SYSTEM DECOMPOSITION METHOD BASED ON PARETO AND THE APPLICATION TO THE OPTIMIZATION OF THE AIRCRAFT CONFIGURATION DESIGN

Xia Lu*, Gao Zhenghong*, Li Tian**
***School of Aeronautics, Northwestern Polytechnical University,**
****Shenyang Aircraft Design & Research Institute**

Keywords: *Multi-objective optimization, system decomposition method, Pareto method, system decomposition method based on Pareto*

Abstract

In this paper, the multi-objective optimization methods are investigated for complicated problem as aircraft configuration design. System decomposition method is the common method in engineering field to reduce the complexity of the system. The inherence problem of traditional system decomposition method in disposing the multi-objective problem is that the multiple objectives have to be turned to a single one and only one optimal result will be obtained after optimization. In this paper, system decomposition method based on Pareto is put forward to solve the problem, which takes the advantage of Pareto method and to obtain a set of evenly spaced solutions on a Pareto front. As an example, an aerodynamic-stealth integration optimization design for a wing-body is carried out by using the method. After optimization, the aerodynamic and stealth performance of the set of solutions on a Pareto front are improved, which showed, by using the method, the anticipant design requirement is achieved and the analysis and the compare could be carried out in the set of solutions to choose the needed result.

1 General Introduction

Aircraft configuration design is a multi-disciplinary, multi-objective, multi-variable optimization problem. Usually, for such engineering problem, not only the number of design variables is considerable, but also the disciplinary objectives are disparate and conflicting. All of these increase the complexity of the optimization problem. It's difficult to obtain a convergent effective optimum solution by using common searching algorithms.

In this paper, the system decomposition method of multidisciplinary design optimization is introduced. The advantage and disadvantage of traditional decomposition method based on Nash equilibrium are studied. In order to overcome the drawback of traditional method, system decomposition method combined with the method to generate Pareto solutions is brought forward. By applying system decomposition method based on Pareto, a wing-body configuration is optimized to obtain low observability and high aerodynamic efficiency.

2 System Decomposition Method

Consider the multi-objective optimization problem of the following mathematical programming statement: search design variables $X = (x_1, x_2, \cdots, x_n)^T$ to

$$\min: \quad [f_i(X)] \qquad i = 1, \cdots, s$$

$$\text{s. t.} \quad \begin{array}{ll} h_k(X) = 0, & k = 1, \cdots, m \\ g_j(X) \ge 0, & j = 1, \cdots, p \end{array} \qquad (1)$$

where *n,s,p,m* are the number of design variables、objective functions 、inequation and equation constrains， X is a vector composed of the design variables, the analysis of constrains

h_k、g_j and objective function f_i involves multiple disciplines.

For a complex optimization problem such as aircraft configuration design, the generally accepted approach is system decomposition method. System decomposition is natural in an engineering organization: to decompose a complex system into several coupling subsystems [1,2]. These subsystems are smaller and simpler. Such decomposition can reduce the complexity of the system by changing the organizational structure of the optimization design problem, which makes it easer to obtain a satisfying optimal solution for a complex system. Moreover, those subproblems keep collaborative and can be analyzed parallel, which reduce the cost and time of the optimization problem.

2.1 System Decomposition Method Based on Nash equilibrium

Traditional system decomposition method is applied to single objective problems. If the problem is multi-objective, the multiple objectives are usually turned into a single one. And then, the system is decomposed. A kind of traditional system decomposition is based on Nash equilibrium, tool from Game Theory.

Nash equilibrium is the solution of a non-cooperative strategy of multi-objective optimization first introduced by J. F. Nash in 1957. For an optimization problem with N objectives, the corresponding Nash game keeps N players, each in charge of one objective and able to modify their subgroups of variables. During the game, each player looks for the best strategy (the optimal values for the variables he is in charge of) and exchanges it with each other. When no player can further improve his criterion, the system has reached a state of Nash equilibrium [3].

Fig. 1 shows a system decomposed into several subsystems by using system decomposition methods based on Nash equilibrium: every rectangle box represents the analysis and optimization of a subsystem. The analysis information is transmitted from the top level to the bottom level. $X_1=(x_1,x_2,\cdots,x_{n1})^T$, $\cdots$, $X_l=(x_{nl+1},x_{nl+2},\cdots,x_n)^T$ represent the subgroups of design variables $X=(x_1,x_2,\cdots,x_n)^T$ decomposed in the top system. When a subsystem is optimized, the parameters $X_1^0, X_2^0,\cdots$, which transmitted from the superior levels are regarded as constant while design variables of the subgroup are regarded as local variables, so the optimization is carrying out inside the subsystem. After a round parallel optimization in these subsystems, in order to collaborate, the optimal result $X_1^1, X_2^1,\cdots$, exchange with each other. And then the next round of optimization is carrying on till the system reaches Nash equilibrium.

