# PRELIMINARY STUDY OF NON-LINEAR AEROELASTIC PHENOMENA IN HYPERSONIC FLOW

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## Abstract

*Analyzing velocity potential equation, we get the source of aerodynamic non-linearity at high Mach numbers. The non-linearity of the various engineering methods for hypersonic aerodynamic loads is validated. By numerical method, the aeroelasticity of hypersonic wing is simulated in time domain. Some non-linear phenomena have been found as follows: (1) The system doesn't apply the superposition principle; (2) The stability of the aeroelasticity is effected by the original conditions; (3) Distortion appears in the time domain response, that is the response in critical conditions is not a harmonic; (4) The LCOs in hypersonic aeroelasticity; (5) The non-linear relations between critical velocity and angle of attack/ Mach numbers.*

## 1 General Introduction

With the step of the mankind's exploring the universe and exceeding the ultimate, some new concept aircrafts such as hyper-X program become the vision-vehicle for the future. There is thus a great need to investigate the aeroelastic problem at hypersonic speed. Aeroelastictiy is a crossed subject that researches the interaction of aerodynamic force, elastic force and inertia. Flutter is a typical aeroelastic problem of aircraft that can cause an unstable vibration and an instant crash. Because of the blunt leading edge of the hypersonic vehicle and the high angle of attack flight, there is a serious non-linearity of aerodynamic loads and brings many difficulties in solving hypersonic aeroelasticity. Litter research has done for the non-linear phenomena of hypersonic aeroelasticity. The crash of the hypersonic vehicle X-43 may due to the failure of the control surface because of flutter. The non-linear aeroelasticity in hypersonic flow needs serious consideration in a design process.

## 2 Analysis of the non-linear aerodynamic loads of hypersonic flow

Because of thin shock-layer viscous interaction, high temperature and the change of the structure modes due to the high temperature, the hypersonic aeroelasticity is a complex non-linear system. We just consider the aerodynamic non-linearity in this paper. Analyzing velocity potential equation, we get the source of aerodynamic non-linearity at high Mach numbers, and can find the aerodynamic loads of slender body at small angle of attack is still non-linear in hypersonic flow[1].

Consider the two-dimensional, irrotational, isentropic flow, the x component velocity, u, and the y component velocity, v. $u = V_\infty + \hat{u}$, $v = \hat{v}$, where $\hat{u}$ and $\hat{v}$ are called the perturbation velocities. We define the velocity potential $\Phi$, and the perturbation velocity potential, $\hat{\Phi}$, such that, $\Phi = V_\infty x + \hat{\Phi}$, where

$$\frac{\partial \hat{\Phi}}{\partial x} = \hat{u}, \quad \frac{\partial \hat{\Phi}}{\partial y} = \hat{v}$$

Hence,

$$\frac{\partial \Phi}{\partial x} = V_\infty + \frac{\partial \hat{\Phi}}{\partial x}, \quad \frac{\partial \Phi}{\partial y} = \frac{\partial \hat{\Phi}}{\partial y}, \quad \frac{\partial^2 \Phi}{\partial x^2} = \frac{\partial^2 \hat{\Phi}}{\partial x^2},$$

$$\frac{\partial^2 \Phi}{\partial y^2} = \frac{\partial^2 \hat{\Phi}}{\partial y^2}, \quad \frac{\partial^2 \Phi}{\partial x \partial y} = \frac{\partial^2 \hat{\Phi}}{\partial x \partial y}$$

Substituting the above definitions into velocity potential equation, we get the perturbation velocity potential

$$[a^2-(V_\infty+\hat{u})^2]\frac{\partial\hat{u}}{\partial x}+(a^2-\hat{v}^2)\frac{\partial\hat{v}}{\partial y}-2(V_\infty+\hat{u})\hat{v}\frac{\partial\hat{u}}{\partial y}=0 \qquad (1)$$

From energy equation we get

$$\frac{a_\infty^2}{\gamma-1}+\frac{V_\infty^2}{2}=\frac{a^2}{\gamma-1}+\frac{(V_\infty+\hat{u})^2+\hat{v}^2}{2} \qquad (2)$$

