

MODIFIED PROPORTIONAL NAVIGATION FOR A MISSILE WITH VARYING VELOCITY - COMPARISON WITH THE OPTIMAL GUIDANCE -

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Abstract

This paper deals with the guidance law for a missile with varying velocity, especially during the boost phase. First, the two methods to predict the missile flight distance during the boost phase are shown. Second, the new guidance law that makes the missile fly on the collision course with the target is proposed. The proposed guidance law takes into account the future intercept point and is mechanized by combining the proportional navigation and the pure pursuit navigation with the mixture ratio. Since the performance of the guidance law depends on the ratio, the discussion is focused on the determination of the ratio. Finally, some simulation results show that the proposed guidance law can make the miss distance small with the small lateral divert requirements over the wide range of off-boresight angle and has the similar results achieved with the optimal collision course.

Nomenclature

a_M : deceleration due to air drag
 C_{D0} : zero-lift drag coefficient
 a_{MP}, C_{DP} : design parameter
 $C_{L\alpha}$: lift curve slope
 D : air drag
 \vec{F} : desired acceleration command vector
 g : gravitational acceleration
 I_{SP} : specific impulse
 k : mixture ratio
 L : lift
 m : mass of a missile

N : navigation constant
 N_e : effective navigation constant
 \vec{P} : position vector of the intercept point with respect to the present target position, $|\vec{P}| = P$
 \vec{Q} : position vector of the intercept point with respect to the present missile position, $|\vec{Q}| = Q$
 \vec{R} : relative distance vector, $|\vec{R}| = R$
 S : reference area
 T_h : thrust
 t_B : burnout time
 t_F : intercept time
 t_{go} : time-to-go
 V_c : closing velocity along the LOS
 \vec{V}_M : missile velocity vector, $|\vec{V}_M| = V_M$
 \vec{V}_{MC} : correct missile velocity vector, $|\vec{V}_{MC}| = V_{MC}$
 $\Delta\vec{V}_M$: $\vec{V}_M - \vec{V}_{MC}$
 \vec{V}_T : target velocity vector, $|\vec{V}_T| = V_T$
 x, y : down and cross range
 α : angle of attack
 γ : flight-path angle
 $\dot{\gamma}$: flight-path rate vector
 ε : off-boresight angle
 κ : induced drag coefficient
 ρ : air density
 σ : LOS angle
 $\dot{\sigma}$: LOS rate vector
 $\tau(\)$: time constant
 ϕ_M : the angle between \vec{V}_M and the LOS
 ϕ_T : the angle between \vec{V}_T and the LOS

1 Introduction

The proportional navigation (PN) is widely used as the guidance law for a missile because of the effectiveness and the easiness of the implementation. If the velocities of a missile and a target are constant and the target does not perform an evasive maneuver, PN is the most suitable guidance law for a missile [1]. However, if the velocity of a missile varies significantly, the performance of PN is seriously degraded because the collision course of the varying speed missile with a target is not the same as that of PN. Some papers on the guidance problem of variable speed missiles have been published. Riggs [2] used the linear optimal control theory to solve the problem. Cho et al. [3],[4] also used the linear optimal control and showed a new guidance law that had the similar structure to the augmented proportional navigation. On the other hand, the authors presented a new guidance law for a missile with varying velocity [5],[6], which is not based on an optimal guidance. This guidance law is derived by estimating the external force exerted on the missile after launch, computing the future intercept point and guiding the missile on the collision course to the point. This is mechanized by combining PN and the pure pursuit navigation (PPN) with the mixture ratio and thus the performance of the law strongly depends on the ratio.

There are two cases where the short range air-to-air missile (SRAAM) speed changes largely. One is the boost phase and the other is the deceleration phase after the rocket motor burns out. Since the missile speed change during the boost phase is larger than the deceleration phase, this paper deals with the boost phase. First, we estimate the future position of the missile and the target and then calculate the future intercept point during the boost phase. Second, the guidance law for a missile to the future intercept point is presented. Finally, some simulations are performed using a two-dimensional engagement model to show the effectiveness of the guidance law presented. Especially let us focus the discussion on the

determination of the mixture ratio and the comparison with the optimal collision course.