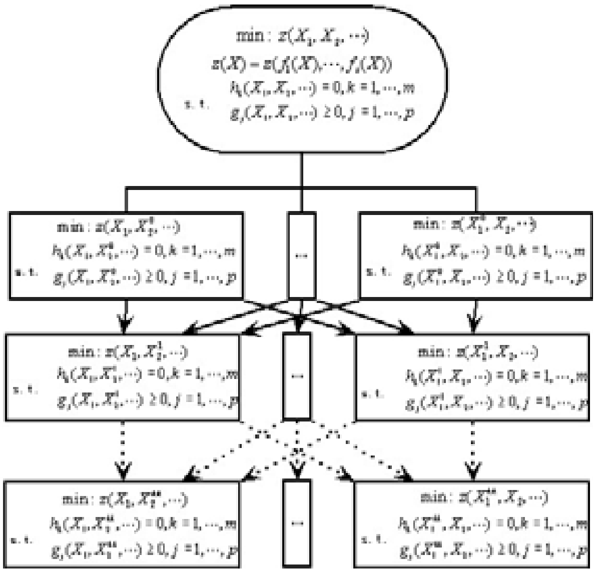

Figure 1 System decomposition method based on Nash equilibrium

However, the inherence drawback of those traditional system decomposition methods in disposing the multi-objective problem is that the multiple objectives have to be turned into a single one and only one optimum result will be obtained after optimization. That means, first, since the optimum result is very sensitive to the combination or the transformation of the multiple objectives, the forming multiple objectives to a single one is important and difficult; second, after numerous operations, only one optimum solution is obtained. In order to overcome the drawback of traditional one, in this paper, system decomposition method based on Pareto is brought forward.

2.2 System Decomposition Method Based on Pareto

Pareto methods can effectively generate Pareto solutions. A Pareto solution is one where any improvement in one objective can only take place if at least one other objective worsens. In other word, these solutions are non-dominated or non-inferior [4]. Mathematically, for a maximization problem with n objectives $F = (f_1, ..., f_n)$, a feasible vector X^* is a Pareto optimum (non-dominated) if and only if there exists no feasible vector X such that:

$$\forall_i f_i(X) \geq f_i(X^*)$$

$$\exists_i f_i(X) > f_i(X^*) \quad i = 1, \cdots, n$$

The collection of non-dominated points in a design space is often called Pareto set, and the curve (surface or hyper-surface) describes by these points in objective space is often called Pareto front. The define shows that it is not possible to distinguish which is the better one over the rest of the solutions. So, the better way to dispose the multi-objective problem is to find out the Pareto set so that the trade-offs between objectives can be fully examined [5].

System decomposition method based on Pareto combine system decomposition method with the method to generate Pareto solutions, which take the advantage of decomposition and overcome the drawback of traditional usage. As showed in Fig. 2, system decomposition method based on Pareto aiming at the characteristics of the complex system, decomposes the design variables $X = (x_1, x_2, \cdots, x_n)^T$ into several subgroups $X_1, X_2, \cdots$, each subgroup with the multiple objectives and constraints is formed to an optimization subsystem. In the subsystem, only the design variables of the corresponding subgroup are local variables and other design variables are regarded as constant. Pareto method is applied to optimize each subsystem. For exchanging the information of different subsystems, the Pareto sets $\{X_{11}, X_{12}, \cdots, X_{1n_1}\}$, $\{X_{21}, X_{22}, \cdots, X_{2n_2}\}$, $\cdots$ obtained from the subsystems are transferred to the top system. In fact, the optimization of the top system is to search the optimum combination of the Pareto solutions from the subsystems and so the number of design variables of the top system is the number of the subsystems. The Pareto solutions $(X_{1i_1}, X_{2j_1}, \cdots)$ from the top system replace the original value $(X_1^0, X_2^0, \cdots)$ to carry out next round of optimization till not a new Pareto solution is generated from the top system, which means the system reaches Nash equilibrium.