Substitution Equation (2) into Equation (1), we obtain

$$(1-M_\infty^2)\frac{\partial\hat{u}}{\partial x}+\frac{\partial\hat{v}}{\partial y}=M_\infty^2[(\gamma+1)\frac{\hat{u}}{V_\infty}+\frac{\gamma+1}{2}\frac{\hat{u}^2}{V_\infty^2}+\frac{\gamma-1}{2}\frac{\hat{v}^2}{V_\infty^2}]\frac{\partial\hat{u}}{\partial x}$$

$$+M_\infty^2[(\gamma-1)\frac{\hat{u}}{V_\infty}+\frac{\gamma+1}{2}\frac{\hat{u}^2}{V_\infty^2}+\frac{\gamma-1}{2}\frac{\hat{u}^2}{V_\infty^2}]\frac{\partial\hat{v}}{\partial y}$$

$$+M_\infty^2[\frac{\hat{v}}{V_\infty}(1+\frac{\hat{u}}{V_\infty})(\frac{\partial\hat{u}}{\partial y}+\frac{\partial\hat{v}}{\partial x})] \qquad (3)$$

Ignored the right-hand side, we can get the linear velocity potential equation. For the transonic flow or hypersonic flow, the right-hand side is not small in comparison with the left hand. This is why the hypersonic flow is difficult to be linearized even in the small perturbation condition.

Various engineering methods for aerodynamic loads have been presented according to the characteristic of the supersonic and hypersonic flow[1~4], such as Newtonian theory, piston theory, tangent-wedge/tangent-cone method, shock-wave and expansion-wave method. These methods are connected to solve a general shaped body and extended to solve the unsteady aerodynamic loads.

Modified Newtonian theory:

$$Cp=Cp_{\max}\times\sin^2\theta,$$

$$Cp_{\max}=\frac{2}{\gamma M_\infty^2}(\frac{p_{02}}{p_\infty}-1)$$

Piston theory: $p=p_\infty(1+\frac{\gamma-1}{2}\frac{w}{a_\infty})^{\frac{2\gamma}{\gamma-1}}$

Tangent-wedge/tangent-cone method:

$$Cp_{wedge}=2\theta^2[\frac{\gamma+1}{4}+\sqrt{(\frac{\gamma+1}{4})^2+\frac{1}{(M_\infty\theta)^2}}]$$

$$Cp_{cone}=\frac{4\sin^2\theta(2.5+8\sqrt{M_\infty^2-1}\sin\theta)}{1+16\sqrt{M_\infty^2-1}\sin\theta}$$

The formulas of shock-wave and expansion-wave method are little complex, which can be found in reference[3]**.** From the comparing with the experimental results or Euler results shown in Fig. 1 and Fig. 2, we can find the engineering methods has good precision in their use range. Just because of the good precision and high efficiency, these methods are widely used in engineering[2,4,5].

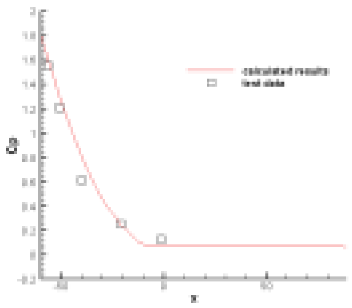
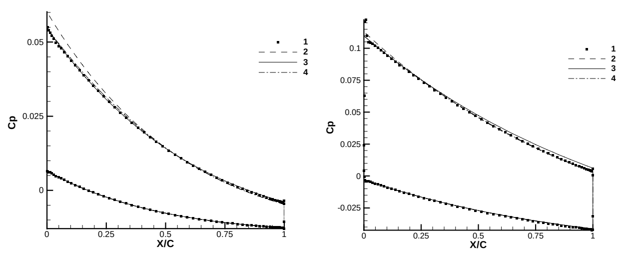


Fig.1 Pressure coefficient distribution of a blunt body




(a) $M$=10, $\alpha=3^\circ$ (b) $M$=5, $\alpha=5^\circ$

Fig. 2 Pressure coefficient distribution of a 4% thickness round airfoil
1—Euler，2—Precise piston, 3—Shock-expansion method，4—tangent-wedge

In this program, Newtonian theory, piston theory, tangent-wedge/tangent-cone method, shock-wave and expansion-wave method are connected to solve the random body's aerodynamic loads in hypersonic flow. Newtonian theory is used in high airflow-hitting angle surface, tangent-wedge/tangent-cone method is used in small airflow-hitting angle surface, shock-wave and expansion-wave method (piston theory) is used in the leeward. The connection of Newtonian theory and tangent-wedge/tangent-cone method use dahem-bark formula[3]. The connected engineering method is used to solve supersonic or hypersonic unsteady loads in time domain.

