteristic depending upon the torque Γ applied around the nose landing gear :

$$\dot{\theta}_{NW} = k_1(|\Gamma|) \times \sqrt{\frac{1}{1 + k_2 \cdot I^2}} \times I = f_1(I, \Gamma) \qquad (5)$$

The SCU control loop can be considered as a proportional controller with input and output saturations corresponding to a maximum steering authority and a servovalve spool travel :

$$I = \gamma \times (\theta_{NW\,d} - \theta_{NW}) \qquad (6)$$

with, $|I| \le I_{max}$ and $|\theta_{NW}| \le \theta_{NW\,max}$

This leads to a fast first order non-linear differential equation with steering rate and angular saturations.

3 The Yaw Rate Control Design

The main goal of the study is to design a sufficiently accurate and stable control law insuring good ground handling qualities. The maneuverability criterion can be defined by limit behaviors inside which the controlled system must stay. In the case of the on-ground yaw rate control, these limit behaviors can mainly be summed up in terms of response time and damping (no overshoot).

The proposed solution approach makes use of the theory of feedback linearization [12, 13 and 14]. Lets start by a first brief review of this theory.

3.1 Theory of feedback linearization

A single-input/single-output (SISO) non-linear systems is considered :

$$\frac{dx}{dt} = f(x) + g(x, u) \qquad (7)^{1}$$
$$y = h(x)$$

where f and g are smooth vector fields on R^n ; h is a smooth function defined on R^n ; u and y are scalar input and output. The term smooth means that the function (the vector field) has continuous partial derivatives of any required order.

The relative degree of the system is defined by a non negative integer ρ which satisfies the following conditions :

$$L_g L_f^k h(x, u) = 0 \quad , \; k = 0, \ldots, \rho - 2 \qquad (8)$$
$$L_g L_f^{\rho-1} h(x, u) \neq 0$$

with : $L_f h = \nabla h \, f$
(Lie derivative of h with respect to f)

and, $\nabla h = \frac{\partial h}{\partial x}$ (∇h is the gradient of h)

It is assumed that the relative degree is well defined and that the inverse of $L_g L_f^{\rho-1}$ exists.

Then, a state transformation can be found :

$$\hat{x} \equiv [\xi \; \eta]^T \equiv \Phi(x), \qquad (9)$$

This transformation is a diffeomorphism such that :

$$\xi \equiv \left[h(x), \cdots, L_f^{\rho-1} h(x)\right]^T \in \Re^{\rho} \qquad (10)$$

The state space description of the system in the new coordinates is given as follows.

$$\Sigma_1 : \left[\begin{array}{l} \dot{\xi}_1 = \xi_2 \\ \quad \vdots \\ \dot{\xi}_\rho = L_f^\rho h\left(\Phi^{-1}(\hat{x})\right) + L_g L_f^{\rho-1} h\left(\Phi^{-1}(\hat{x}), u\right) \\ \quad = A\left(\Phi^{-1}(\hat{x})\right) + B\left(\Phi^{-1}(\hat{x}), u\right) \\ \quad = A(x) + B(x, u) \end{array}\right.$$
$$y = \xi_1$$
$$\Sigma_2 : \quad \dot{\eta} = f_\eta(\xi, \eta) + g_\eta(\xi, \eta, u) \qquad (11)$$

The I/O model behavior is linearized using the following feedback control law :

$$u = B_u^{-1}\left(x, v - A(x)\right) \qquad (12)$$

where B_u^{-1} is the inverse function of B with regard to u, such as, $B\left(x, B_u^{-1}(x, z)\right) = z$

Thus :

$$\Sigma_1 : \left[\begin{array}{l} \dot{\xi}_1 = \xi_2 \\ \quad \vdots \\ \dot{\xi}_\rho = v \end{array}\right. \qquad (13)$$
$$y = \xi_1$$
$$\Sigma_2 : \quad \dot{\eta} = f_\eta(\xi, \eta) + g_\eta\left(\xi, \eta, B_u^{-1}\left(\Phi^{-1}(\hat{x}), v - A\left(\Phi^{-1}(\hat{x})\right)\right)\right)$$

The system dynamics are decomposed into an external (input/output) part Σ_1, and an internal part Σ_2.

