ROBUST FLUTTER MARGINS OF A TYPICAL WING SECTION USING WINDTUNNEL EXPERIMENT DATA : A PRELIMINARY STUDY

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Abstract

Flutter prediction methods usually rely on tracking modal damping trends,estimated from flight/experimental data, which are not always accurate indicators of flutter onset. This methods is based on a finite element model of the aircraft and does not directly consider flight/experimental data from the physical model. A new approach to computing flutter instability boundaries based on the structured singular value is presented. This approach is developed that utilizes a theoretical model while directly accounts for the variations using the experimental data. The aeroelastic stability problem is formulated in a fremework suitable for well-developed robust stability theory by parameterizing around velocity and introducing uncertainty operators to account for modeling errors. Experimental data can be used to validate the robust system model and increase accuracy of the flutter margin estimate.
Parameterization around velocity allows the generalized equation of motion to be a linear function of wind tunnel flow-speed so that perturbations to this parameter can be entered in the form of linear fractional transformation. The μ-analysis method will treat the perturbation as a system uncertainty. Two uncertainty operators are used to describe the modeling uncertainties in the linear aeroelastic model. The first uncertainty operator is associated with the state matrix of aeroelastic linear model. This uncertainty models variations in both the natural frequency and damping values for each mode. The second uncertainty operator is a multiplicative uncertainty on the force input to the linear model. This uncertainty is used to cover nonlinearities and unmodeled dynamics. The level of both uncertainty is determined from reasoning of the modeling process and analysis on the wind tunnel experiment data.
Using this method on an aeroelastic wing section system gives a flutter prediction that is closer to the experimental result, which means it can give a better prediction from safety point of view.

1. Introduction

Aeroelastic system is a combination of elastic structure and aerodynamics system. Aeroelastic is a critical system, since it can lead to an unstable condition. Several analysis techniques for predicting the behaviour and stability of an aeroelastic system have been developed. Most of those techniques based their analysis on mathematical representation of the system, whether analytical models or empirical ones. In other words, the analysis and investigations will entirely depend on the quality of the model representation and/or the technique in obtaining data from experimental test for an empirical model.

As a combination of structural and aerodynamics system, an aeroelastic model must be able to represent the characteristics of each part and the interaction between them.

Modeling of an aeroelastic system is a difficult task, since in the real system there are many uncertainties involved. Some non-linearities will be found on both the structural and aerodynamics part. As these forces interact each other, some other uncertainties will also occur.

These non-linearities are difficult to be modeled, since there are some constraints that should be taken into account in building a mathematical model, the limitation of model order is one of them. If most of these non-linearities are included in the model, a very high order model will be obtained, which is not practical for engineering use, like for control apllications. In other hand, neglecting most most of them will diverge the model from its real system and thus will make the analysis becomes incorrect, a result that is very dangerous considering the critical behaviour of aeroelastic system.

2. Aeroelastic Nominal Model

The aeroelastic nominal model can be derived by combining the representation of structural dynamic force, which involves inertia, stiffness, and damping forces, with the unsteady aerodynamics forces. The result is an aeroelastic equation of motion as follows :

$$[M]\{\ddot{z}\}+[C]\{\dot{z}\}+[K]\{z\}=\{F_{aero}\} \qquad (1)$$

where [M], [C], [K], {z}, {Faero}, denote the inertia matrix, the damping matrix, the stiffness matrix, the variable of motion, and the aerodynamic force vector respectively. Using two degree of freedom of the system as the variables, and employing steady aerodynamics forces approach for the external aerodynamic load, the following aeroelastic system formulation can be obtained :

$$\left[\overline{M}_S-\frac{\bar{q}Sc}{2U_0^{\,2}}\overline{M}_a\right]\begin{Bmatrix}\ddot{h}\\ \ddot{\alpha}\end{Bmatrix}+\left[\overline{D}_S-\frac{\bar{q}S}{U_0}\overline{D}_a\right]\begin{Bmatrix}\dot{h}\\ \dot{\alpha}\end{Bmatrix}+\left[\overline{K}_S-\bar{q}S\overline{K}_a\right]\begin{Bmatrix}h\\ \alpha\end{Bmatrix}=0 \qquad (2)$$

where,

$\overline{M}_S=\begin{bmatrix} m & S_{h\theta}\\ S_{h\theta} & I_\theta\end{bmatrix}$ is the structural mass matrix

$\overline{D}_S=-\overline{M}_S\begin{bmatrix} 2\zeta_h\omega_h & 0\\ 0 & 2\zeta_\alpha\omega_\alpha\end{bmatrix}$ is the struct.damping

$\overline{K}_S=\begin{bmatrix} k_h & 0\\ 0 & k_\alpha\end{bmatrix}$ is the struct. stiffness

The above formulation gives a model representation of aeroelastic system in a low and narrow reduced frequency range near steady condition.