## 3 The time-based simulation of the aeroelasticity[2,6~8]

The time-based simulation is an important method for the engineer. It is the most expedient to construct, the most capable of handing complex modeling issues, and the most understandable with an engineer's physical intuition. For example, a non-linear aeroelastic system is often easily solved by a time-based approach.

Assuming the deformation of the wing is very small, the elasticity of the wing is linear. We use the traditional modes to describe the wing's oscillation, $z(x,y,t)=\sum_{i=1}^{N}\Phi_i(x,y)\cdot q_i(t)$, the N is the orders of the modes, $\phi_i(x,y)$ is the $i$th order mode. $q_i(t)$ is the generalized coordinate corresponding $\phi_i(x,y)$.

Basing on the Lagrange equation, the wing's movement equations can be written in matrix form:

$$[M]\{\ddot{q}\}+[G]\{\dot{q}\}+[K]\{q\}=\{F\}, \qquad (4)$$

[M] is the mass matrix, [G] is the structural damp matrix, [K] is the stiffness matrix, [F] is the generalized aerodynamic loads. $F_i=\iint_s p(x,y,t)\phi_i(x,y)dxdy$ , the unsteady pressure p(x,y,t) is solved by the unsteady aerodynamic solver at every time step.

In order to solve Eq. 4 by Ronge-Kutta time discretization, we introduce a variable—{E}, {E}= $[\{q\}^T,\{\dot{q}\}^T]^T$ = $[q_1,q_2,\cdots,q_N,\dot{q}_1,\dot{q}_2,\cdots,\dot{q}_N]^T$, Eq. 4 becomes:

$$\{\dot{E}\}=\begin{bmatrix}[O] & [I]\\ -[M]^{-1}[K] & -[M]^{-1}[G]\end{bmatrix}\{E\}+\begin{bmatrix}[O]\\ [M]^{-1}\end{bmatrix}\{F\} \qquad (5)$$

By the fourth order Ronge-Kutta discretization, we can get the wings pressure distribution which is needed for the generalized aerodynamic loads matrix at every time steps.

If there is an initial angle of attack, the wing will have a steady deformation under the steady aerodynamic loads except the dynamical deformations. The steady deformation has no effections to the aeroelasticity for the linear system. But in the hypersonic flow , the aerodymanic loads are non-linear, the steady deformation has effects on the aeroelastic problem. In order to consider the effects, the steady deformation can be calculated by Eq.6: $[K][q]_s=\{F\}$, which is get from Eq.5 by remove the dynamic term. Subscript "s" means steady deformations.

The detailed processes for the supersonic/hypersonic aeroelastic problems are:

(1) Calculating the aerodynamic loads on the wing, from the elastic equations and aerodynamic equations, we can get the steady deformation of the wing. At the base of the angle of attack, we add the deformation, solve the new aerodynamic loads, then solve the new steady deformation, time and time until the steady deformation converge to a stagnation data. This is the steady deformation at the angle of attack.

(2) When we get the steady deformation, there is a balance between the elastic force and the aerodynamic force. Adding an impulse at the balance, the wing will turn into a dynamic response. By analyzing the response, we can get the characteristics of the aeroelastic problem. Changing the Mach number, the angle of attack and the free stream dynamic pressure, we can get all the aeroelastic characteristics.

Table 1 shows that computed flutter speeds are about 10% of experimentally determined ones[2].