Since the external part consists into a linear differential relation between y and v, it becomes easier to design a control law for v. For instance, a dynamic of the ρ^{th} order can be followed by output y when taking :

$$v = k_0 \cdot (y_d - y) - \sum_{i=1}^{\rho-1} k_i \cdot y^{(i)} \qquad (14)$$

Then, $$\frac{y}{y_d} = \frac{1}{1 + \frac{k_1}{k_0} p + \cdots + \frac{k_{\rho-1}}{k_0} p^{\rho-1} + \frac{1}{k_0} p^{\rho}}$$

At this time, the stability of the internal dynamic, which are affected by the I/O control law has to be checked to ensure that the internal states remain bounded.

3.2 Application to aircraft yaw rate control

To design the control law, a simplified model of the aircraft on-ground lateral dynamics is used. This model is a SISO 2-degree of freedom model :

$$\left\{\begin{array}{l} \dot{r} = \frac{1}{Izz}\left[Mr_{NW} + Mr_{MLG}\right] \\ \dot{V}y = \frac{1}{m}\left[Fy_{NW} + Fy_{MLG}\right] - r \cdot Vx \end{array}\right. \qquad (15)$$

[1] Observe that the representation is not necessarily an affine form

The two state variables are the yaw rate and the lateral speed, while the longitudinal speed is considered as an exogenous parameter and the steering order as the input. Here the inner loop and the actuator dynamics and saturations are not taken into account. The aircraft is assumed to keep a horizontal attitude and the aerodynamic forces and moments are neglected, since here the speed is supposed to remain small.

Within the feedback linearization approach, the selected output is : $y = r$

thus, $\dot{y} = \dot{r} = Izz^{-1}\left[Mr_{MLG} + Mr_{NW}\right]$

$$\text{where : } Mr_{NW} = L_{NW} \times Fy_{NW}\left(\beta_{NW}, Vx\right)$$
$$\beta_{NW} = \beta_F - \theta_{NW} \tag{16}$$

Introducing the auxiliary input v, the yaw rate dynamic associated to the expressions *(16)* can be linearized by using the steering angle command :

$$\theta_{NW_c} = \beta_F - \beta_{NW_c} \tag{17}$$

$$\text{with, } \beta_{NW_c} = Fy_{NW}^{-1}\left(\frac{Mr_{NW_c}}{L_{NW}}, Vx\right)$$

$$\text{and, } Mr_{NW_c} = Izz \cdot v - Mr_{MLG}$$

where Fy_{NW}^{-1}, the inverse function of Fy_{NW} with regard to the nose wheel steering angle (θ_{NW}), is assumed to exist.

The yaw rate dynamic then reduces to $\dot{r} = v$ and thus, the yaw rate order can be forced to follow a first order dynamic, by using a proportional controller.

$$v = \frac{1}{\tau}\left(r_d - r\right) \tag{18}$$

It appears that the non-linear internal dynamics, associated to the present output choice are naturally stable (understeer property of commercial aircrafts).

The diagram in figure 1 describes the yaw rate control law so defined.

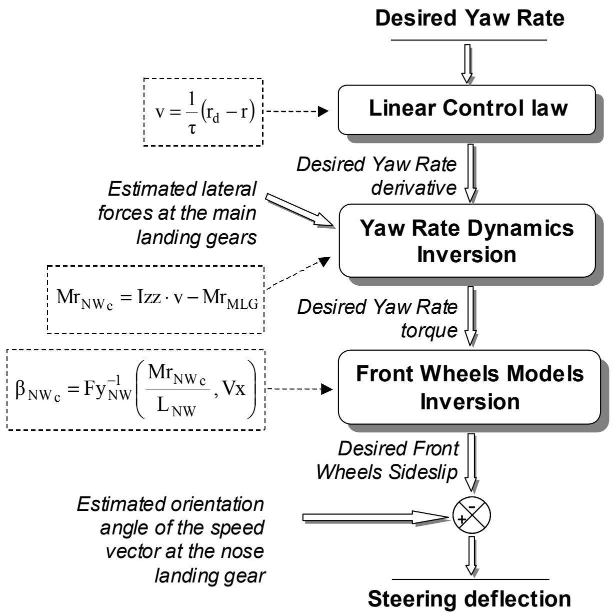

Fig. 1. Architecture of the feedback linearizing yaw rate control law.

