AIRCRAFT CONTROL USING THE ENGINES (JET PROPULSION SYSTEMS)

Romulus LUNGU*, Alexandru N. TUDOSIE*
***University of Craiova, Romania**

Keywords: *aircraft control, jet engine, pitch angle, sliding, banking, thrust*

Abstract

The paper presents the possibility of aircraft control using its engines (propulsion systems), when the flight commands are inactive (or damaged). An automatic system can be imagined and realized, which can assure the aircraft (airplane) control; the stick and the rudder bar are no more the input signal sources for the aerodynamic surfaces (elevator, ailerons, rudder), but realize a complex input signal for the engines' regulators (controllers), using a signal forming block (BFS) and an on-board computer. The studied system can be applied for twin or more engines aircraft (airplanes).

One studies the aircraft movements (longitudinal and lateral), taking into account the engines' operating effects, which leads to the classic mathematical model's modifying. For both of the models one elaborates the linearised non-dimensional forms.

In order to estimate the performances, one has studied a passenger airplane's behavior.

The authors have established the new mathematical models, the command laws for engines and also have performed the simulation for the case of a line airplane (passenger plane).

1 Introduction

It is well known that an airplane (aircraft) horizontally flying, with all flight commands on neutral positions, if the engine's thrust grows, it tends to climb; similarly, thrust decreasing leads to the opposite effect, aircraft's descent.

If the plane is twin engine (or multi-engine), having the engines symmetrical mounted (on the wings, or on the rear fuselage), if one engine's thrust grows and the other one's diminishes (or is kept constant), the plane banks slowly and executes a slide-slip turn, toward the engine which thrust is lower.

For both of the situations, the thrust modifying generates the torque variation, which leads to the balance modifying and, consequently, the flight attitude modifying.

2 System presentation

Starting of these observations, one can realize the airplane's (aircraft's) control using the engines, when the flight commands are stucked up or inactive. Normally, if for a reason or another (hydraulic failure, servo-amplifier's stucking, or some command's chain's elements stucked up) the aircraft control becomes impossible and the crush is imminent. Even so, the pilot can regain the control and land safely if he could synchronize

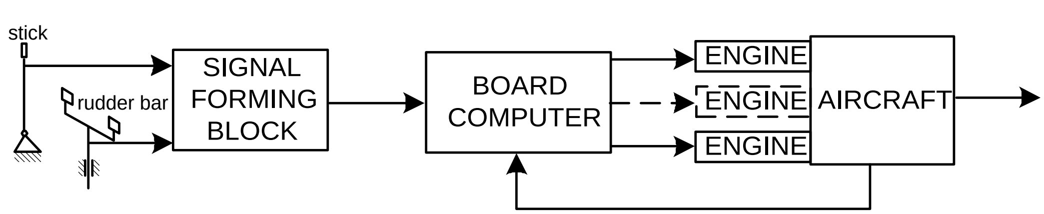

Fig.1 Control system's block diagram

the engines in order to realize the climbing/descent and turns. Although, it is very difficult for the pilot to determine properly the thrust exact values, the exact throttles positioning, because of the huge quantity of information which must be processed.

A new automatic system can be imagined, based on an unique command of the airplane-engines ensemble; using a board computer and specific software, the pilot can control the airplane (aircraft) using the engines. So, the stick and rudder-bar don't realize the input signals for the aerodynamic control surfaces, but the input signals for a complex signal forming block, which output represents the input signal for the board computer (engine's auto-pilot).

This one realizes the signals for the engines, that means the throttles positioning is less precisely and could lead to uncontrollable effects (because the engine's response time is bigger then the aerodynamic command's response time).

A practical solution for the control's improvement is the commands transmissions ratio's modifying; the stick's and rudder-barrel's tasks are undertaken by a control panel equipped with trimmers (in fact small control wheels, thumb-wheels meaned to preset the command parameters: flight speed, pitch angle, banding angle, flight course etc). These signals (parameters) will be processed by the board computer, together with the airplane's (aircraft's) flight parameters furnished as feed-beck signals.

From the pilot's point of view, this one will preset the input arguments (parameters) using the board panel or the stick's and rudder bar's displacement. The board computer realizes the transforming into signal for engines, with respect to the airplane's (aircraft's) flight altitude and attitude.

In order to emphasize the aircraft behavior when the engine's thrusts is modified, the system's mathematical model must be re-written, including the engine's operating effect.

3 Aircraft control

3.1 Longitudinal motion's control

The linearised non-dimensional mathematical model is [2]

$$\left[\frac{\mathrm{d}\hat{v}}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\alpha}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\theta}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\hat{\omega}_y}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\hat{H}}{\mathrm{d}\hat{t}}\right]^T = \hat{A}\left[\hat{v}\ \alpha\ \theta\ \hat{\omega}_y\ \hat{H}\right]^T + \hat{B}\left[\delta_m\ \delta_p\ z_1\ z_2\ z_3\ \upsilon_z\right]^T, \tag{1}$$

where

$$\hat{t} = \frac{t}{\tau_a},\ \hat{v} = \frac{\Delta v}{v^*},\ \hat{\omega}_y = \frac{\bar{b}}{v^*}\omega_y,\ \hat{H} = \frac{\Delta H}{\tau_a v^*}; \tag{2}$$

τ_a is the aerodynamic time constant, v*=const. – uniform horizontal flight's speed, v-flying speed, α-attack angle, θ-pitch angle, $\omega_y = \dot{\theta}$ - pitch speed, H-flight altitude, δ_m- engine's command,, δ_p- elevators angle, z_1, z_2, z_3-disturbances, $\upsilon_z = \frac{v_{vz}}{v}$, v_{vz}- wind vertical speed, $\hat{A}, \hat{B}$- matrix (5x5), respectively (5x6).