2 Calculation of Future Intercept Point

2.1 Calculation of Time-to-go

Figure 1 shows the geometrical relationship between a missile and a target. The points M and T represent the actual position of a missile and a target, the point I is the future intercept point and thus the triangle TMI is the collision triangle. The following equation must be satisfied in order to form the collision triangle.

$$Q^2 - P^2 - R^2 - 2PR \cos \phi_r = 0 \quad (1)$$

Since Q and \bar{P} are functions of time-to-go, Eq.(1) is the basic equation for computing the time-to-go. Here, the relative distance vector \bar{R} , the missile velocity vector \vec{V}_M and the target flight direction are to be measured but the target position vector \bar{P} and the missile flight distance Q must be estimated.

2.2 Prediction of Target Position

First, let us predict the target position \bar{P} at the time-to-go. If the target has constant speed and does not perform evasive maneuvers on the two-dimensional plane of the constant altitude, \bar{P} can be written as:

$$\bar{P} = \vec{V}_T t_{go} \quad (2)$$

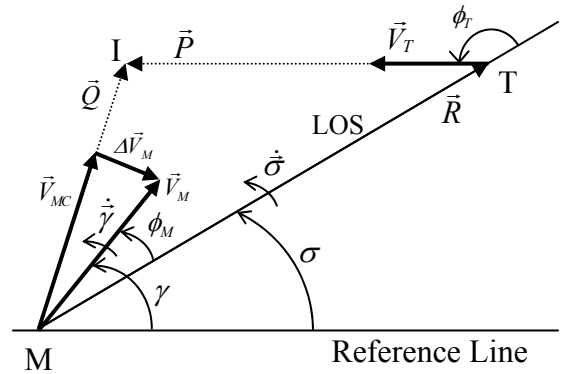


Fig. 1. Geometrical Relationship between Missile and Target

2.3 Prediction of Missile Flight Distance

Next, let us predict the missile flight distance Q during the time-to-go. As we are considering a constant-altitude missile in this paper, the gravity can be neglected. The external forces acting on the missile during the boost phase are the thrust and the air drag. Thus, we have

$$m(t)\dot{V}_M = T_h - \frac{1}{2}\rho V_M^2 S C_D(V_M) \quad (3)$$

where

$$\begin{aligned} m(t) &= m_0 + \dot{m}t \\ &= m_0 - \frac{T_h}{I_{SP}g}t \end{aligned} \quad (4)$$

I_{SP} is a specific impulse and m_0 is an initial mass of the missile. The thrust T_h , the air density ρ and the time derivative of the missile mass \dot{m} are constant during the boost phase. Since the drag coefficient $C_D(V_M)$ changes with the missile velocity, the correct closed-form solution of the missile flight distance Q can not be derived. So let us derive the approximate closed-form solution. The two methods are shown as follows.

(a) Method 1

From Eq.(3), we have

$$\dot{V}_M = \frac{T_h}{m(t)} - a_M \quad (5)$$

where

$$a_M = \frac{1}{2m(t)}\rho V_M^2 S C_D(V_M) \quad (6)$$

Assuming that the deceleration a_M is constant, we can integrate Eq.(5) and obtain

$$V_M(t) = V_M(0) + I_{SP}g \log \left| \frac{t_R(0)}{t_R(0) - t} \right| - a_M t \quad (7)$$

where

$$t_R(t) = \frac{I_{SP}m(t)g}{T_h} = \frac{I_{SP}m_0g}{T_h} - t \quad (8)$$

$$I_{SP} = -\frac{T_h}{\dot{m}g} \quad (9)$$

Integrating both sides of Eq.(7), we obtain the flight distance Q during the time-to-go t_{go} as follows:

$$\begin{aligned} Q &= V_M(0)t_{go} - \frac{a_M}{2}t_{go}^2 \\ &+ I_{SP}gt_{go} \left\{ 1 - \frac{t_R(0) - t_{go}}{t_{go}} \log \left(\frac{t_R(0)}{t_R(0) - t_{go}} \right) \right\} \end{aligned} \quad (10)$$

Equation (10) gives the good estimate for the flight distance if the missile velocity change is small. When the velocity change is large, however, since we cannot assume that a_M is constant, Eq.(10) does not show the good estimate for the flight distance.