For normal rolling and ground maneuvers, algebraic analysis and simulation runs [2] show that the feedback linearizing control of the yaw rate can produce very satisfactory results. Nevertheless, it also point out that, when adhesion between tires and ground reduces, the response time increase and a static error appear.

By representing the tire/ground friction forces reduction through a multiplicative model error coefficient δ, the yaw rate aircraft behavior can then be estimated by the equation :

$$\dot{r} \approx \Delta_1(\delta) \times \frac{1}{\tau}\left(r_d - r\right) + \Delta_2(\delta) \tag{19}$$

where Δ_1 and Δ_2 are functions of the model error.

An example of those results is displays figures 2a and 2b for 4 different cases, form δ=1 (corresponding to dry conditions) to δ=0.3 (corresponding to snowy conditions).

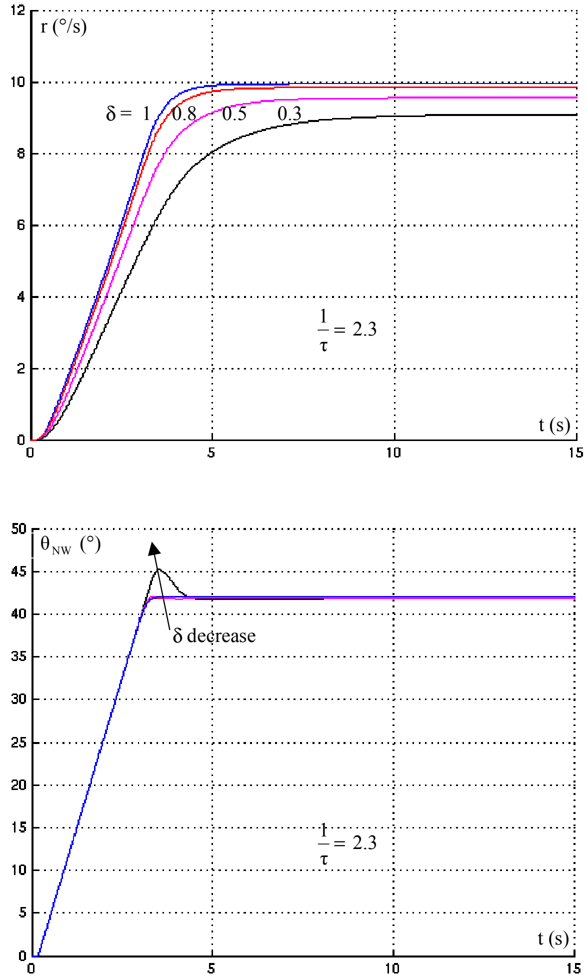

Fig. 2a and 2b. Responses to a yaw rate step command from 0 to 10°/s.

4 Control Law Robustness Improvement

In this section, the improvement of the previous control law robustness is described. It mainly aims to cancel any static error that could occur. To do so, the proposed approach makes use of the sliding mode control methodology.

4.1 Sliding Mode Control

Sliding mode control [14, 15, 16, 17, 18 and 19] is a largely developed nonlinear control method that has its roots in the 1970's, when it first appears in the literature (in English). The appeal of this methodology is based on its ability to treat nonlinear systems with bounded uncertainties and disturbances.

This controller design method is based on two steps :

- the definition of a 'sliding surface' : $S(t) : s(x\ ;\ t)=0$, over which the system state (x) is driven towards the desired equilibrium.
- the determination of a control law such as s converges to 0.

4.1.1 General theory of sliding mode control

The single-input/single-output nonlinear system, defined in 3.1 (equation (7)) is considered. Through the diffeomorphism (10), this system can be transformed into the system (13), decomposed in the subsystem Σ_1 (in companion form) and the subsystem Σ_2, which is assumed to be stable.