For a simplified form of the system, one ca renounce at "^", so the mathematical model becomes equivalent to

$$(\mathrm{s} - a_{11})v - a_{12}\alpha - a_{13}\theta = b_m\delta_m(\mathrm{s}) + z_1, \tag{3}$$

$$-a_{21}v + (\mathrm{s} - a_{22})\alpha - (\mathrm{s} - a_{23})\theta = z_2, \tag{4}$$

$$-a_{41}v - a_{42}\alpha + (\mathrm{s}^2 - a_{44}\mathrm{s} - a_{43})\theta = b_p\delta_p(\mathrm{s}) + b_0 z_2 + z_3 \tag{5}$$

$$\mathrm{s}H = \theta - \alpha + \upsilon_z , \qquad (6)$$

The command law's components can be chosen as:

$$\delta_m(\mathrm{s}) = k_m^v \overline{v} - k_{mv}^v(\mathrm{s}) v - k_{m\theta}^v(\mathrm{s})\theta - \\ -k_{mH}^v(\mathrm{s}) H, \qquad (7)$$

$$\delta_p(\mathrm{s}) = k_p^\theta \overline{\theta} - k_{p\theta}(\mathrm{s})\theta - k_{pv}(\mathrm{s}) v - k_p^H H \qquad (8)$$

One assume that the airplane (aircraft) has the flight commands stucked up on the neutral position, so the elevator's angle $\delta_p = 0$, then from (8) one obtains:

$$H = k_H^\theta \overline{\theta} - k_{H\theta}(\mathrm{s})\theta - k_{Hv}(\mathrm{s}) v , \qquad (9)$$

where the co-efficient are:

$$k_H^\theta = \frac{k_p^\theta}{k_p^H}, k_{H\theta}(\mathrm{s}) = \frac{k_{p\theta}(\mathrm{s})}{k_p^H}, k_{Hv}(s) = \frac{k_{pv}(\mathrm{s})}{k_p^H} \qquad (10)$$

Substituting H from Eq (9) into (7), one obtains:

$$\delta_m(\mathrm{s}) = k_m^v \overline{v} - k_H^\theta k_{mH}(\mathrm{s}) \overline{\theta} - (k_{mv}(\mathrm{s}) - \\ -k_{mH}(\mathrm{s}) k_{Hv}(\mathrm{s})) v - (k_{m\theta}(\mathrm{s}) - \\ -k_{mH}(\mathrm{s}) k_{H\theta}(\mathrm{s}))\theta. \qquad (11)$$

Supplementary, from Eq. (6) and Eq. (9) α becomes

$$\alpha = -k_H^\theta \mathrm{s} \overline{\theta}(1 + \mathrm{s} k_{Hv}(\mathrm{s}))\theta + \mathrm{s} k_{H\theta}(\mathrm{s}) v + \upsilon_z . \qquad (12)$$

One eliminates δ_m between Eq. (4) and Eq. (11). One also eliminates α between the first and the second equation of the mathematical model (Eqs.(3) and (4)), respectively Eq. (12); it results

$$[\mathrm{s} - a_{11} - a_{12}\mathrm{s} k_{Hv}(\mathrm{s}) + b_m(k_{mv}(\mathrm{s}) - \\ -k_{mH}(\mathrm{s}) k_{Hv}\mathrm{s})] v + [(a_{12} + a_{13}) + \\ +a_{12}\mathrm{s} k_{H\theta}(\mathrm{s}) - b_m(k_{m\theta}(\mathrm{s}) - \\ -k_{mH}(\mathrm{s}) k_{H\theta}(\mathrm{s}))]\theta = b_m k_m^v - \\ -k_H^\theta(a_{12}\mathrm{s} + b_m k_{mH}(\mathrm{s}))\overline{\theta} + z_1, \qquad (13)$$

$$[-a_{21} + \mathrm{s}(\mathrm{s} - a_{22}) k_{Hv}(\mathrm{s})] v + \\ +[-(a_{22} + a_{23}) + \mathrm{s}(\mathrm{s} - a_{22}) k_{H\theta}(\mathrm{s})]\theta = \\ = k_H^\theta \mathrm{s}(\mathrm{s} - a_{22})\overline{\theta} + z_2 - (\mathrm{s} - a_{22})\upsilon_z . \qquad (14)$$

So, the above equation system (13) and (14) can be expressed by

$$P_{1v}(\mathrm{s}) v + P_{1\theta}\theta = f_1(\mathrm{s}), \qquad (15)$$

$$P_{2v}(\mathrm{s}) v + P_{2\theta}\theta = f_2(\mathrm{s}), \qquad (16)$$

Identifying properly the arguments, one obtains:

$$P_{1v}(\mathrm{s}) = (\mathrm{s} - a_{11}) + b_m k_{mv}(\mathrm{s}) - k_{Hv}(\mathrm{s}) \\ (b_m k_{mH}(\mathrm{s}) + a_{12}(\mathrm{s})), \qquad (17)$$

$$P_{1\theta}(\mathrm{s}) = -[(a_{12} + a_{13}) - b_m k_{m\theta}(\mathrm{s}) + k_{H\theta}(\mathrm{s}) \\ (b_m k_{mH}(\mathrm{s}) + a_{12}(\mathrm{s})), \qquad (18)$$

$$P_{2v}(\mathrm{s}) = -a_{21} + \mathrm{s}(\mathrm{s} - a_{22}) k_{Hv}(\mathrm{s}), \qquad (19)$$

$$P_{2\theta}(\mathrm{s}) = -(a_{22} + a_{23}) + \mathrm{s}(\mathrm{s} - a_{22}) k_{H\theta}(\mathrm{s}); \qquad (20)$$

$$f_1(\mathrm{s}) = b_m k_m^v \overline{v} - k_H^\theta(b_m k_{mH}(\mathrm{s}) + a_{12}\mathrm{s})\overline{\theta} + z_1, \qquad (21)$$

$$f_2(\mathrm{s}) = k_H^\theta \mathrm{s}(\mathrm{s} - a_{22})\overline{\theta} + z_2 - (\mathrm{s} - a_{22})\upsilon_z . \qquad (22)$$