(b) Method 2

Though a_M is assumed to be constant in the Method 1, let us consider C_D is constant and integrate Eq.(3) in the Method 2. From Eq.(3), we obtain

$$\frac{\dot{V}_M}{T_h - \omega V_M^2} = \frac{1}{m(t)} \quad (11)$$

where

$$\begin{aligned} \omega &= 0.5\rho S C_D \\ &\doteq 0.5\rho S C_{D0} \end{aligned} \quad (12)$$

Since ω is considered to be constant, both sides of Eq.(11) can be integrated as follows:

$$\int_{V_M(0)}^{V_M(t)} \frac{dV_M}{T_h - \omega V_M^2} = \frac{1}{\dot{m}} \int_{m_0}^m \frac{dm}{m} \quad (13)$$

The integration of the left term of Eq.(13) is given by

$$\int_{V_M(0)}^{V_M(t)} \frac{dV_M}{T_h - \omega V_M^2} = \frac{-1}{2\sqrt{\omega T_h}} \log \left| \frac{V_M - \eta}{V_M + \eta} \right| + C_1 \quad (14)$$

where

$$\eta = \sqrt{\frac{T_h}{\omega}} \quad (15)$$

$$C_1 = \frac{1}{2\sqrt{\omega T_h}} \log \left| \frac{V_{M0} - \eta}{V_{M0} + \eta} \right| \quad (16)$$

The right term of Eq.(13) is integrated as follows:

$$\frac{1}{\dot{m}} \int_{m_0}^m \frac{dm}{m} = \frac{t_M}{m_0} \log \left(\frac{t_M}{t_M - t} \right) \quad (17)$$

where

$$t_M = \frac{I_{SP} m_0 g}{T_h} \quad (18)$$

From Eq.(14) and Eq.(17), we have

$$\frac{-1}{2\sqrt{\omega T_h}} \log \left| \frac{V_M - \eta}{V_M + \eta} \right| + C_1 = \frac{t_M}{m_0} \log \left(\frac{t_M}{t_M - t} \right) \quad (19)$$

Considering $V_M < \eta$, Eq.(19) can be transformed to

$$\frac{-\eta + V_M}{\eta + V_M} = p^{-h} \times e^{2C_1 \sqrt{\omega T_h}} \quad (20)$$

where

$$h = 2t_M \sqrt{\omega T_h} / m_0 \quad (21)$$

$$p = 1 - t / t_M \quad (22)$$

Let us define ν as follows:

$$\nu = e^{2C_1 \sqrt{\omega T_h}} = \frac{\eta - V_{M0}}{\eta + V_{M0}} \quad (23)$$

Substituting Eq.(23) into Eq.(20) and rearranging it, we obtain V_M as follows:

$$V_M(t) = \frac{\eta}{1 + \nu p^h} - \frac{\eta \nu p^h}{1 + \nu p^h} \quad (24)$$

Integrating both sides of Eq.(24), we obtain the flight distance Q as follows:

$$Q = \eta t_M \xi \int_{p_{t_{go}}}^1 \left[\frac{1}{p^h + \xi} - \frac{\nu p^h}{p^h + \xi} \right] dp \quad (25)$$

where

$$\xi = \frac{1}{\nu} \quad (26)$$

$$p_{t_{go}} = 1 - \frac{t_{go}}{t_M} \quad (27)$$

Since h , ξ and ν are constant, the integration of Eq.(25) can be done using a hyper-geometric function G , that is,

$$Q = \eta t_M \left\{ p_{t_{go}} - 1 + 2G \left(\frac{1}{h}, 1, 1 + \frac{1}{h}; -\nu \right) - 2p_{t_{go}} G \left(\frac{1}{h}, 1, 1 + \frac{1}{h}; -\nu p_{t_{go}}^h \right) \right\} \quad (28)$$

Substituting Eq.(2) and Eq.(10) or Eq.(28) into Eq.(1) and solve it, we obtain t_{go} and thus \bar{P} and

Q are determined. This means that the future intercept point is predicted.