The sliding surface can be defined by :

$$s \equiv \left(\frac{d}{dt}+\lambda\right)^{\rho-1} e \qquad (20)$$

$$= \sum_{k=0}^{\rho-1} C_{\rho-1}^{k} \cdot \lambda^{\rho-1-k} \cdot e^{(k)}$$

with, $e = (y_d - y)$ and $\lambda > 0$

ρ is the relative degree of the system and,

$$C_n^p = \frac{n!}{p!(n-p)!}$$

A sufficient condition for the convergence of s to 0 and for exponential stability is :

$$\frac{1}{2}\cdot\frac{d}{dt}s^2 = s\cdot\dot{s} \le -\eta\cdot|s|, \qquad \eta > 0 \qquad (21)$$

or $\quad \dot{s} \le -\eta\cdot \mathrm{sign}(s), \qquad \eta > 0 \qquad (22)$

where $\dot{s} = y_d^{(\rho)} - y^{(\rho)} + \sum_{k=1}^{\rho-1} C_{\rho-1}^{k-1} \cdot \lambda^{\rho-k} \cdot e^{(k)} \qquad (23)$

The condition (22) can be satisfied using a control law such as :

$$u = B_u^{-1}(x, v - A(x)) \qquad (24)$$

with, $v = y_d^{(\rho)} + \sum_{k=1}^{\rho-1} C_{\rho-1}^{k-1} \cdot \lambda^{\rho-k} \cdot e^{(k)} + \eta\cdot \mathrm{sign}(s)$

Considering nominal values $\hat{A}$ and $\hat{B}$, the robust exponential stability in the presence of model errors (bounded along the desired trajectory) can be guaranteed using the control law [16] :

$$u = \hat{B}_u^{-1}\left(x\,,\,v - \hat{A}(x)\right) \tag{25}$$

with, $v = y_d^{(\rho)} + \sum_{k=1}^{\rho-1} C_{\rho-1}^{k-1} \cdot \lambda^{\rho-k} \cdot e^{(k)} + K \cdot \mathrm{sign}(s)$

where K is given by :

$$K \equiv \beta \cdot (\Delta A + \eta) + (\beta - 1) \cdot \left|v - \hat{A}\right|$$

with, $\left|A(x) - \hat{A}(x)\right| < \Delta A(x;t)$

$$\beta^{-1}(x;t) \le \frac{\hat{B}(x;u)}{B(x;u)} \le \beta(x;t)$$

4.1.2 Quasi-sliding control using continuous control laws

The previous control law is discontinuous across the sliding surface S and thus, induces control ‘chattering’.
Such a high-frequency control activity can be avoided by smoothing out the discontinuity in a thin boundary layer neighbouring the switching surface [14].

This can be achieved by using the equation :

$$v = y_d^{(\rho)} + \sum_{k=1}^{\rho-1} C_{\rho-1}^{k-1} \cdot \lambda^{\rho-k} \cdot e^{(k)} + K \cdot \mathrm{sat}\left(\frac{s}{\lambda^{\rho-1}\varepsilon}\right) \tag{26}$$

where ε is the boundary layer width (cf. figure 3) and the ‘sat’ function is such as if |y|<1 sat(y)=y and otherwise sat(y)=sign(y).

The induced modifications of the control law (25) guarantees that all the trajectories outside the boundary layer will reach it in finite time : the layer is attractive, hence (positively) invariant (invariant manifold).
Without model errors, the control law guarantees global asymptotic stability.

$$\frac{1}{2} \cdot \frac{d}{dt} s^2 = s \cdot \dot{s} \le -\eta \cdot \mathrm{sat}\left(\frac{s}{\lambda^{\rho-1}\varepsilon}\right) \cdot s \quad < 0 \tag{27}$$

Nevertheless, in case of model errors, the guaranteed tracking precision will be limited by the layer width.

The tracking precision results from a intuitive balance ratio between the bandwidth to the power of ρ and the parametric uncertainty along the desired trajectory.

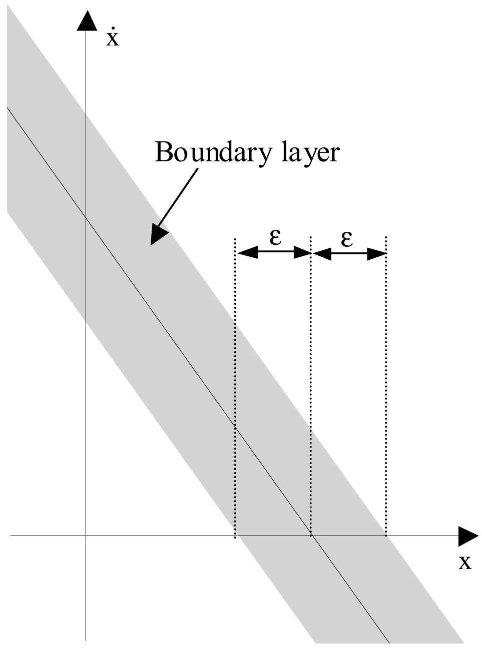

Fig. 3. Boundary layer in the case that n = ρ = 2.