Imposing that the two channels (pitch angle's and longitudinal speed's channel) are independent, in order to eliminate any interaction between, it leads to the simultaneous fulfilling of the conditions

$$P_{1\theta} = 0, \quad P_{2v} = 0. \qquad (23)$$

Consequently, the system described by Eq. (15) and (16) becomes:

$$P_{1v}(\mathrm{s})v = f_1(\mathrm{s}), \qquad (24)$$

$$P_{2\theta}(\mathrm{s})\theta = f_2(\mathrm{s}). \qquad (25)$$

The first equation of (23) can be expressed as:

$$\begin{aligned}&[b_m k_{m\theta}(\mathrm{s}) - (a_{12} - a_{13})] +\\ &+k_{H\theta}(\mathrm{s})[b_m k_{mH}(\mathrm{s}) + a_{12}\mathrm{s}] = 0,\end{aligned} \qquad (26)$$

equivalent to the equation system

$$b_m k_{m\theta}(\mathrm{s}) = b_m k_m^{\theta} = a_{12} + a_{13}, \qquad (27)$$

$$b_m k_{mH}(\mathrm{s}) = b_m k_m^{\dot{H}}(\mathrm{s}) = -a_{12}\mathrm{s}. \qquad (28)$$

For the second equation of (23) one can impose $k_{Hv}(\mathrm{s}) = k_H^v$; so

$$\mathrm{s}^2 - a_{22}\mathrm{s} - \frac{a_{21}}{k_H^v} = 0 \Leftrightarrow \mathrm{s}^2 + 2\xi_2\omega_2\mathrm{s} + \omega_2^2 = 0. \qquad (29)$$

Imposing the ω_2 frequency and properly identifying, one obtains,

$$k_H^v = -\frac{a_{21}}{\omega_2^2}, \quad k_H^{\dot{v}} = 0 \qquad (30)$$

Taking into account the Eq. (28) form, Eq. (24) and Eq. (25) become:

$$[\mathrm{s} - a_{11} + b_m k_{mv}(\mathrm{s})]v = b_m k_m^v \bar{v} + z_1, \qquad (31)$$

$$\begin{aligned}&[\mathrm{s}(\mathrm{s} - a_{22})k_{H\theta}(\mathrm{s}) - (a_{22} - a_{23})]\theta =\\ &= k_H^{\theta}\mathrm{s}(\mathrm{s} - a_{22})\bar{\theta} + z_2 - (\mathrm{s} - a_{22})\upsilon_z\end{aligned} \qquad (32)$$

Analyzing the left member's terms in the above equations, it results that the transmission functions can have the forms

$$k_{mv}(\mathrm{s}) = k_m^v, \quad k_{H\theta}(\mathrm{s}) = k_H^{\theta} + k_H^{\dot{\theta}}\mathrm{s}. \qquad (33)$$

Imposing also the frequency range in the speed's channel $\omega_1 = \frac{1}{T_1}$, where T_1- transfer function's time constant of

$$\begin{aligned}&H_v^{\bar{v}}(\mathrm{s}) = \frac{v(\mathrm{s})}{\bar{v}(\mathrm{s})} =\\ &= \frac{b_m k_m^v}{\mathrm{s} + (b_m k_m^v - a_{11})} = \frac{b_m k_m^v}{\mathrm{s} + \omega_1},\end{aligned} \qquad (34)$$

one obtains:

$$b_m k_{mv}(\mathrm{s}) = b_m k_m^v = a_{11} + \omega_1. \qquad (35)$$

If the transfer function $H_{\bar{\theta}}^{\theta}$ has a Vishnegradsky form [see 3,5], that means

$$H_{\bar{\theta}}^{\theta}(\mathrm{s}) = \frac{\theta(\mathrm{s})}{\bar{\theta}(\mathrm{s})} = \frac{k_H^{\theta}\mathrm{s}(\mathrm{s} - a_{22})}{\mathrm{s}^3 + A_2\omega_0\mathrm{s}^2 + A_1\omega_0^2\mathrm{s}\omega_0^3} \qquad (36)$$

