STUDY OF THE EFFECTS OF SYNTHETIC JET ACTUATORS ON THE DELAY OF TRANSITION OF THE BOUNDARY LAYER DEVELOPED ON A FLAT PLATE

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Abstract

This work has as a fundamental objective the study of the effects of synthetic jet actuators on the flow of the boundary layer developed on a flat plate. The present work means to obtain computational and experimental data that indicate how this effect may be used as a means of flow control. The interaction of synthetic jets with traverse flow can lead to an apparent modification in the aerodynamic shape of blunt bodies and, in that way, supply a means of control of transition within the boundary layer. The main focus of the current study is in the description of the dynamics of the synthetic jet in the presence of external flow. Recent studies demonstrate that different types of flows may be produced by the actuator, depending on the amplitude of oscillation of the membrane. The case of flow generated using a synthetic jet actuator with a circular orifice is investigated. In the scaling of the geometric characteristics of the cavity and of the orifice, some results obtained in other studies are measured, that consider the case of the diameter of the actuator being the same as that of the cavity. The flow in the cavity will be included as part of the direct numeric simulation of the synthetic jet flow. The fundamental advantage of this method is that the entire geometry of the synthetic jet, including the oscillating diaphragm, is modeled within the meshes of the numeric simulations.

1. Introduction

Several studies demonstrate that flow control can be used to delay the transition of the boundary layer. Experimental works has been done to demonstrate and evaluate drag reduction techniques that have definite possibilities of influencing, for instance, the project of an aircraft. The passive means are the forms of operation that do not interact dynamically with the flow, and the active means act in a dynamic way in order to modify with the flow. Through active flow control, synthetic jet actuators have been used; several experiments have demonstrated that it is possible this way to delay the transition within the boundary layer of the flow over airfoils. Other studies showed the use of the synthetic jet actuators in the control of separation of the boundary layer on airfoils.

The development of jets with zero liquid flow mass was the result of several investigations that employed several experimental and computational techniques and methods. Both methods, experimental and computational, are used in a complementary way. The experimental data provides not only the physical base for the study, but also a database, against which the computational data may be tested. The validated computational model may be used to examine details of the flow that cannot be solved experimentally.

The synthetic jet actuator may be considered an attractive device for future use for several reasons. The device is made-up by a cavity with a movable membrane connected to an orifice; it may be manufactured in small dimensions. It is of simple construction that it does not depend on of micro machining. With the above-mentioned attributes, it is easy to manufacture the actuators and to modify their geometric parameters. Also, they operate with zero net mass flow, that is, they synthesize a jet from the flow field an external source of fluid. Thus, no piping system is necessary for the actuator, which greatly simplifies installation.

2. Flow control with an synthetic jet actuator

2.1 The evolution of the synthetic jet actuators in the flow control

The first studies of the use of synthetic jet actuators in active flow control were executed by Smith and Glezer [1]. This device when applied in a flow field, results in exclusive effects that are not possible with suction or blowing means in a fixed or pulsed process. The synthetic jet appeared as one of the most useful devices of active flow control, with a potential application in several techniques of flow control.

Some of these techniques are seen in studies that include transition control in Lorkowski et al. [2]. Separation delay, Sinha and Pal [3]. Effective vortex control, Roos [4]. Vectorial propulsion of jet motors, Smith and Glezer [5]. Mixture upgrading for active control of separation, Davis and Glezer [6] and turbulence in boundary layers, Crook et al. [7]. At this time, of all the flow control actuators being studied, most of these studies are addressed to the use of synthetic jets actuator.

Synthetic jets were used recently for the application to effective vortex control. However, there were a few investigations of those made by Smith and Glezer [8], in which they noticed that synthetic jets generated using rectangular plane orifices, exhibited a constant vortex close to the exit of the orifice. This was explained as being due to the turbulent dissipation of the centers of the vortices. They also noticed that auto-simulation of the flow is established closer to the orifice than for a fixed jet. This can be verified in studies on acoustic propagation, where this phenomenon appears when sound waves induce unidirectional flow, and it has a typically very weak effect unless the sound waves somehow attenuated. For synthetic jets, the reduction is provided by the orifice and also through viscous dissipation. However, the flow of the synthetic jet is not induced only by acoustic propagation, but also by the movement of a membrane.