4.1.3 Integral sliding mode control

The control law precision can be improved using integral sliding mode control, where the variable of interest becomes : $\chi = \int e\,d\tau$.

The sliding surface is defined by :

$$s = \left(\frac{d}{dt} + \lambda\right)^{\rho} \chi \tag{28}$$

$$= \sum_{k=0}^{\rho} C_\rho^k \cdot \lambda^{\rho-k} \cdot \chi^{(k)}$$

$$= \int \left(\sum_{k=0}^{\rho} C_\rho^k \cdot \lambda^{\rho-k} \cdot e^{(k)}\right) d\tau$$

Here : $\dot{s} = y_d^{(\rho)} - y^{(\rho)} + \sum_{k=0}^{\rho-1} C_\rho^k \cdot \lambda^{\rho-k} \cdot e^{(k)}$ (29)

Now, the desired control law can be obtained by defining v as :

$$v = y_d^{(\rho)} + \sum_{k=0}^{\rho-1} C_\rho^k \cdot \lambda^{\rho-k} \cdot e^{(k)} + K \cdot \mathrm{sat}\left(\frac{s}{\lambda^\rho \varepsilon}\right) \quad (30)$$

4.1.4 Sliding mode control versus feedback linearization

Assuming that the desired output is constant, the auxiliary input v is then equal to :

$$v = -\sum_{k=1}^{\rho-1} C_\rho^k \cdot \lambda^{\rho-k} \cdot y^{(k)} \quad (31)$$
$$+ \lambda^\rho \cdot (y_d - y) + K \cdot \mathrm{sat}\left(\frac{s}{\lambda^\rho \varepsilon}\right)$$

which can be written :

$$v = k_0 \cdot (y_d - y) - \sum_{i=1}^{\rho-1} k_i \cdot y^{(i)} \quad (32)$$
$$+ K \cdot \mathrm{sat}\left(\frac{s}{\lambda^\rho \varepsilon}\right)$$

with, $\quad k_i = C_\rho^i \cdot \lambda^{\rho-i}$

Integral sliding mode control is thus equivalent to the previous feedback linearizing control (equation (14)), completed by a specific integral feedback term that improves stability and robustness.

Remark :

This integral feedback can be linearized using :

$$v = k_0 \cdot (y_d - y) - \sum_{i=1}^{\rho-1} k_i \cdot y^{(i)} + \frac{K}{\lambda^\rho \varepsilon} \cdot s \quad (33)$$

Inside the boundary layer, the control law is unchanged. Outside the layer, s behaves as :

$$\frac{1}{2} \cdot \frac{d}{dt} s^2 = s \cdot \dot{s} \leq -\eta \cdot \frac{s^2}{\lambda^{\rho-1} \varepsilon} \leq -\eta \cdot |s| \quad (34)$$

and the robust stability remains guaranteed.

The combination of (33) and (28) leads to :

$$v = k_0 \cdot (y_d - y) - \sum_{i=1}^{\rho-1} k_i \cdot y^{(i)} \quad (35)$$
$$+ \frac{K}{\lambda^\rho \varepsilon} \cdot \int \left(\sum_{k=0}^{\rho} C_\rho^k \cdot \lambda^{\rho-k} \cdot e^{(k)} \right) d\tau$$

which can be written as :

$$v = k_0 \cdot (y_d - y) - \sum_{i=1}^{\rho-1} k_i \cdot y^{(i)} \quad (36)$$
$$+ \frac{K}{\lambda^\rho \varepsilon} \cdot \int \left(-y^{(\rho)} - \sum_{i=1}^{\rho-1} k_i \cdot y^{(i)} + k_0 \cdot (y_d - y) \right) d\tau$$

This integral feedback corresponds to the integration of errors between the real system dynamics and the reference ones which have been used to design the feedback linearizing controller.