identifying member by member Eq. (36) and Eq. (32), one obtains the system

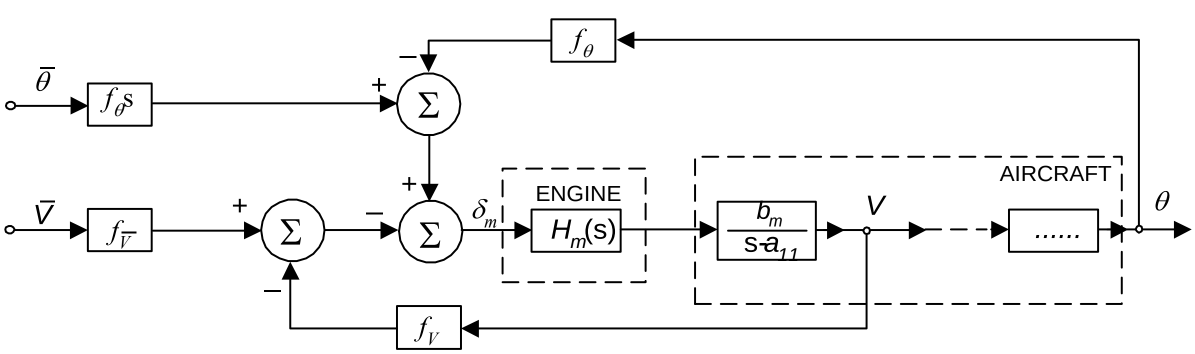

Fig.2.a Longitudinal (pitch angle) control system's block diagram

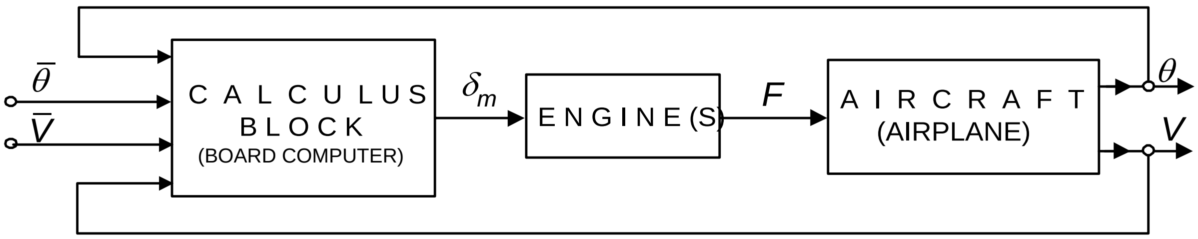

Fig. 2.b Equivalent block diagram

$$\omega_0^3 = -\frac{a_{22}+a_{23}}{k_H^{\dot{\theta}}}, \tag{37}$$

$$A_2\omega_0 = 2\xi\omega_0 = -a_{22} + \frac{k_H^{\theta}}{k_H^{\dot{\theta}}}$$

Imposing ω_0 and ξ, from Eq. (37) one obtains the values of the transitions ratio k_H^{θ} and $k_H^{\dot{\theta}}$.

Introducing Eq. (27), (28), (30), (33) and (35) into (11), one obtains the command law as

$$\begin{gathered} b_m\delta_m = (a_{11}+\omega_1)\overline{v} - \frac{a_{12}(a_{22}+a_{23})}{\omega_0^3} \\ (a_{22}+2\xi)\mathrm{s}\overline{\theta} - \left(a_{11}+\omega_1 - \frac{a_{12}a_{21}}{\omega_2^2}\mathrm{s}\right)v - \\ [(a_{12}+a_{13}) - \frac{a_{12}(a_{22}+a_{23})}{\omega_0^3}\mathrm{s} \\ (\mathrm{s}+a_{22}+2\xi\omega_0)]\theta \end{gathered} \tag{38}$$

so, this one has a specific form

$$\delta_m = f_{\overline{v}}\overline{v} + f_{\overline{\theta}}\mathrm{s}\overline{\theta} - f_v v - f_\theta\theta \tag{39}$$

If $\overline{\theta} = const.$, then $s\overline{\theta} = \dot{\overline{\theta}} = 0$ so Eq. (39 becomes).

$$\delta_m = f_{\overline{v}}\overline{v} - f_v v - f_\theta\theta. \tag{40}$$

The system's block diagram is shown in fig.2.a, and its equivalent form –in fig.2.b.

The engine's transfer function has one of the following forms [2,5]

$$H_m(\mathrm{s}) = \frac{1}{\tau\mathrm{s}+1},\ H_m(\mathrm{s}) = \frac{1}{\mathrm{s}(\tau\mathrm{s}+1)}, \tag{41}$$

$$H_m(\mathrm{s}) = \frac{k_M(\tau'\mathrm{s}+1)}{\mathrm{s}(\tau\mathrm{s}+1)}$$

where $\tau = \frac{\tau_m}{\tau_a}$ - time constant; τ_m- engine's time constant with respect to the engine's thrust and τ_a-airplane's (aircraft's) aerodynamic time constant.

3.2 Lateral motion's control

The linearised non-dimensional mathematical model of an airplane (aircraft is [2]

$$\begin{gathered} \left[\frac{\mathrm{d}\beta}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\hat{\omega}_x}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\hat{\omega}_z}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\varphi}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\psi}{\mathrm{d}\hat{t}}\ \frac{\mathrm{d}\hat{y}}{\mathrm{d}\hat{t}}\right]^T = \\ = \hat{A}\left[\beta\ \hat{\omega}_x\ \hat{\omega}_z\ \varphi\ \psi\ \hat{y}\right]^T + \\ +\hat{B}\left[z_1\ \ z_2\ \ z_3\ \ \delta_e\ \ \delta_d\ \ \upsilon_y\right]^T, \end{gathered} \tag{43}$$

where

$$\hat{t} = \frac{t}{\tau_a},\ \hat{\omega}_x = \tau_a\omega_x,\ \hat{\omega}_z = \tau_a\omega_z,\ \hat{y} = \frac{\Delta y}{\tau_a v*}; \tag{44}$$

τ_a is the aerodynamic time constant, $v*$- flight speed's horizontal component, β- sliding angle, φ- banking angle, $\omega_x = \dot{\varphi}$- banking speed, ψ- flight course, $\omega_z = \dot{\psi}$- gyration speed rate, y- lateral displacement;

$\hat{A}, \hat{B}$ - matrix (6x6); δ_e - aileron angle, δ_d - rudder angle, $z_1, z_{2,} z_3$ - disturbances and $\upsilon_y = \frac{v_{vy}}{v}, v_{vy}$ - lateral wind speed..

Renouncing at "^", the liberalized non-dimensional mathematical model of the lateral motion becomes

$$\dot{\beta} = a_{11}\beta + a_{12}\dot{\varphi} + a_{13}\dot{\psi} + a_{14}\varphi + z_1, \quad (45)$$

$$\ddot{\varphi} = a_{21}\beta + a_{22}\dot{\varphi} + a_{23}\dot{\psi} + z_2 + b_{2e}\delta_e, \quad (46)$$

$$\ddot{\psi} = a_{31}\beta + a_{22}\dot{\varphi} + a_{33}\dot{\psi} + z_3 + b_{3e}\delta_e + \\ + b_{3d}\delta_d + b_M\delta_M \quad (47)$$