3 Modified Proportional Navigation

Since the future intercept point is predicted, let us drive the guidance law which guides the missile to the point. From Fig.1, the LOS rate is given by the following vector equation.

$$\dot{\sigma} = \frac{\bar{R} \times (\bar{V}_T - \bar{V}_{MC})}{R^2} + \frac{\bar{R} \times (-\Delta \bar{V}_M)}{R^2} \quad (29)$$

The first term of Eq.(29) represents the LOS rate of the missile flying on the collision course and the second term represents the one due to the deviation from the collision course. If a missile is guided with a flight-path rate in proportion to the second term of Eq.(29), assuming no missile dynamic lags, the flight-path rate of the missile becomes

$$\dot{\gamma} = N \frac{\bar{R} \times (-\Delta \bar{V}_M)}{R^2} \quad (30)$$

where N is the navigation constant, which is related to the effective navigation constant N_e as follows:

$$N = \frac{N_e V_c}{V_M \cos \phi_M} \quad (31)$$

Thus, the required lateral acceleration command for a missile to fly along the correct collision course is given by the following equation [5],[6]:

$$\bar{F} = \frac{N}{R^2} \left\{ \left(\bar{V}_M - \frac{V_M}{Q} \bar{P} \right) \times \bar{R} \right\} \times \bar{V}_M \quad (32)$$

In general, however, the missile uses the LOS rate as the guidance information. So, Equation (32) is not an adequate form to implement. Substituting Eq. (2) into Eq. (32) and arranging it, we obtain

$$\bar{F} = Nk \frac{(\bar{V}_M - \bar{V}_T) \times \bar{R}}{R^2} \times \bar{V}_M + N(1-k) \frac{\bar{V}_M \times \bar{R}}{R^2} \times \bar{V}_M \quad (33)$$

where the mixture ratios k is defined by the following equations:

$$k = \frac{V_M t_{go}}{Q} \quad (34)$$

The first term of Eq. (33) represents PN with the navigation constant Nk and the second term represents PPN with the navigation constant $N(1-k)$. Figure 2 shows the block diagram for this guidance law. Since this is considered to be one of the modifications of PN, we call this the modified proportional navigation (MPN) for convenience. As is obvious from Eq. (33) or Fig.2, the performance of MPN largely depends on the mixture ratio k , which is given by Eq. (34) and changes with time. This means that it is very difficult to determine the mixture ratio k directly. Since the mixture ratio k is a function of a_M or C_{D0} , however, the parameters a_M and C_{D0} can be used as the design parameters to determine the mixture ratio k . So we use a_{MP} and C_{DP} instead of a_M and C_{D0} , respectively, to confirm that a_{MP} and C_{DP} are the design parameters for k . As is shown later, changing the parameter a_{MP} or C_{DP} depending on the position and the launch angle makes the miss distance nearly zero over the wide range of off-boresight angle.

4 Optimal Trajectory

One of the most interesting missile flight trajectory is the optimal trajectory which minimize the following performance index

$$J = wR(t_F) + \int_0^{t_F} F^2 dt \quad (35)$$

where w is the weight on the miss distance $R(t_F)$. In order to compare the trajectory of MPN with the optimal trajectory, we solve the

Table 1. Parameters of Missile

$m_0 = 94$ kg	$t_B = 5.5$ s
$m_{tB} = 63$ kg	$N_e = 3.5$
$T_h = 12740$ N	Load limit = ± 30 G
$S = 0.0127$ m ²	Disable time = 0.5 s
$I_{SP} = 230.6$ s	Blind distance = 100 m
$\tau_\alpha = 0.4$ s	$\dot{m} = -5.64$ kg/s

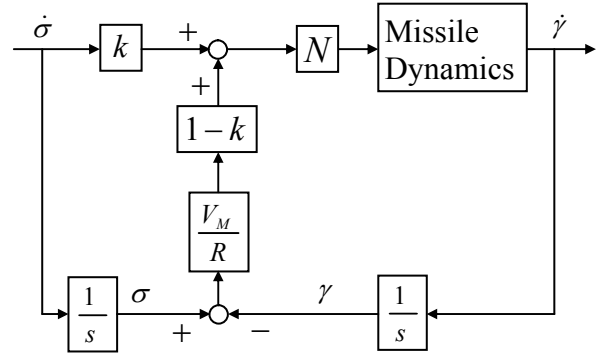


Fig. 2. Block Diagram for MPN

nonlinear programming problem (NLP) which is the minimization of Eq.(35). The practical computation is performed using the sequential quadratic programming (SQP) algorithm, EZopt of U.S.A. AMA Company.