Carpenter et al. [9] used a vorticity and velocity method to compute the whole flow. They ignored viscous effects and thought after a wrong initial estimate, that the velocity in the centerline of the flow underwent softened oscillations. In agreement with Kral et al. [10], numeric simulations of the flow of a synthetic jet, neglecting the cavity, showed in better agreement with the data of Smith and Glezer [5], when the flow was assumed to be completely turbulent. Cain et al. [11] extended this study to include compressibility effects. They verified that as the jet Mach number increased, the acoustic waves induced by the jet became much stronger. Bremhorst and Hollis [12] realized a study with flow control being executed through the use of pulsed fixed jets.

External acoustic excitement for separation control in an airfoil was investigated by Ahuja and Burrin [13], in which they explored the acoustic resonance in the test section of a wind tunnel to induce disturbances in the velocity of the flow, in a direction perpendicular to the flow.

According to Amitay et al. [14], synthetic jet actuators may be used for the control of flow separation on a circular cylinder. Depending on the azimuth position of the jets, a new aerodynamic profile for the cylinder was evident as an increase of base pressure and an asymmetrical pressure distribution, which grew around of the cylinder. This improved profile had a pressure drag coefficient 30% lower and a stable peek of 0.9 for the lift coefficient.

2.2 The mechanism of synthetic jet performance

The synthetic jet actuator consists of a membrane located at the base of a small cavity that possesses an orifice in the opposite face to that of the membrane. The membrane is forced to oscillate through electrical, magnetical or mechanical means. In the flow field created by the synthetic jet actuator, during a single cycle of operation, a succession of important events occur that may be observed. In a single oscillation cycle, fluid is expelled by the orifice as the membrane moves upward, as shown in the figure 1.

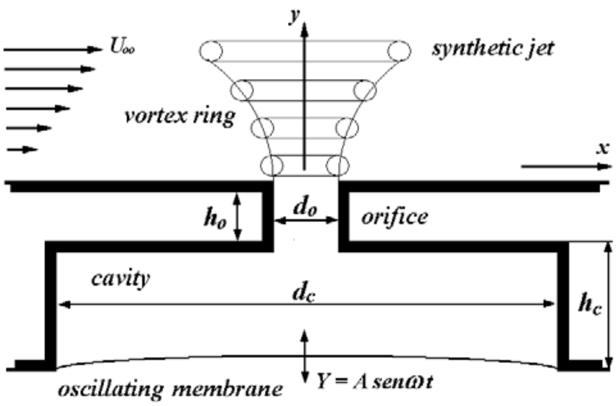

Fig.1. Operation of the synthetic jet actuator.

When the volume of the cavity decreases, the synthetic jet is in the expulsion cycle. During this interval of time, fluid begins to flow outside from the cavity through the orifice. This induces a vortex ring at the extremity of the orifice due to the separation of the flow. A boundary layer grows on the walls of the orifice. This boundary layer separates at the extremity of the orifice. The layer free from recently formed shearing moves as a vortex ring or, depending on the geometry of the orifice, as a pair of vortexes. The vortex ring moves in an outward direction, under its own pulse.
After the vortex ring moves a certain distance from the surface, the volume of the cavity begins to expand. The jet begins its suction cycle. When the membrane moves down, air is entrained into the cavity. The vortex ring continues moving further away from the extension of exit of the actuator. Fluid is being extracted from the atmosphere to fill the volume of the cavity that expands. If the vortex ring is at a certain distance from the orifice, this will not be influenced by the fluid entraining into the cavity. Once the volume of the cavity reached maximum, the cycle is ready to begin again. This cycle is repeated-exhaustion, ring formation, ring translation, suction-countless times in sequence of 100 to 2000 times a second. In this way, on top of a single period of oscillation of the membrane, although there is zero net mass flow inside or outside of the cavity, the average transfer of momentum is not null. The flow produced, where a transfer of momentum occurs; it is indeed a jet that was synthesized from the ambient fluid.

3. Experimental procedure

3.1 Variation of the geometric parameters

The synthetic jet will be generated in a way similar to previous investigations, using a construction device that only requires simple machining processes. In this way, it is easy to manufacture the actuators and to modify their geometric parameters. The dimensions of the actuator should vary with regard to the cavity and the orifice, according to the variables d_o, d_c, h_o and h_c, in figure 2.