$$\dot{y} = \psi - \beta + \upsilon_y. \quad (48)$$

Similarly, assuming that all command surfaces are stucked up

$$\delta_e = 0, \delta_d = 0. \quad (49)$$

The command law's components are chosen as

$$\delta_d(\mathrm{s}) = k_d^{\psi}\overline{\psi} - k_{d\varphi}(\mathrm{s})\varphi - \\ k_{d\psi}(\mathrm{s})\psi - k_{d\varepsilon}(\mathrm{s})\varepsilon - k_d^y y; \quad (50)$$

$$\delta_M(\mathrm{s}) = k_M^{\psi}\overline{\psi} - k_{M\varphi}(\mathrm{s})\varphi - \\ k_{M\psi}(\mathrm{s})\psi - k_{My}(\mathrm{s})y; \quad (51)$$

where the co-efficient $k_d^{\psi}, k_M^{\psi}, k_d^y, k_M^y$ are transmission ratios and the others are transmissions functions:

$$k_d^{\psi}(\mathrm{s}) = k_d^{\varphi} + k_d^{\dot{\varphi}}\mathrm{s}, \ k_{d\psi}(\mathrm{s}) = k_d^{\psi} + k_d^{\dot{\psi}}, \quad (52)$$
$$k_{M\varphi}(\mathrm{s}) = k_M^{\varphi} + k_M^{\dot{\varphi}}\mathrm{s}, \ k_{M\psi}(\mathrm{s}) = k_M^{\psi} + k_M^{\psi}\dot{\psi}$$
$$k_{My}(\mathrm{s}) = k_M^y + k_M^{\dot{y}}\mathrm{s};$$

ε - radio beacon's direction.
Imposing that $\delta_d = 0$, one obtains

$$y = k_y^{\psi}\overline{\psi} - k_{y\varphi}(\mathrm{s})\varphi - k_{y\psi}(\mathrm{s})\psi - k_{y\varepsilon}(\mathrm{s})\varepsilon, \quad (53)$$

where

$$k_{y\varphi}(\mathrm{s}) = \frac{k_{d\varphi}(\mathrm{s})}{k_d^y} = k_y^{\varphi} + k_y^{\dot{\varphi}}\mathrm{s}, \quad (54)$$

$$k_{y\psi}(\mathrm{s}) = \frac{k_{d\psi}(s)}{k_d^y} = k_y^{\psi} + k_y^{\dot{\psi}}\mathrm{s}, \quad (55)$$

$$k_{y\varepsilon}(\mathrm{s}) = \frac{k_{d\varepsilon}(\mathrm{s})}{k_d^y} = k_y^{\dot{\varepsilon}} + \frac{1}{\mathrm{s}}k_y^{\varepsilon}. \quad (56)$$

Substituting y from (53) in Eq. (51), this one becomes:

$$\delta_M(\mathrm{s}) = (k_M^{\psi} - k_M^y k_y^{\psi})\overline{\psi} + \\ + k_{My}(\mathrm{s})k_{y\varepsilon}(\mathrm{s})\varepsilon - (k_{M\varphi}(\mathrm{s}) - \\ \_k_{My}(\mathrm{s})k_{y\varphi}(\mathrm{s}))\varphi - (k_{M\psi}(\mathrm{s}) - k_{My}(\mathrm{s}) \\ k_{y\varphi}(\mathrm{s}))\varphi - (k_{M\psi}(\mathrm{s}) - k_{My}(\mathrm{s})k_{y\psi}(\mathrm{s}))\psi \quad (57)$$

and Eq. (48), considering Eq. (52), also becomes:

$$\beta = -k_y^{\psi}\mathrm{s}\overline{\psi} + (1 + \mathrm{s}k_{y\psi}(\mathrm{s}))\psi + \\ + \mathrm{s}k_{y\varphi}(\mathrm{s})\varphi + sk_{y\varepsilon}(\mathrm{s})\varepsilon + \upsilon_y \quad (58)$$

Assuming that $\delta_e = \delta_d = 0$, from Eq. (47) and (49), written in complex, one obtains

$$a_{13}\mathrm{s}\psi + (a_{12}\mathrm{s} + a_{14})\varphi - (\mathrm{s} - a_{11})\beta = -z_1 \quad (59)$$

$$(\mathrm{s}^2 - a_{33}s)\psi - a_{32}\mathrm{s}\varphi - a_{31}\beta - b_M\delta_M = \\ = -z_3 \quad (60)$$

Eliminating δ_M and β between Eq. (57), (58), (59) and (60); a new system is obtained

$$P_{1\psi}(\mathrm{s})\psi + P_{1\varphi}(\mathrm{s})\varphi = g_1(\mathrm{s}), \quad (61)$$

$$P_{2\psi}(\mathrm{s})\psi + P_{2\varphi}(\mathrm{s})\varphi = g_2(\mathrm{s}), \quad (62)$$

$$P_{1\psi}(\mathrm{s}) = (\mathrm{s}^2 - a_{33}\mathrm{s} - a_{31}) + b_M k_{M\psi}(\mathrm{s}) - \\ -k_{y\psi}(\mathrm{s})(b_M k_{My}(\mathrm{s}) + a_{31}\mathrm{s}), \quad (63)$$

$$P_{1\varphi}(\mathrm{s}) = -a_{32}\mathrm{s} + b_M k_{M\varphi}(\mathrm{s}) - \\ -k_{y\varphi}(\mathrm{s})(b_M k_{My}(\mathrm{s}) + a_{31}\mathrm{s}), \quad (64)$$

$$P_{2\psi}(\mathrm{s}) = a_{11} - \mathrm{s}(\mathrm{s} - a_{11})k_{y\psi}(\mathrm{s}), \quad (65)$$

$$P_{2\varphi}(\mathrm{s}) = a_{12}\mathrm{s} + a_{14} - \mathrm{s}(\mathrm{s} - a_{11})k_{y\varphi}(\mathrm{s}); \quad (66)$$

$$g_1(\mathrm{s}) = [-k_y^{\psi} a_{31}\mathrm{s} + b_M(k_M^{\psi} - k_M^{y} + k_y^{\psi})]\overline{\psi} \cdot \\ +k_{y\varepsilon}(\mathrm{s})(b_M k_{my}(\mathrm{s}) + a_{31}\mathrm{s})\varepsilon - z_3 + a_{31}\upsilon_y, \quad (67)$$

$$g_1(\mathrm{s}) = -k_y^{\psi}\mathrm{s}(\mathrm{s} - a_{11})]\overline{\psi} + \mathrm{s}(\mathrm{s} - a_{11}) \\ k_{y\varepsilon}(\mathrm{s})\varepsilon + z_1 + (\mathrm{s} - a_{11})\upsilon_y. \quad (68)$$