5 Simulation

5.1 Calculation of Missile Flight Distance

Some simulations are performed to examine the accuracy of the two approximate solutions for missile flight distance Q presented in section 2.3. We assume that the missile is flying on the straight line at the constant altitude of 5000m. The initial Mach number is set to 0.75. The parameters of the missile for this simulation are shown in Table 1 and Fig.3. The parameter C_{DP} is set to 1.54 which is the mean value of the aerodynamic coefficient C_{D0} of Fig.3. The parameter a_{MP} is set to 4.42 using the following

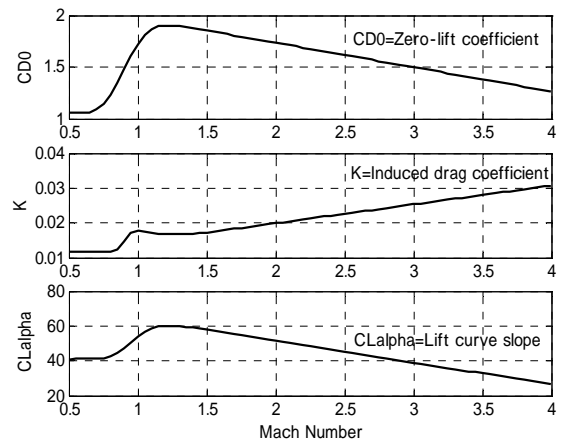
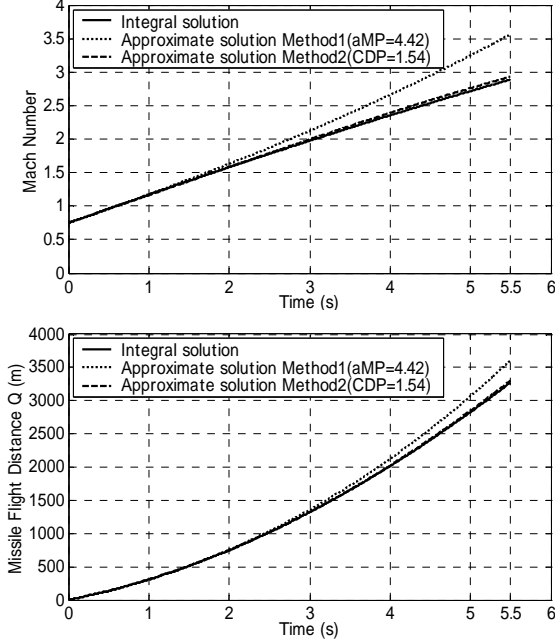


Fig. 3. Aerodynamic Coefficients of Missile


 Fig.4. Mach Number and Q

equation:

$$a_{MP} = \frac{\rho V_{M0}^2 S C_{DP}}{2m_0} \quad (36)$$

Figure 4 shows the time histories of the Mach number and the missile flight distance Q . From Fig.4, we see that the approximate solutions by Method 2 (dashed lines) show the good agreement with the integral solutions (solid lines), that is, the true solutions. On the other hand, the errors of the approximate solutions by Method 1 (dotted lines) increase gradually as the flight time increases.