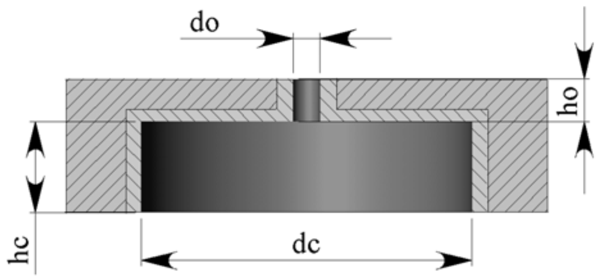

Fig. 2. Geometric parameters of the actuator.

With regard to the orifice, the diameter d_o will have the values: 1.5mm, 2.0mm and 2.5mm. The height of the orifice h_o should have the following dimensions: 1.5mm, 2.5mm and 3.5mm. With regard to the cavity, the diameter d_c should have the following values: 17.5mm, 22.5mm and 27.5mm. The height of the cavity

h_c will have the following dimensions: 3.5mm, 5.5mm and 7.5mm. These dimensions for the geometric parameters of the actuator will be used in certain combinations for comparison with experimental and numeric results of other studies.

3.2 Variation of the frequency and amplitude

The frequency of oscillation of the synthetic jet actuator should vary over a range of 200Hz to 800Hz. In several studies published on the use of synthetic jet actuators in active flow control, the highest frequencies did not surpass 2000Hz. In this study different frequencies of oscillation of the membrane of the synthetic jet actuator will be used, and will then be discussed the influences of this parameter on the result of the operation of the actuator. With regard to the amplitude of oscillation of the membrane of the actuator, in this study three different oscillation amplitudes will be used: 25mm, 50mm and 75mm. The variation of the oscillation amplitude is able to modify the flow in the cavity of the actuator of the synthetic jet and to influence in the resulting flow in the boundary layer. Studies of the influence of this variation demonstrate that high amplitudes produce a flow similar to a jet, while low amplitudes produce flow with a predominant suction. These values of the oscillation amplitude should be obtained for a definition of the frequencies to be used in the experiments.

3.3 Measurement of the amplitude

The oscillation of the membrane will be obtained by the use of vibration excitation using variable oscillation frequency and amplitude variable. The frequency that will be used for the vibration equipment will be produced by a digital sign generator that will then be amplified using an oscillation amplifier. The oscillation of the membrane produces a flotation of the pressure field in the cavity and in the orifice, causing these to act periodically as a source or a whirlpool. The amplitude of oscillation of the membrane should be measured optically using a high-velocity image. Displacement will be measured through digital images of the membrane in its maximum and minimum position of oscillation with the use of image processing software. The membrane of the actuator is a brass foil firmly attached to the perimeter. The oscillation will be executed using a piezo ceramic disk attached to the lower surface of the membrane. The reception for the piezo ceramic disk will be provided by a sign generator with a particular voltage of fixed and constant emission for each frequency used.

4. Numeric procedure

4.1 Parameters in the modeling of the meshes

The effects produced by the synthetic jet actuator in the resulting flow in the boundary layer depend on the parameters of the jet and of the orifice, however not, of course, effecting the parameters of the external flow present in the flow of the jet and in the flow in the cavity. This requires a necessary simulation of the flow inside of the cavity of the jet itself and this is one of the objectives of the present study. In the study of Rizzetta et al. [15], the diaphragm was modeled assuming a piston type movement, and this can lead to a significantly different flow inside the cavity. In the present simulations, the diaphragm is modeled in a more realistic way as the fundamental oscillations of a plate. Like this, the diaphragm has its maximum deflection at the center and null deflection at both extremities. In the simulations, the parameters that have a larger influence on the characteristics of the jet are focused. The parameters to be varied in the present study are the amplitude of the diaphragm, diameter and height of the orifice, diameter and height of the cavity and the Reynolds number in the thickness of the boundary layer of the external flow. The interaction of a synthetic jet modeled numerically as a boundary layer on a flat plate is investigated using a numeric method utilizing the two-dimensional incompressible Navier-Stokes equations. The equation of the continuity for the incompressible two-dimensional flow simulated in this study should be:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