3. Robust μ Framework

As it has been mentioned earlier, building a "good" aeroelastic model is a difficult task. Analytically, it is difficult to model all dynamics of a system which contains high frequency dynamics and uncertainties. Other technique that can be used is by employing system identification scheme to obtain an empirical model. This technique principally works by fitting a model formulation to the input-output data of the real system. Since it uses input-output data, this technique depends on the quality of the data and/or the process of producing the data. This means that the technique is susceptible to measurement noise (SNR) in experimental data, and/or the simulation technique to generate the data. Considering all those problems, a new approach has been introduced which uses the term of "validation" rather than "identification" in describing the technique. This technique is based on the robust μ framework which considers the characteristics of a system model under the influence of perturbation. The perturbation are used to model any "difficult to model" parts of the system like unmodelled dynamics, unmeasured forces, non-linearities, and parameters uncertainties. An operator , Δ, is used to include each perturbation to the system and contained in a set Δ. This set Δ is norm-bounded to anticipate the limits of the size of the perturbation range :

$$\Delta = \{\Delta : \|\Delta\|_\infty \leq 1\} \tag{3}$$

This uncertainty operator is connected to the nominal system in feedback manner. This relation is called the linear fractional transformation (LFT) which can accommodate multiple systems and uncertainties, and formed them as a plant with associated uncertainty operators.

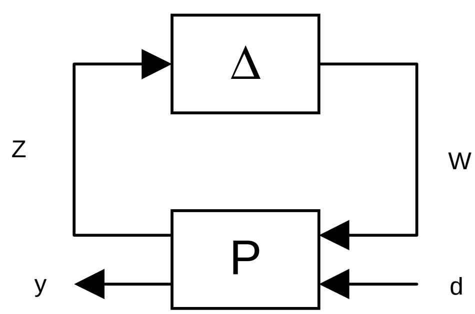

Fig. 1. Robust Stability Analysis Block Diagram

The robust stability of a system P interconnected with uncertainty set **Δ** can be guaranteed if

$$\mu(P) \; \langle \; 1 \tag{4}$$

where :

$$\mu(P) = \frac{1}{\min\limits_{\Delta\in\Delta}\{\overline{\sigma}(\Delta) : \det(I - P\Delta) = 0\}} \tag{5}$$

The structured singular value μ is a measure of robustness of P with respect to **Δ.** If there is no Δ exist such that det(I - PΔ) = 0, then μ = 0. The inverse of μ can be thought of as the magnitude of the smallest perturbation Δ which can make the system P becomes unstable. μ is an exact measure of robustness of a system with structured uncertainty.

This μ approach can be used to form a realistic model of a real system. To obtain a realistic model, it requires that the uncertainty description of the systems must be realistic too. Too much uncertainty level will make the obtained system becomes too conservative, and in the other hand lack of uncertainty will give a model which doesn't represent the true error in the model. To "adjust" the uncertainty level, a model validation algorithm is developed which can indicate when the uncertainty level is "fit" with the realistic error in the model. This algorithm uses experimental data as the "guidance" in determining the range of uncertainty level which is realistic for the model as the representation of the real system. It can be seen here that this algorithm uses both the nominal model, which is obtained analytically, and the experimental data to "adjust" the uncertainty connected to the nominal model, not the parameters of the model itself. So, the difference with the identification technique, which entirely uses real data to form the "full" model, is quite clear. It can also be explained that in identification technique, structured formulations for the unknown elements of the system are assumed at the initial stage, and the data will be used to provide the magnitude for these structures. In validation technique, both the structures and initial estimate for every element of the system are assumed, and then the data is used to improve the structured elements.

4. Uncertainty in Aeroelastic Model

4.1. Parametric Uncertainty

Parametric uncertainties are associated with specific parameters in the aeroelastic system. In the aeroelastic model developed in this paper, this type of uncertainty is used for describing the modeling errors in the inertia, stiffness, and damping parameters of the aeroelastics. Despite describing the error in the value of each parameters, in this paper the error is included in term of the eigen value error. This uncertainty operator is included for each mode considered in the system. This can be done by substituting the block matrix of each mode with the related block describing its natural frequency and damping, as follows :

$$A_i = \begin{bmatrix} e_r & e_i \\ -e_i & e_r \end{bmatrix} \qquad (6)$$

where :

$$\begin{aligned} \omega_i &= \sqrt{e_r^{\,2} + e_i^{\,2}} \\ \zeta_i &= -e_r / \omega_i \end{aligned} \qquad (7)$$

The uncertainty for the eigen value error can be inserted by defining :

$$\begin{aligned} \bar{e}_r &= e_r (1 \pm w_r \delta) \\ \bar{e}_i &= e_i (1 \pm w_i \delta) \end{aligned} \qquad (8)$$

Scalar weightings, w_r and w_i are used to affect the amount of uncertainty in each matrix element which represent the amount of variation in the natural frequency and damping. These variation are determined by the magnitude of these scalar weightings.