5.2 Engagement Simulation

The proposed guidance law is applied to the model of SRAAM and examined through mathematical simulations. The simulation results are compared with those achieved with PN. We assume that both a missile and a target are point mass. Their trajectories are limited on a two-dimensional plane at the constant altitude of 5000m. The missile dynamics in Fig.2 is given by the first-order lag with time constant τ_α . The initial Mach number of the missile is set to 0.75. On the other hand, the target's Mach number is constant at 0.75 and the target does not perform evasive maneuvers. Figure 5 shows

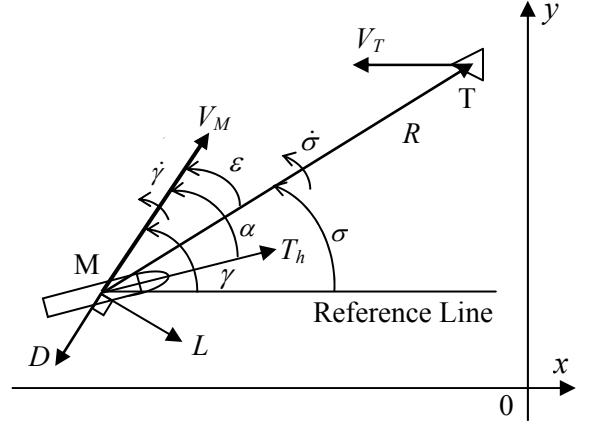


Fig. 5. Symbols for Missile and Target

symbols for the missile and the target and the equations of motion of the missile are as follows:

$$\dot{x}_M = V_M \cos \gamma \quad (37)$$

$$\dot{y}_M = V_M \sin \gamma \quad (38)$$

$$m\dot{V}_M = T_h \cos \alpha - D \quad (39)$$

$$mV_M \dot{\gamma} = T_h \sin \alpha + L \quad (40)$$

$$m = m_0 + \dot{m}t \quad (41)$$

$$\dot{\alpha} = -\frac{1}{\tau_\alpha} \alpha + \frac{1}{\tau_\alpha} \alpha_c \quad (42)$$

$$L = 0.5 \rho V_M^2 S C_{L\alpha} \alpha \quad (43)$$

$$D = 0.5 \rho V_M^2 S (C_{D0} + \kappa C_{L\alpha}^2 \alpha^2) \quad (44)$$

$$\alpha_c = 2mF / (\rho V_M^2 S C_{L\alpha}) \quad (45)$$

From Eq.(33) or Fig.2, the commanded acceleration F for MPN is given by

$$F = NkV_M \dot{\sigma} + N(1-k)V_M \frac{V_M \sin(\sigma - \gamma)}{R} \quad (46)$$

Here, k is determined from Eq.(34) but there are two methods to compute Q , that is, the Method 1 of Eq.(10) and the Method 2 of Eq.(28). The Method 1 needs the value of a_{MP} and the Method 2 needs the value of C_{DP} . In order to distinguish these two methods, the MPN which uses the Method 1 to compute k is called MPN1 and the MPN which uses the other method is called MPN2.

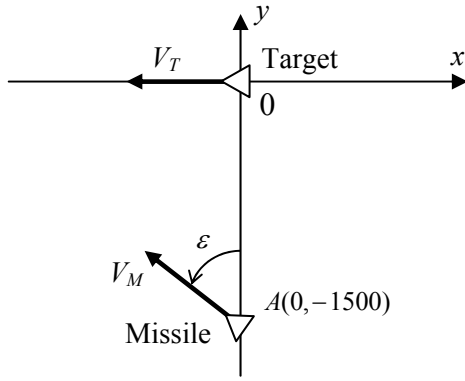


Fig. 6. Initial Geometrical Conditions for Missile and Target

On the other hand, F for MPN is given by

$$F = NV_M \dot{\sigma} \quad (47)$$

As mentioned before, the data for simulations are given in Table 1 and Fig.3. Figure 6 shows the initial geometrical conditions for the missile and the target.