In the numeric simulations realized in this study the equations are used in the formulation vorticidade/velocity. Adopting vorticidade as being the negative of the rotational of the velocity, the equation of vorticidade transport can be obtained:

$$\frac{\partial \omega_z}{\partial t}=-u\frac{\partial \omega_z}{\partial x}-v\frac{\partial \omega_z}{\partial y}+ +\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\omega_z}{\partial x^2}+\frac{\partial^2\omega_z}{\partial y^2}\right) \quad (2)$$

and a Poisson equation for the velocity v:

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=-\frac{\partial \omega_z}{\partial x} \quad (3)$$

4.2 Derivatives Time

In this study it is intended to capture the transient effects, and in order to simulate these effects appropriately, it is necessary to use increments of time Δt in relatively small numerical simulations, of the same order of greatness as that necessary for the numerical stability criteria to be satisfied. In this way, the explicit method of the Runge-Kutta type of fourth order is opted for. The equation of vorticidade transport is used to determine the value of the vorticidade components at each point of the computational domain in the time $t^{n+1}=t^n+\Delta t$, except within the contours. The method here described is executed in four steps, according to Ferziger [16], in agreement with the equations below:

$$\omega_z^{(n+1/2)*}=\omega_z^n+\frac{\Delta t}{2}f\left(t^n,\omega_z^n\right) \quad (4)$$

$$\omega_z^{(n+1/2)**}=\omega_z^n+\frac{\Delta t}{2}f\left(t^{n+(1/2)},\omega_z^{n+(1/2)*}\right) \quad (5)$$

$$\omega_z^{(n+1)*}=\omega_z^n+\Delta t f\left(t^{n+(1/2)},\omega_z^{n+(1/2)**}\right) \quad (6)$$

$$\omega_z^{(n+1)}=\omega_z^n+\frac{\Delta t}{6}[f\left(t^n,\omega_z^n\right)+ +2f\left(t^{n+(1/2)},\omega_z^{n+(1/2)*}\right)+ +2f\left(t^{n+(1/2)},\omega_z^{n+(1/2)**}\right)+ +f\left(t^{(n+1)},\omega_z^{(n+1)*}\right)] \quad (7)$$

4.3 Derivatives Spatial

For the calculation of the derivatives spatial in the directions x and y, compact finite differences were used. In this study a sixth order scheme was adopted that results in a tridiagonal array. For the closed point of the contour sixth order approaches are adopted and for points within the contour, fifth order approaches are used. For the calculation of the first and second derivatives, adopted matrices are used for the calculation of the derivatives in the x and y directions.

4.3.1 Calculation of the first derivatives

For a point on the contour, i=1, an off centre fifth order approach was adopted:

$$f'_1+4f'_2=\frac{1}{24h}(-74f_1+16f_2+72f_3+ -16f_4+2f_5)+O(h^5) \quad (8)$$

For a point close to the wall, i=2, an off centre sixth order approach was adopted:

$$f'_1+6f'_2+2f'_3=\frac{1}{120h}(-406f_1+ \\ -300f_2+760f_3-80f_4+30f_5+ \\ -4f_6)+O(h^6) \qquad (9)$$

For central points the following approach was adopted:

$$f'_{i-1}+3f'_i+f'_{i+1}=\frac{1}{12h}(-f_{i-2}-28f_{i-1}+ \\ +28f_{i+1}+f_{i+2})+O(h^6) \qquad (10)$$

For the case i=N and i=N-1 the approaches are similar to those obtained for the points i=1 and i=2, with inversion of the sign on the left-hand side of the equation.

4.3.2 Calculation of the second derivatives

For a point on the contour, i=1, an off centre fifth order approach was adopted:

$$13f''_1+137f''_2=\frac{1}{120h^2}(9775f_1+ \\ -20285f_2+11170f_3-550f_4-145f_5+ \\ +36f_6)+O(h^5) \qquad (12)$$

For a point close to the wall, i=2, an off centre sixth order approach was adopted:

$$f''_1+12f''_2+3f''_3=\frac{1}{360h^2}(4834f_1+ \\ -8424f_2+1890f_3+2320f_4-810f_5+ \\ +216f_6-26f_7)+O(h^6) \qquad (14)$$

For central points the following approach was used:

$$2f''_{i-1}+11f''_i+2f''_{i+1}=\frac{1}{4h^2}(3f_{i-2}+ \\ +48f_{i-1}-102f_i+48f_{i+1}+3f_{i+2})+ \\ +O(h^6) \qquad (15)$$

For the case i=N and i=N-1 the approaches are similar to obtained them for the points i=1 and i=2.