One assume that the two cannels are independent; imposing this condition, equivalent to

$$P_{1\varphi}(\mathrm{s}) = 0, \quad P_{2\psi}(\mathrm{s}) = 0, \quad (69)$$

the system (58) and (59) can be described by two independent sub-systems, so it becomes

$$P_{1\psi}(\mathrm{s})\psi = g_1(\mathrm{s}), \quad (70)$$

$$P_{2\varphi}(\mathrm{s})\varphi = g_2(\mathrm{s}). \quad (71)$$

The firs condition of (69) can be written as

$$(-a_{32}\mathrm{s} + b_M k_{M\varphi}(\mathrm{s})) - \\ -k_{y\varphi}(\mathrm{s})(b_M k_{My}(\mathrm{s}) + a_{31}\mathrm{s}) = 0, \quad (72)$$

which is equivalent to the system

$$b_M(k_M^{\varphi} - k_M^{\dot\varphi}\mathrm{s}) = a_{32} \Leftrightarrow k_M^{\varphi} = 0, \\ b_M k_M^{\dot\varphi} = a_{32}, \quad (73)$$

$$b_M(k_M^{y} - k_M^{\dot y}\mathrm{s}) = -a_{31} \Leftrightarrow k_M^{y} = 0, \\ b_M k_M^{\dot y} = -a_{31}. \quad (74)$$

The second condition (69) is equivalent to

$$\mathrm{s}^2 + 2\xi_2\omega_2\mathrm{s} + \omega_2^2 = 0. \quad (75)$$

Identifying properly, one obtains

$$k_y^{\psi} = -\frac{a_{11}}{\omega_2^2},\ k_y^{\dot\psi} = 0. \quad (76)$$

Considering Eq. (73), the system based on (70) and (71) becomes:

$$[(\mathrm{s}^2 - a_{33}\mathrm{s} - a_{31}) + b_M k_{M\psi}(\mathrm{s})]\psi = \\ = (-k_y^{\psi} a_{31}\mathrm{s} + b_M k_M^{\psi})\overline{\psi} - z_3 + a_{31}\upsilon_y, \quad (77)$$

$$[(a_{12}\mathrm{s} + a_{14}) - \mathrm{s}(\mathrm{s} - a_{11})k_{y\theta}(\mathrm{s})]\varphi = \\ -k_y^{\psi}\mathrm{s}(\mathrm{s} - a_{11})\overline{\psi} + \mathrm{s}(\mathrm{s} - a_{11})k_{y\varepsilon}(\mathrm{s})\varepsilon + \\ +z_1 + (\mathrm{s} - a_{11})\upsilon_y. \quad (78)$$

Analyzing the left member's terms structure of the above system, the transmission functions can be properly chosen as

$$k_{M\psi}(\mathrm{s}) = k_M^{\psi} + k_M^{\dot\psi}\mathrm{s},\ k_{y\varphi}(\mathrm{s}) = k_M^{y} + k_M^{\dot y}\mathrm{s} \quad (79)$$

Imposing ζ and ω_0, and identifying, from

$$H_{\overline{\psi}}^{\psi}(\mathrm{s}) = \frac{\psi(\mathrm{s})}{\overline{\psi}(\mathrm{s})} = \\ = \frac{-k_y^{\psi} a_{31}\mathrm{s} + b_M k_M^{\psi}}{\mathrm{s}^2 + \left(b_M k_M^{\dot\psi} - a_{33}\right)\mathrm{s} + (b_M k_M^{\psi} - a_{31})} = \\ = \frac{-k_y^{\psi} a_{31}\mathrm{s} + b_M k_M^{\psi}}{\mathrm{s}^2 + 2\xi\omega_0\mathrm{s} + \omega_0^2} \quad (80)$$

it results

$$b_M k_M^{\psi} = a_{31} + \omega_0^2,\ b_M k_M^{\psi} = a_{33} + 2\xi\omega_0 \quad (81)$$

substituting $k_{y\varphi}(\mathrm{s})$ in Eq. (78), and imposing ξ_1 and ω_1 values, it results

$$k_y^{\dot\varphi} = -\frac{a_{14}}{\omega_1^3},\ k_y^{\varphi} = k_y^{\dot\varphi}(a_{11} + 2\xi_1\omega_1) \quad (82)$$

Because $k_{My}(s) = k_M^{\dot y}s$, $k_{y\varepsilon}$ of (57) can be chosen as

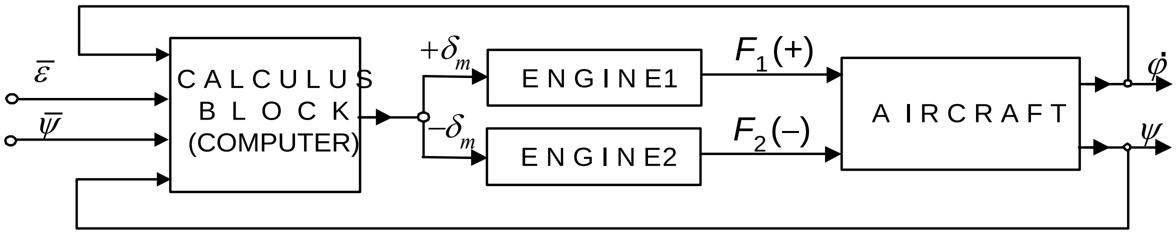

Fig.3 Lateral Control system's equivalent block

$$k_{y\varepsilon}(s) = k_y^{\dot{\varepsilon}} + \frac{1}{s} k_y^{\varepsilon}. \qquad (83)$$