Since equation (14) is sufficient to ensure system stability, equation (36) will ensure it too.

At equilibrium, s stays in the boundary layer,

$$s = -k_1 \cdot e + k_0 \cdot \int e \, d\tau , \quad (37)$$

and the tracking error tends to zero.
Therefore, the above integral sliding mode control law guarantees static error cancellation.

4.2 Application to Yaw Rate Control

According to the previous paragraph, the yaw rate control law robustness can be improved using integral sliding mode control to design an additional integral feedback.

The sliding surface is defined by :

$$s = (r_d - r) + \frac{1}{\tau} \cdot \int (r_d - r) \, d\tau \quad (38)$$

Then, the initial control law is changed by defining v as :

$$v = \frac{1}{\tau}(r_d - r) + \tau \cdot \frac{K}{\varepsilon} \cdot s \quad (39)$$
$$= \frac{1}{\tau}(r_d - r) + \tau \cdot \frac{K}{\varepsilon} \cdot \left((r_d - r) + \frac{1}{\tau} \cdot \int (r_d - r) \, d\tau \right)$$

According to above theoretical results, such a control law adaptation should improve robustness and cancel static errors.

In this case, the value of K/ε is limited by the neglected steering system dynamics (simulation runs are used to determine this limit).
For a given tuning of K/ε, a reduction of the tire/ground friction coefficient induces a boundary layer enlargement and thus, slows down the integral feedback term effect.

Adaptation to actuator saturations

During on-ground manoeuvres, the aircraft nose wheel steering system often reach saturation. However, saturations cannot be taken into account in the theoretical developments of paragraphs 3 and 4 (B(x;u) must be inversible).

Reference [2] shows that, adopting an adequate tuning of the feedback linearizing control law, saturation does not induce unacceptable system behavior. Nevertheless, its action on the additional integral feedback introduced in relation (39) can be far more harmful.

The use of such a term in presence of saturations requires to add an anti-windup protection. For the aircraft yaw rate control application, such a protection can be easily provided using specific systems acquisitions.

The two main saturations (in steering angle and rate) can be directly detected using steering angle and steering electric order measurements. The anti-windup protection can then be achieved by "disconnecting" the integrator, while the system stays at saturation.

Out of saturation, the control law remains unchanged, however, at saturation, it will acts as the initial feedback linearizing control.

Simulation runs drawn on figures 4a and 4b show that such control law acts as expected and, whatever the conditions, the static errors are cancelled.
The response time stays nearly unchanged. This is mainly due to the steering system efficiency limitation. During most of the response time, the system stay at saturation and the additional integral feedback is not active.

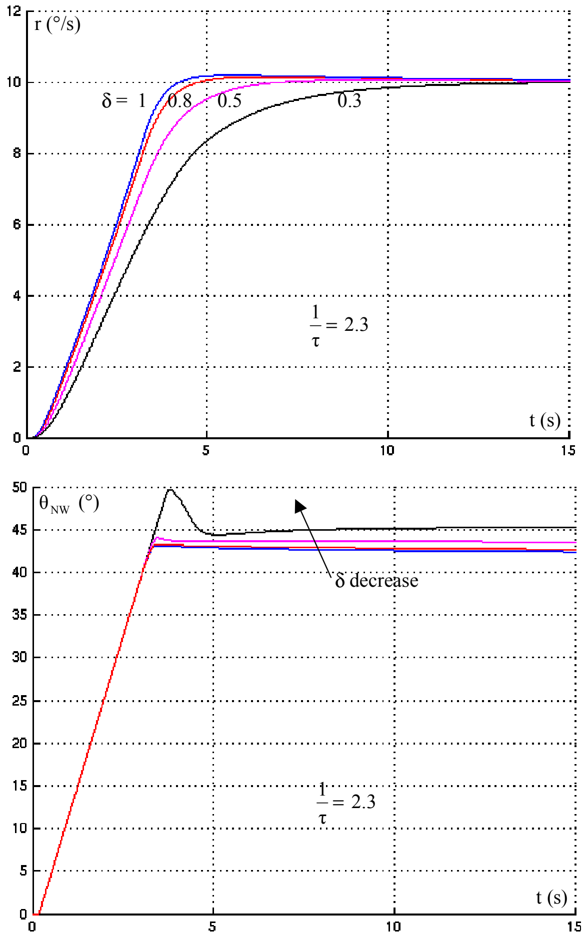

Fig. 4a and 4b. Responses to a yaw rate step solicitation with the additional integral feedback term.