4.4 Modeling of meshes with movable areas

The current numerical effort describes the flow field inside the cavity of the actuator, as well as directly outside of it, by obtaining solutions for the two-dimensional incompressible Navier-Stokes equations. This is achieved for the internal flow by the use of a disturbed mobile sectorial mesh system. Solutions for the external jet flow field are considered in two spatial dimensions that use a compact implicit a high order finite difference scheme. The computational procedure will be evaluated, details of the calculations are presented, and the precision of the numerical results is evaluated by resolution of the mesh and studies of time step dimensioning. Characteristics of the resultants flow fields are examined, and comparison is made with experimental data.

5. Analysis of the results

5.1 Objectives of the analysis of operation of the synthetic jet actuator

This research effort focuses directly on the events that occur during the operation of a synthetic jet in the proximity of the orifice, with and without a cross-flow. In this sense, the objective is to understand the physics of the flow involved in the formation of the flow field of the synthetic jet and its interaction inside of the boundary layer on a flat plate. An increase in our knowledge with regard to this interaction will help in the development of actuators with increased effectiveness in flow control and it will also provide data that may be used in the development of these actuators for better flow control performance.

5.2 Comparison of the synthetic jet actuator with a fixed jet

Experimental and computational data indicate that the flow during the operation with synthetic jet actuators is self-similar up to a distance of approximately ten times the diameter of the orifice from the surface of the plate, according to the study of Mallinson et al. [17]. This contrasts with the fixed jet for which self-similar flow is not established until approximately forty times the diameter of the orifice. These observations were also made by Rathnasingham and Breuer [18] for actuators with a circular section, and Smith and Glezer [19] for actuators with rectangular sections.
Bremhorst and Hollis [12] showed that for a pulsed jet, in which the velocity varies ciclically between zero and some fixed value, the oscillations in the flow cause an increase in the entraining of ambient fluid in the jet that leads to a larger rate of mass flow. The data of Bremhorst and Hollis [12] are shown in the figure 3. These results indicate that the oscillations in the flow induced by the synthetic jet have a slightly larger effect on the flow volume than that of the pulsed jet.

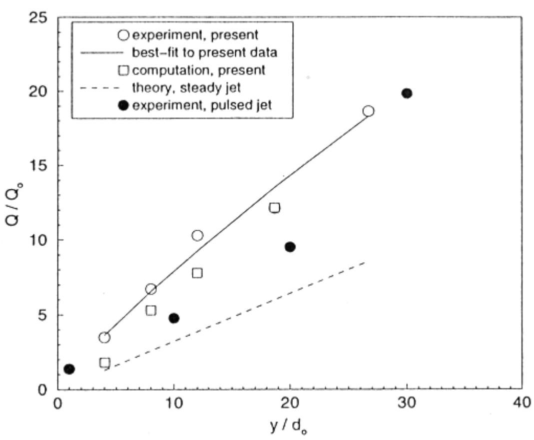

Fig. 3. Mass flow rate distribution.

Figure 4 of the study of Gilarranz et al. [20] displays a comparison of the velocity profiles at the moment of suction and maximum expulsion for each one of the two operational conditions of the actuator. It may be observed that for operation at an amplitude of 75mm, and a frequency of 10Hz, the velocity profile at maximum expulsion is very different from that corresponding to the maximum suction. In reality, for this case, the expulsion is only distributed in the area of flow directly below the orifice, reaching its maximum close to the centreline, while the suction velocity reaches its maximum close to the perimeter of the orifice wall, with its minimum close to the centreline. On the other hand, during operation at an amplitude of 25mm, and a frequency of 100 Hz, the velocity profiles for both conditions are very similar, reaching a maximum at the centreline of the orifice. During the expulsion cycle, the velocity profile also reaches its maximum close to the centreline. However, this is not the case in the 10Hz condition where maximum suction is reached close to the orifice wall instead of its centreline.