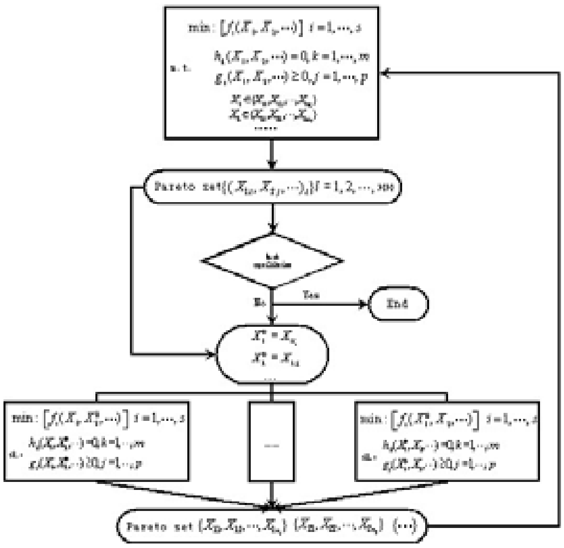

Figure 2 System decomposition method based on Pareto

In engineering application, if the number of Pareto solutions in some round is great, the computational cost is great when all the solutions replace original ones into next round. In this paper, the idea of finite element is introduced. According to the shape of Pareto front, it is divide into some parts and a key point of every part is chosen to represent it. The optimal solutions corresponding to the key points on the Pareto front replace original ones into next round of optimization. In this way, the computational cost is reduced, the primary information of the Pareto solutions is guaranteed, and only the well distribution of Pareto solutions may worsens.

For aircraft configuration design, first, since the aircraft is composed of many different parts, the number of design variables is large; second, the performance analysis is usually complicated. All of these increase the complexity of the optimization problem and it's difficult to obtain a convergent effective optimum solution using common searching algorithms. However, applying system decomposition method based on Pareto, the system is easy to decomposed rationally according to different parts and so the complexity of such system is reduced. Since the number of design variables of the top system is the number of the subsystems, the optimal search is easier. These take full advantage of system decomposition method. Furthermore, it achieves an optimization in a genuine multi-objective sense. In other words the optimum Pareto set corresponding to all kinds of the weight assignment can be achieved, which could provide some reference results for engineering designers.

3 Results and Discussion

To validate the feasibility of system decomposition method based on Pareto, computational analysis of an example for the design of a wing-body is carried out to obtain low observability and high aerodynamic efficiency. According to the characteristics of design variables, they are decomposed into two subgroups: parameters describing the wing planform such as the length of the root, the tip, the span and the value of the sweepback, and parameters describing the cross section shape of body such as Bezier curve parameters, etc.

3.1 Optimization Model

The optimization design of the wing-body is carrying out at Mach number $M = 0.6$, angle of attack $\alpha = 4°$ and Rader wave frequency $f = 9GHz$. The objective functions are maximizing the lift-to-drag ratio (L/D) and minimizing RCS head average value from $0°$ to $60°$. According to structure, intensity, the feature of the objective functions and the engineering limits, the constrains are decided: the low and up bounds of the design variables; the range of the area of the planform of the wing; the ranges of the area of the control cross sections of the body; the low bound of the lift coefficient.

3.2 Optimum Search Algorithm

In this paper, Pareto genetic algorithm which combines genetic algorithm and Pareto method, is used as search algorithm to generating Pareto solutions. In order to generate well-distributed solutions, skills such as Pareto solution filter, population ranking [5,6], etc are used.

3.3 Performance Analysis and Computation Method

Since Pareto genetic algorithm is taken as search algorithm, it usually requires thousands of analysis runs to search the design space for even fairly simple cases. In order to keep the efficiency of the optimization, Green's function method is adopted as the analysis method for aerodynamic performance. Green's function method with strict theoretical proof is applicable to the aerodynamic analysis of complex shape, the analysis result is exact and the computational expense is less [7,8]. The analysis method for RCS in this paper is based on physical optics method. Considering that the aircraft often with deferent wings, which are the stronger edge diffraction source, while physical optics method does not deal with edge diffraction, the equivalent current method is adopted to computing the influence of the edge diffraction[8,9].

3.4 Results

In Fig.3, the Pareto solutions and the original value are showed in objective space. The coordinates are two objective functions which are nondimensioned. Compared with the original value, not only the aerodynamic performance but also the stealth performance of all the solutions are improved. From the solutions, According to the different requirements of aerodynamic and stealth performance, the needed results are easy to choose. For example, A and B are two optimum results in the Pareto set. The comparison between the performances is showed in Fig. 4, Fig. 5 and Table 1. As showed in Fig.3 and Table 1, obviously, the aerodynamic performance of A is better than that of B. At the design point, the objective, lift-to-drag ratio of A increases 20.6% while that of B increases 11.3%. Whereas, the stealth performance of B is better than that of A. At the design point, the objective, RCS head average value from 0° to 60° of B reduces 78.4% while that of A reduces 59.4%. These show the characteristics of Pareto set: the optimum set corresponding to all kinds of the weight assignment.