Taking into account the transmission functions forms, Eq. (57) becomes

$$\begin{aligned}
&b_M \delta_M(s) = b_M k_M^{\psi} \bar{\psi} + b_M k_M^{\dot{y}} \psi (k_y^{\varepsilon} + k_y^{\dot{\varepsilon}} s)\varepsilon \\
&+\left[b_M \left(k_M^{\dot{y}} k_y^{\varphi} - k_M^{\dot{\varphi}} \right) + b_M k_M^{\dot{y}} k_M^{\dot{\varphi}} s \right] \dot{\varphi} - \\
&-\left[b_M k_M^{\psi} + \left(b_M k_M^{\psi} - b_M k_M^{\dot{y}} k_y^{\psi} \right) s \right] \psi = \\
&= (a_{31}\omega_0^2)\bar{\psi} + (a_{33} + 2\xi\omega_0)(k_y^{\varepsilon} + k_y^{\dot{\varepsilon}} s)\varepsilon + \\
&+\left[\frac{a_{31}a_{14}}{\omega_1^3}(s + a_{11} + 2\xi_1\omega_1) - a_{32} \right] \dot{\varphi} - \qquad (84) \\
&\left[(a_{31} + \omega_0^2) + \left(a_{33} + 2\xi\omega_0 - \frac{a_{11}a_{31}}{\omega_2^2} s \right) \right] \psi.
\end{aligned}$$

System's block diagram is presented in fig.3.

4 Numerical simulation

In order to estimate the performance, a passengers airplane was studied, which has the mathematical model's co-efficient [3]

$a_{11} = -0.12;\ a_{12} = 0.28;\ a_{13} = -0.4;$

$a_{21} = 0.8;\ a_{22} = -2,5;\ a_{23} = -0.02;$,

$\omega_1 = 0.25;\ \omega_0 = 0.7;\ \xi = 0.6;\ \omega_2 = 0.5.$,

and the transmission function are

$f_{\bar{v}} = 0.13;\ f_{\bar{\theta}} = 0.3\ f_v = 0.13 - 0.81s;$

$f_{\theta} = -0.12 + 2s(s - 1.9)$

If one uses for k_H^V a PID-type law, the results are not acceptable, because the stabilizing process is divergent. Renouncing at the integrator component, k_H^V becomes PD-type, so (considering P_{2v}=0), k_H^V becomes $k_H^V = \frac{a_{21}}{s(s - a_{22})}$, and subsequently the engine's command law becomes:

$$\begin{aligned}
&b_m \delta_m = (a_{11} + \omega_1)\bar{v} - \\
&-\frac{a_{12}(a_{22} + a_{23})}{\omega_0^3}(a_{22} + 2\xi)s\bar{\theta} - \\
&-\left(a_{11} + \omega_1 - \frac{a_{12}a_{21}}{\omega_2^2} s \right) v - \qquad (85) \\
&\left[(a_{12} + a_{13}) - \frac{a_{12}}{s - a_{22}}(s + a_{22} + 2\xi\omega_0) \right] \theta.
\end{aligned}$$

One has considered that the airplane (aircraft) has a constant flight speed, without any disturbances ($\bar{V} = 0, \upsilon = 0$), and $\bar{\theta}$ as a input argument for the estimation. The best results were obtained using a transfer function for the engines as shows Eq. (41); the results are presented in fig. 4, which shows the system behavior, for a positive $\bar{\theta}$ input argument.

The main parameters evolve in acceptable range and after acceptable laws; the flight altitudine an the climbing speed are growing (4.a), but the climbing speed' slope is constantly decreasing (especially for the last part of the trajectory, when the altitude has grown enough); the engine's thrust grows spectacularly in the first 1..1,5 seconds, in order to obtain a properly attack angle for the airplane (aircraft), which must raise the nose, than decreases until reaches , after 4..5 sec, the new base level (2% bigger than the initial thrust); furthermore, the thrust growing continues (but with a smooth slope) in order to assure the climbing (fig.4b).