The eigen block for each mode becomes

$$\tilde{A}_i = \begin{bmatrix} \bar{e}_r & \bar{e}_i \\ -\bar{e}_i & \bar{e}_r \end{bmatrix} \qquad (9)$$

The real part that represents damping of the aeroelastic modes is usually lower compared to the imaginary part. Due to the experimental technique which usually identifies the natural frequency better than damping value, the choosing of weighting of the real part is expected to be larger than that of the imaginary part.[3] This is shown by the observed modal parameters in the windtunnel experiment data. The natural frequencies show variations of ± 5 % from the theoritical model whereas the uncertainty in the damping ±15 %.

4.2. Dynamic Uncertainties

Dynamics uncertainty is a type of uncertainty usually used for describing any errors in modeling which is not to be represented by parametric uncertainty. This uncertainty can represent error in both magnitude and phase of signal. Modeling error like negligence of high frequency dynamics can be modeled with dynamic uncertainty, which will give a model with lesser conservatism than that given by parametric uncertainty. A multiplicative uncertainty, Δ_M, can be used to anticipate a high frequency mode which is not included in the linear model. Using this uncertainty, then the model can be represented as :

$$P = P_o (I + W_M \Delta_M) \qquad (10)$$

where P is the real or "validated" system, P_o is the nominal model, W_M is the weighting function, and Δ_M is the uncertainty operator. The determination of the W_M is based on equation (10) by using the difference between experimental frequency responses function (FRF) and theoritical FRF [1]. The result of experimental FRF data can be seen on the later section.

5. The typical Wing Section Model

The aeroelastic system investigated in this paper is a typical wing section installed on a wind-tunnel test section. This model is mounted on its rig via 8 springs to provide the stiffness of the system. Two accelerometers installed inside the wing body structure are used as the sensors for observing the dynamic response of the system. Excitations can be stored to the system through the control surface or by implementing an impact directly to the structure.

The aeroelastic mathematical model for the typical wing section has been derived using the method mentioned earlier. Two lowest mode, heaving and torsion, are used in building the equation. This nominal model contains some uncertainties caused by limitations in the modeling technique. Some uncertainties are defined for this nominal model such that this model can be “adjusted” to approx the real system. The modeling error associated with the eigen value of the 2 modes used are anticipated by 2 blocks of parametric uncertainties. A dynamics uncertainty is introduced for anticipating an unmodeled dynamics, that is the rolling mode, which is not included in the nominal model although it is present in the real system. Experimental data is used to obtain and shape the weighting factor in the uncertainty blocks. The Robust μ approach then is used to calculate the bound of the uncertainty operator which can validate the model. The model is shown in Fig.2.