5 Conclusion

This study shows that a yaw rate control law based on feedback linearization can allow the achievement of strong requirements. However, in case of important model uncertainties, robustness problems can occur.
The solution, detailed in this article, proposed to improve classical feedback linearizing control by using integral sliding mode control. It leads to design an additional integral feedback that increase overall robustness. In the case of the aircraft-on-ground yaw rate control, it allows to ensure the precision goal even in very harsh conditions.

The evaluation of the proposed control law is still in progress through an accurate validation on the Airbus reference simulator and is to be soon evaluated on aircraft.

References

[1] Villaume F. Contribution à la commande des systèmes complexes : Application à l'automatisation du pilotage au sol des avions de transport. *Ph.D. Thesis, LAAS - CNRS*, 2002.

[2] Duprez J, Mora-Camino F and Villaume F. Control of the Aircraft-on-Ground Lateral Motion During Low Speed Roll and Manoeuvres. *IEEE Aerospace Conference*, Big Sky (MT), paper 7.1302, 2004.

[3] Klyde D.H, Myers T.T, Magdaleno R.E, Reinsberg J.G. Identification of the dominant ground handling characteristics of a navy jet trainer. *AIAA-2000-3903 A00-39679*

[4] Bakker E, Pacejka H.B, Lidner L. A new Tire Model with an Application in Vehicle Dynamics Studies. *SAE paper 890087*.

[5] Bakker E, Nyborg L, Pacejka H.B. Tyre Modelling for Use in Vehicle Dynamics Studies. *SAE paper 870421*.

[6] AGARDograph 333. Enhancement of aircraft ground handling simulation capability. *AGARD-AG-333*.

[7] Clot J et al. Système d'alarme et de sécurité active pour la conduite automobile. *Rapport LAAS - CNRS n°98441*, Oct. 1998.

[8] Anquetin C. Identification et modélisation du comportement des pneumatiques de l'A320 sur piste sèche et mouillée. *Dissertation, ENSEEIHT*, 2002.

[9] Duprez J, Mora-Camino F and Villaume F. Aircraft-on-Ground Lateral Control for Low Speed Maneuvers. *16th IFAC Symposium on Automatic Control in Aerospace*, St. Petersburg (Russia), 2004.

[10] Garcia J.P, Martin D. System Description Note (SDN) - A320 - ATA32-20/40/50 relating to : Nose Landing Gear Wheels and Brakes / Steering. *Airbus Document.*

[11] Duprez J. Optimisation de la loi de commande du système d'orientation des roues avant sur A320. *DEA Dissertation, LAAS - CNRS*, Oct. 2002.

[12] Isidori A, Krener A.J, Gori-Ggiorgi C, Monaco S.V. Nonlinear decoupling via feedback differential geometric approach. *IEEE Transactions on Automatic Control, Vol. 26*, 1981.

[13] Isidori A. *Nonlinear Control Systems.* 3rd edition. Belin : Springer-Verlag, 1995.

[14] Slotine J.J.E, Li W. *Applied Nonlinear Control*, 1st edition. Prentice-Hall, 1991.

[15] Slotine J.J.E. Sliding controller design for non-linear systems. *Int. J. Control,* Vol. 40, No. 2, 421-434, 1984.

[16] Fernandez B. R, HEDRICK J. K. Control of multivariable non-linear by the sliding mode method. *Int. J. Control,* Vol. 46, No. 3, 1019-1040, 1987.

[17] Utkin V.I. Variable structure systems with sliding Mode : a survey. *IEEE Transactions on Automatic Control, Vol. 2*, 1977

[18] Utkin V.I. Sliding Modes and their Application in Variable Structure Systems (1974). Translated from Russian by A. Parnakh. *Automn remote Control, Vol. 44, page 1105*, 1984.

[19] Mora-Camino F, Achaibou A. K. Zero-gravity atmospheric flight by robust nonlinear inverse dynamics. *J. of Guidance, Control, and Dynamics, Vol. 16, pages 604-607,* 1993.