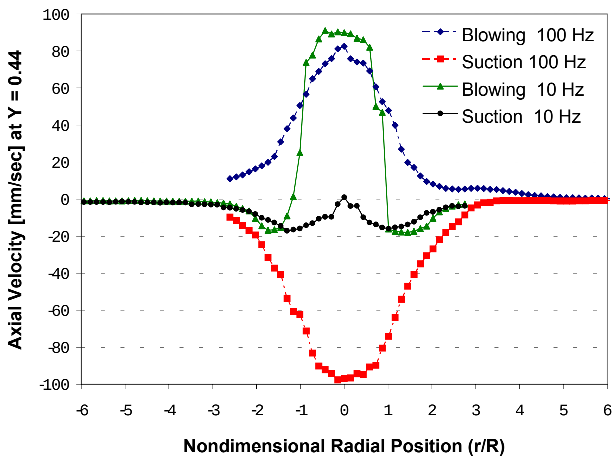

Fig. 4. Comparison between the axial velocity profiles for both operating conditions of the actuator.

5.3 Analysis of numeric results

Rizzetta et al. [15] included the flow of the cavity as part of the direct numerical simulations of synthetic jet flow and they verified that significant differentiation appeared when the flow of the cavity was neglected. Udaykumar et al. [21] developed a method that allows simulation of instable viscous incompressible flows with movable contour. The fundamental advantage of this method for the flow is that the whole geometry of the synthetic jet besides the diaphragm oscillating is modelled in the stationary Cartesian mesh, and an example of this mesh can be seen in the figure 5.

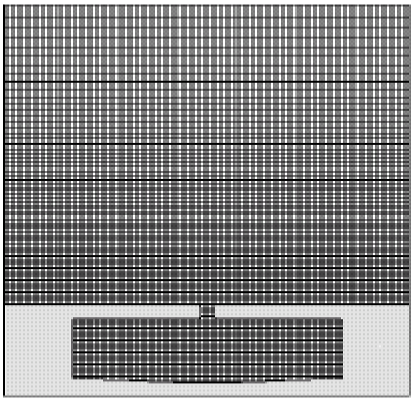

Fig.5. A fixed non-uniform Cartesian grid used in the synthetic jet calculation.

This method allows simulation of instable viscous incompressible flows with movable complex submerged borders in Cartesian meshes. In this way, the mesh does not need to adapt to the complex movable borders and this simplifies the grouping of the mesh in the flow domain. The method uses a central finite difference scheme with second order accuracy for space discretization and a system of explicit and implicit mixed fractional time progress steps. An efficient algorithm of several meshes is used for the solution of Poisson's pressure equation.

In the figure 6 of the study of Mital et al. [22] the current lines corresponding to the main flow are sketched. Some previous studies such as that of Chatlynne et al. [23], assumed the hypothesis that the synthetic jet is capable of altering the effective form of the body for the formation of an area of recirculation in the middle within the external flow that is significantly larger than the size of the jet. In figure 6a, it is observed that the relative velocity of the jet discharge creates a relatively large system of recirculation bubbles on the surface of the plate it glides with a length that is approximately ten times the diameter of the actuator. For the jets with lower velocity, seen in the figures 6b and 6c, no recirculation area is formed and the streamline are only lightly disturbed in the very close neighborhood of the exit of the jet. In this way, the synthetic jet is really capable of forming large areas of flow recirculation. However this capacity depends on the relationship of the jet with the velocity of the external flow.

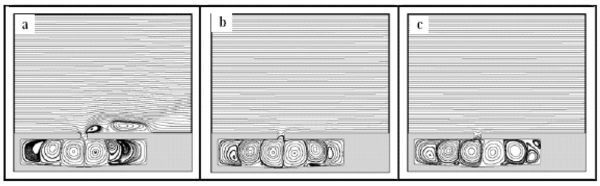

Fig. 6. The mean streamline plot of the average velocity for three Reynolds numbers.

Results of two-dimensional computations indicate that exact geometric elements of the cavity of the actuator were of vital importance in the determination of the profile at the exit of the mouthpiece and the subsequent formation of the synthetic jet. Also, the numerical aspect ratio of the exit of the jet was just one tenth of that of the experiment.

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