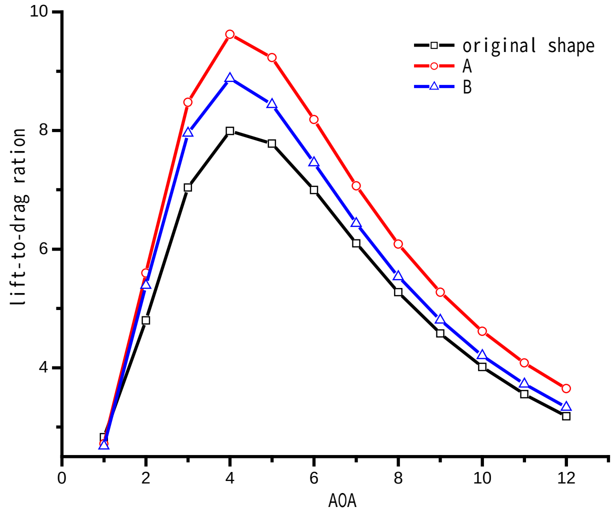

Fig.4 Comparison of aerodynamic performance

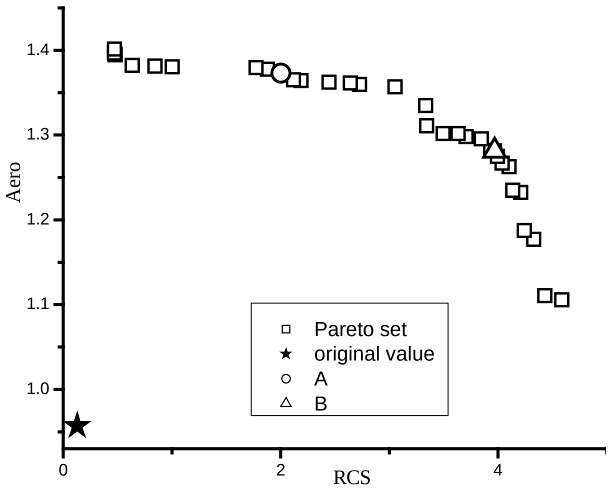

Fig.3 Pareto front

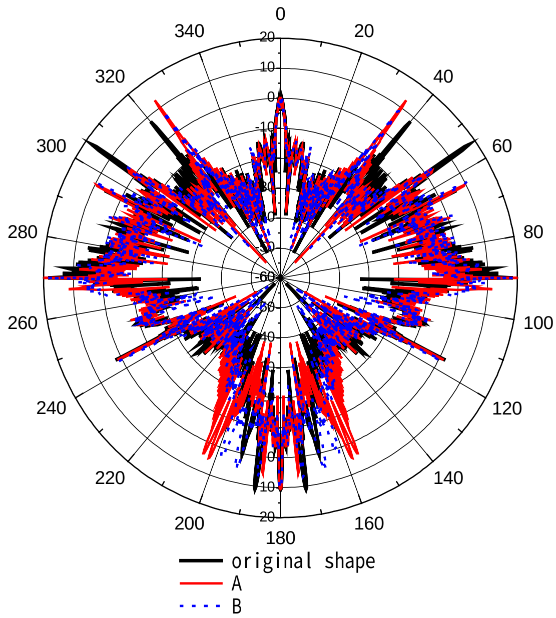

Fig.5 Comparison of stealth performance

Performance	Original shape	A	B
lift Coefficient	0.231	0.246	0.244
drag Coefficient	0.029	0.0256	0.0275
lift-to-drag ratio	7.97	9.61	8.87
RCS head average value	0.32	0.13	0.069

Table 1 The Compare of optimal results to original value

4 Conclusion

(1) For engineering design problem as aircraft configuration, if putting too much emphasis on the global optimal in theory, the method may lack in practicability. The optimization theory should be combined with engineering methods to obtain satisfying optimal results.

(2) System decomposition method is common in engineering design field: the design task of a complex system is assigned to different design teams and then collaboration and weight are carried on.
(3) The example of wing-body design shows that applying system decomposition method based on Pareto, the satisfying Pareto set will be obtained. It's convenient for comparing different solutions to find out the characteristics of the design object and help to build better optimization model in the future.

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