Table 1：The computed results comparing with the experimental results

| $M$ | Experimental velocity/ $m\cdot s^{-1}$ | Computational velocity/ $m\cdot s^{-1}$ | Relative error/% | Experimental frequency/Hz | Computational frequency/Hz |
|---|---|---|---|---|---|
| 1.53 | 424.5 | 454.6 | 7 | 51.2 | 53 |
| 2.01 | 505.0 | 438.8 | 13.1 | 31.2 | 41 |
| 2.01 | 504.8 | 434.5 | 13.9 | Lacked | 46 |
| 2.50 | 566.1 | 506.7 | 10.5 | Lacked | 59.4 |
| 3.01 | 606.0 | 605.0 | 0.2% | 36.6 | 40.8 |

## 4 Several non-linear phenomena in hypersonic aeroelasticity

By simulating the hypersonic wing's aeroelastic responses in time domain, some non-linear phenomena have been found as follows:

(1) The system doesn't apply the superposition principle. This example gives the comparative responses of the wing's first mode at $0^{\circ}$ angle of attack and $5^{\circ}$ angle of attack at 10 Mach numbers. From Fig. 3 we can find the wing's steady deformation has effect on the response. It changes the frequencies, amplitude and the phase of the response. The angles of attack have no effect on the figures of the wing's response in linear aeroelastic system.

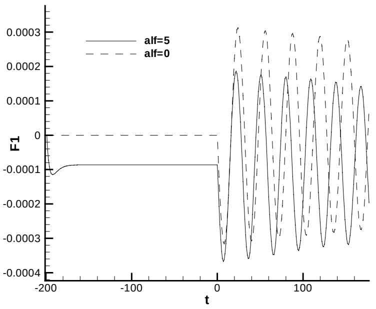



Fig. 3 The effect of the angle of attack on the response

(2) The stability of the aeroelasticity is affected by the initial conditions. The initial turbulence and the angel of attack influence the aeroelastic stability. The critical velocities (V) at both M 2 and M 5 decrease with the increasing of the initial turbulence (DF), but both the comparative value and absolute value of the decreasing range of M 2 is much less than M 5's as shown in Figure 4, that means $\left|\frac{\partial V}{\partial (DF)}\right|_{M=2} << \left|\frac{\partial V}{\partial (DF)}\right|_{M=5}$. This example shows that the non-linearity increases with the Mach numbers in supersonic flow.

(3) Distortion appears in the time domain response. That means the aeroelastic response in critical conditions is a periodic vibration, but not a harmonic vibration[6]. This is a typical phenomenon for non-linear systems. A critical response of a wing at M 10 has been simulated. The responses of the two modes are not harmonic but periodic vibrations as shown in Figure 5.

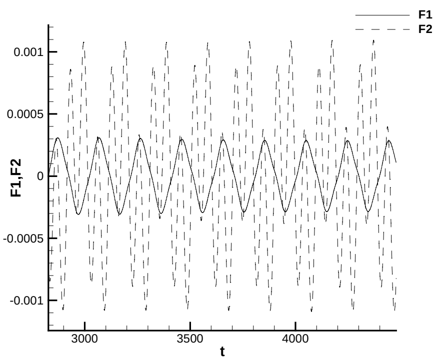
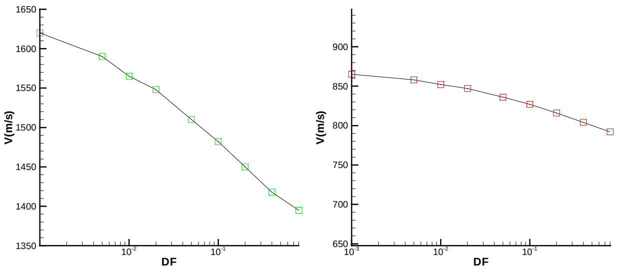



Fig. 5 Time history of a wing's response at *M* 10

(4) The LCOs are typical phenomena in the non-linear system[2]. Fig. 6 and Fig. 7 show the LCOs in hypersonic aeroelasticity at different velocities. From Table 2 we can find the frequencies and amplitudes both increase with the velocity.