First, let us show the concrete way of choosing the design parameters a_{MP} and C_{DP} . Performing a lot of simulations changing the parameters a_{MP} and C_{DP} with respect to the various off-boresight angles ϵ , we obtain Fig.7(a) and Fig.7(b). Figures 7(a) and 7(b) depict the contour lines of the miss distance(MD) in ϵ - a_{MP} plane and ϵ - C_{DP} plane, respectively. From these figures, the appropriate design parameters a_{MP} and C_{DP} for the given ϵ can be chosen so as to make the miss distance nearly zero or small. And also we know the following from Figs.7(a) and 7(b). If the contour lines are sparse, that is, the space between the each other's contour lines is wide, the relevant MPN is robust with respect to the change of ϵ . And if the contour lines are extended vertically, the relevant MPN has good robustness with the parameter a_{MP} or C_{DP} . Therefore, from Fig.7(a) it is said that the MPN1 has good robustness with respect to the change of ϵ but does not have enough robustness with a_{MP} because the contour lines are inclined to the right. On the other hand, Fig.7(b) shows that the MPN2 has better robustness with ϵ and C_{DP} than the MPN1 because the contour lines are sparse and nearly vertical.

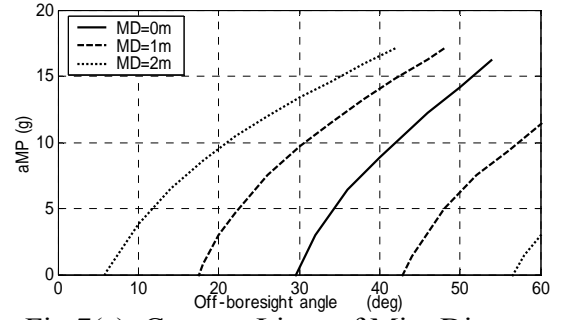


Fig.7(a). Contour Lines of Miss Distance in ϵ - a_{MP} Plane with MPN1

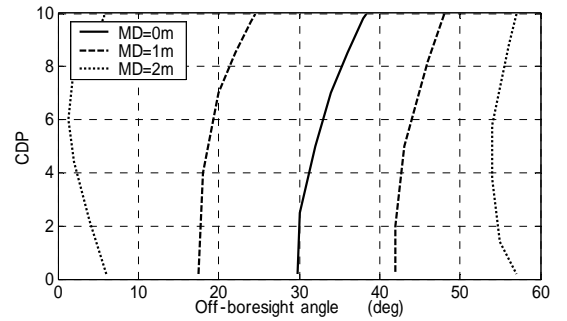


Fig.7(b). Contour Lines of Miss Distance in ϵ - C_{DP} Plane with MPN2

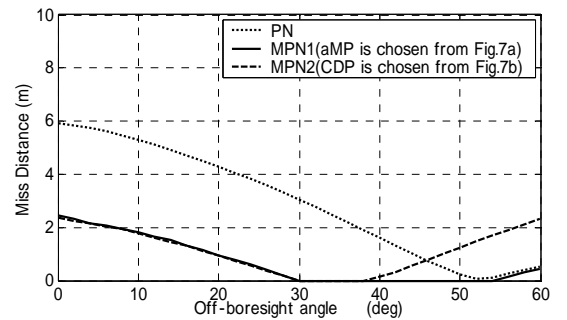


Fig.8(a). Off-boresight Angles vs. Miss distance

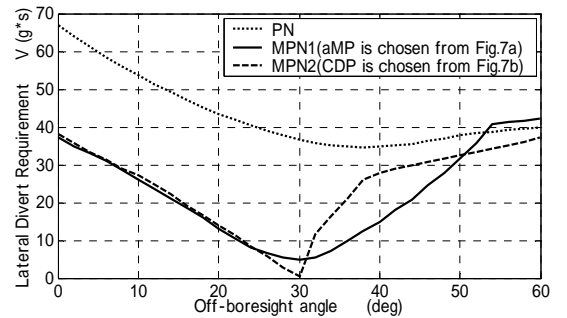


Fig.8(b). Off-boresight Angles vs. Lateral Divert Requirement ΔV

Next, let us compare the off-boresight ability of MPN with that of PN. Figure 8 shows the MD and the lateral divert requirement ΔV for various ϵ . ΔV is the time integration of the missile commanded acceleration as follows:

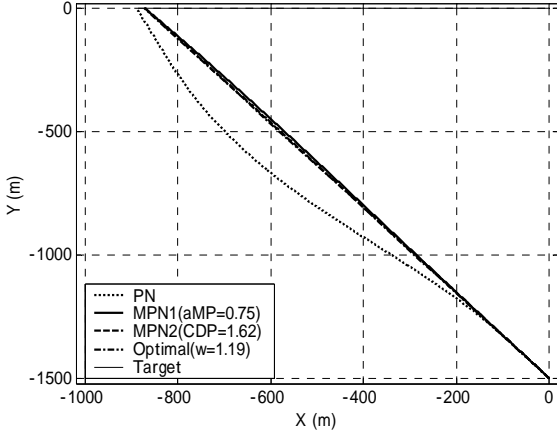


Fig.9(a). Flight Trajectories of Missile and Target ($\varepsilon = 30\text{deg}$)

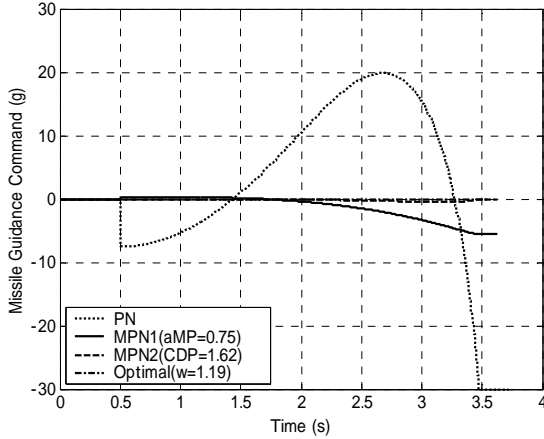


Fig.9(b). Time Histories of Missile Guidance Command ($\varepsilon = 30\text{deg}$)

Table 2. Miss Distance ($\varepsilon = 30\text{deg}$)

PN	3.1 m
MPN1($a_{MP}=0.75$)	0 m
MPN2($C_{DP}=1.62$)	0 m
Optimal($w=1.19$)	0 m

$$\Delta V = \int_0^{t_f} |F| dt \quad (48)$$

From Fig.8(a), we see that the correct off-boresight angle which makes MD zero with PN is only 52 deg. As is also evident from Fig.8, the miss distance with MPN1 is zero over the off-boresight angle range of 30 deg to 54 deg using the appropriate a_{MP} chosen from Fig.7(a). The miss distance with MPN2 is zero over the off-boresight angle range of 30 deg to 38 deg using the appropriate C_{DP} chosen from Fig.7(b). These

results show that MPN1 can make the miss distance zero over the wider range of ε than the MPN2 by adjusting the parameter a_{MP} . However, from Fig.8(b), it is clear that the MPN2 has the straight collision course but the MPN1 and PN do not have such kind of courses, because the ΔV achieved with the MPN2 for the ε of 30deg is zero but the ΔV with MPN1 or PN never turns out to be zero for all ε .

Finally, let us compare the trajectories of MPN1, MPN2 and PN with the optimal trajectory. The off-boresight angle is set to 30deg. Figure 9 displays the flight trajectories of the missile and the target and the time histories of missile guidance command. The miss distances of these simulations are shown in Table 2. From Fig.9, the trajectory and the command with MPN2 coincide well with those of the optimal guidance and the trajectory with MPN1 is slightly curved with the small command. But the trajectory of PN missile curves largely and thus the command of PN is large.

6 Conclusion

In this paper, the two methods to predict the missile flight distance during the boost phase are presented. The first method, Method 1, is obtained assuming the deceleration due to the air drag is constant and the second one, Method 2, assumes that the drag coefficient C_D is constant. Though the errors of the Method 1 increase gradually as the flight time increases, the Method 2 shows the good estimation. These methods are used to predict the future intercept point and then the new guidance law which guides the missile to the point is presented. This guidance law is mechanized by combining the proportional navigation and the pure pursuit navigation with the mixture ratio k . The guidance law presented is called the modified proportional navigation (MPN) and the MPN which uses the Method 1 to compute k is called MPN1 and the MPN which uses the Method 2 is called the MPN2. MPN1 and MPN2 have the high off-boresight ability comparing with the proportional navigation. As for MPN2,

especially, if the off-boresight angle is the correct lead angle, the trajectory achieved with MPN2 coincides with that of the optimal guidance.

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