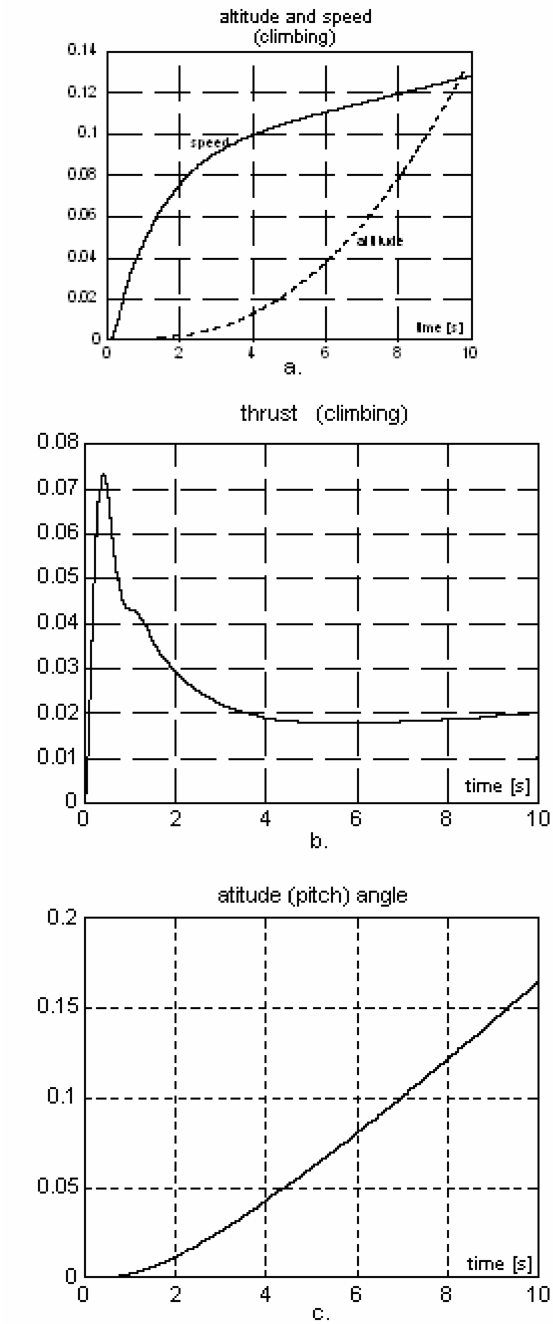

Fig.4 Longitudinal behavior

$a_{11} = -0.9$; $a_{14} = 0.2$; $a_{31} = -2,5$; $a_{32} = -0.01$;
$a_{33} = -0.1$; $\omega_0 = 0.5$; $\xi = 0.5$; $\omega_1 = 0.4$; $\xi_1 = 0.6$;
$a_{22} = -2.31$; $a_{23} = -0.7$

calculated for a low altitude flight regime (airplane's configuration for landing). Fig 5 shows the main parameters behavior, for an input argument $\bar{\psi} \approx 9^o$.One has neglected the disturbance's and also the radio beacon signal influence (the terms containing ε were considered 0 both in the mathematical model and in the command law). The curves in fig.5 show that ψ angle (flight course) is self stabilizing at the preset (input) value. The banking angle φ is growing (as absolute value), then, during the turnback, it tends to return to the initial null-value.

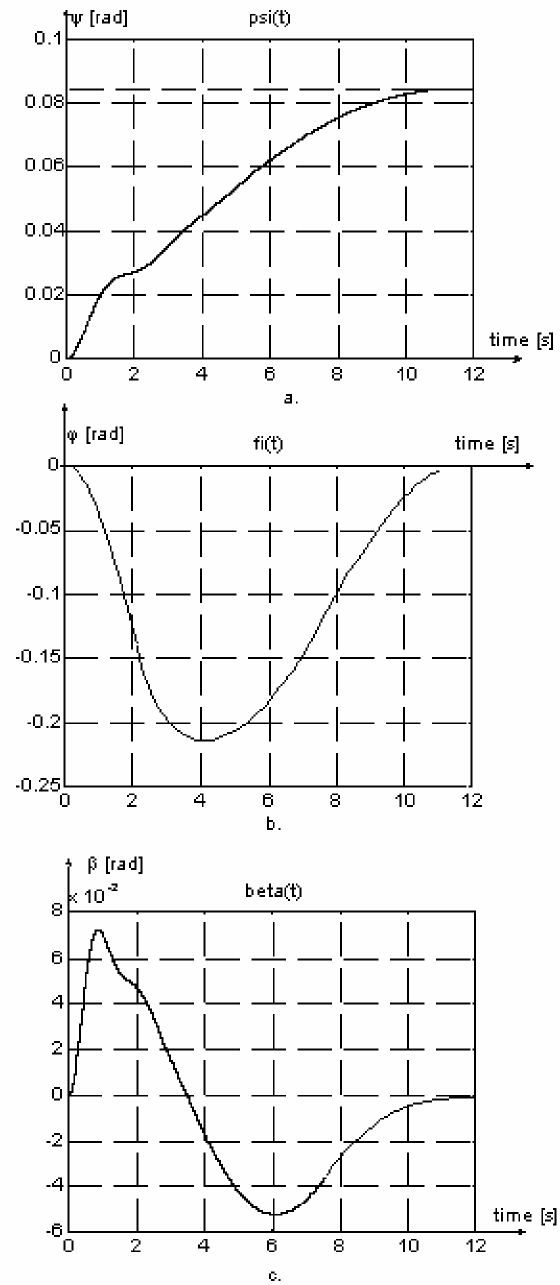

Fig.5 Lateral behavior

The sliding angle β grows at first, following the turning sense, then, in order to bring back the plane on its trajectory, it changes sense and at least it reaturns to its initial null-value. The thrusts of the two engines have opposite senses of variations, in order to realize the airplane's rotation torque during its turn.

5 Conclusions

The paper deals with a possible automatic control system for airplanes (aircrafts) using its jet engines , as back-up system in case of damaged or accidental non-active flight commands.

The authors have established new mathematical models for aircraft's longitudinal and lateral motion, which include the engine(s) operating effects; both models are linearised and transformed in non-dimensional linear systems. Imposing the channels' decoupling, the authors have determined the command laws for each motion, respectively they have determined engines' command laws for both channels.

The performed numerical simulation (which had as studied object a medium-heavy line airplane, twin-engines) has confirmed the designed engines' command laws; the study was realized for low altitude flights, which means that the propulsion flight control can be used for emergency landings.

The paper presents the first results of some specific studies performed by the Avionics Division of the University of Craiova and can be useful for airplane (aircraft) and engine's control systems designers, aeronautical specialists and students.

References

[1] Abraham, R. H. *Complex dynamical systems.* Aerial Press, Santa Cruz, California, 1986.

[2] Bodner, V. A. *Teoria avtomaticeskovo upravlennia poletom.* Izd. Nauka, Moskva, 1964.

[3] Dumitrescu, M. *Comanda automată a aparatelor de zbor.* vol. I, Ed. Academiei Tehnice Militare, Bucureşti, 1974.

[4] Dumitrescu, M. *Comanda automată a aeronavelor.* Ed. Academiei Tehnice Militare, Bucureşti, 1986.

[5] Lungu, R. *Automatizarea aparatelor de zbor.* Ed. Universitaria, Craiova, 2000.