(a) *M*=5 (b) *M*=2

Fig. 4 The effect of initial conditions on the flutter speed

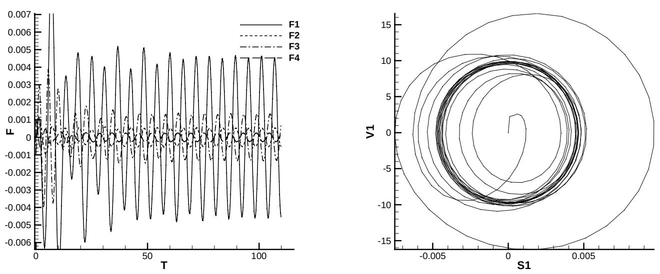



Fig. 6 The wing's response at $V_f$ =2000m/s

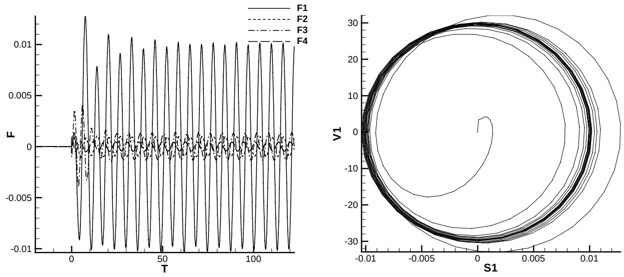
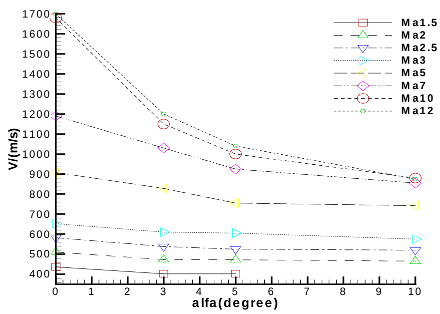



Fig.7 The wing's response at $V_f$=3000m/s

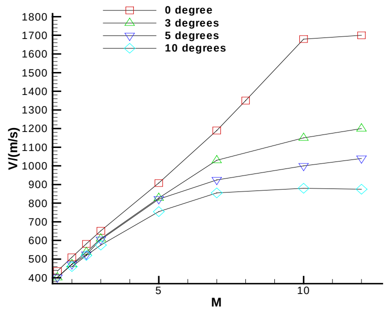
Table 2 The wing's nonlinear responses

| Velocity (m/s) | Type | $\omega_n$ (rad/s) | Amplitude |
|---|---|---|---|
| 1500 | Convergence | | |
| 1600 | LCO | 1850 | 0.0031 |
| 2000 | LCO | 2100 | 0.0045 |
| 2500 | LCO | 2570 | 0.0070 |
| 3000 | LCO | 2990 | 0.010 |
| 4000 | Diffusing | | |

(5) The non-linear relations between critical velocity and angle of attack/ Mach numbers in hypersonic aeroelastic system as shown in Figure 8[2,9]. The critical velocity decreases greatly with the increasing of the angle of attack at high M due to the non-linear aerodynamic loads. But for the low M supersonic flow, the angle of attack almost has no effect on the critical velocity as shown in Figure 8 (a). For low M supersonic flow, the critical velocity increases with the M. But for the high Mach numbers (M>10,Figure 8 (b)), the critical velocities are almost independent of the M. This phenomenon also suits the hypersonic Mach number independence principle. Not considering the effect of M on the structure, the hypersonic aeroelasticity accords with the Mach number independence principle.




(a) Critical velocity vs $\alpha$ at different $M$




(b) Critical velocity vs $M$ at different $\alpha$

Fig.8 The relation between critical velocity and $M/\alpha$

## 5 Conclusions

Analyzing the hypersonic flow, we testify the non-linearity of the aerodynamic loads at high Mach numbers that directly induces the non-linearity of the hypersonic aeroelasticity. By numerical method, the non-linear aeroelasticity is simulated in time-domain and some non-linear phenomena have been found as follows: (1) The system doesn't apply the superposition principle; (2) The stability of the aeroelasticity is effected by input and the original conditions; (3) Distortion appears in the time domain response, that is the response in critical conditions is not a harmonic; (4) The LCOs in hypersonic aeroelasticity; (5) The non-linear relations between critical velocity and angle of attack / Mach numbers. Because of the difficulty of the non-linearity, the work needs further research in the future.

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