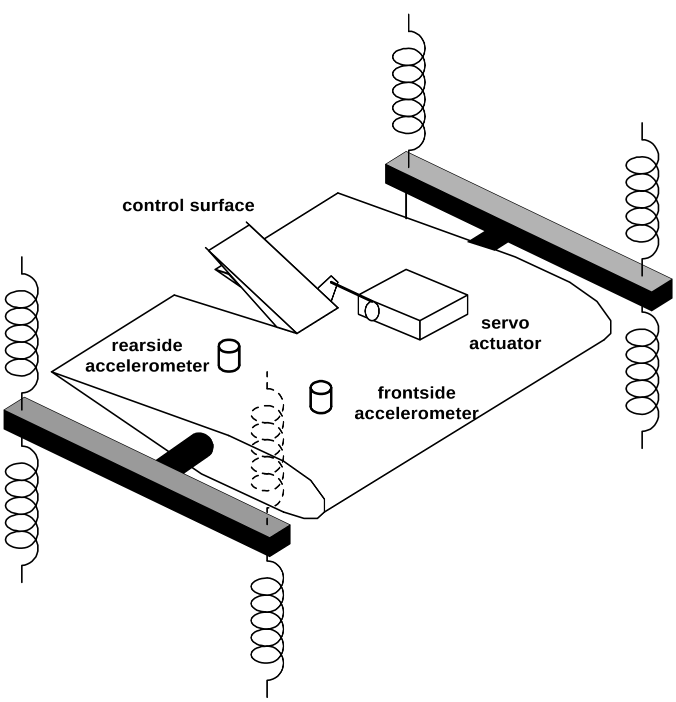

Fig.2 Typical Wing Section Model

6. Experimental set-up

Experimental data is obtained by performing dynamic test on the typical wing section model under various wind loading condition. This can be done by carrying out the test on various wind-tunnel flow speed.
The typical section model is mounted on the wind-tunnel via 8 tension springs which can be adjusted to obtain the desired stiffness. The typical section is equipped with two accelerometers, front and rear location with respect to freestream direction, as motion sensor. It also has a control surface driven by a electroservo actuator.
During the test, the typical section is excited by applying an impact load via a rod extended from the top of the test section. This rod is attached to a force transducer for sensing the load transmitted to the model. The output signal from force transducer, along with the model response signal from the accelerometer, is transformed by conditioning amplifier and then is fed to the Dual Channel Digital Signal Analyzer. The signal analyzer will process the data and calculate the Frequency Response Function (FRF) of the model. Repeating these steps for a variation of flow speed , a set FRF for different flow speeds can be obtained. This set of model FRF then is used for determining the weighting function to construct the model in LFT form.

7. Experimental Data

Some experimental data are generated by performing dynamic test on the typical wing section system. The dynamics behaviour of the system, in term of its Frequency Response Function (FRF), are obtained from the test for a range of wind-tunnel flow speed.

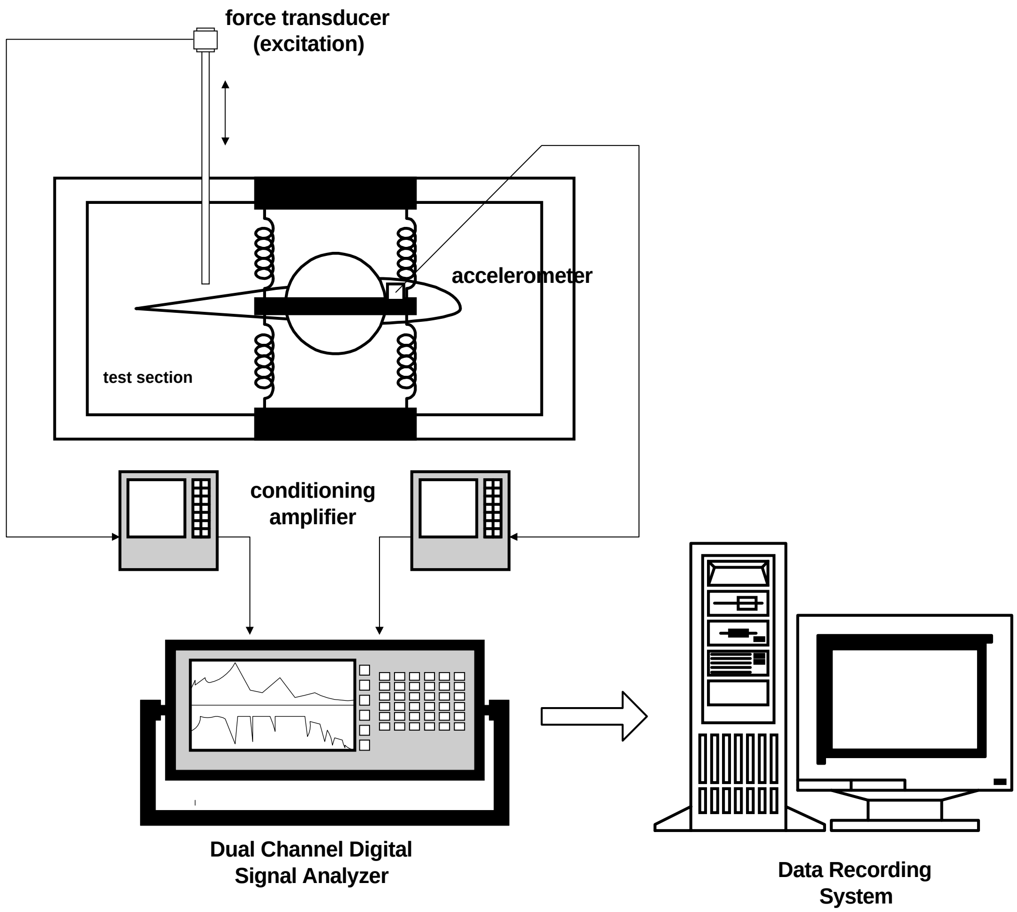

Fig.3. Instrumentation Set-Up

8. Analysis And Conclusions

In the aeroelastic model, one of the sources of errors is in determining the correct value of the system damping. This problem will produce uncertainty in the system. Error in determining the stiffness and inertia of the system can also produce uncertainties. These problems can be represented in error value of each element. Other approach can be used is by representing those errors in terms of the eigen value errors of each mode included.

The result obtained from this work gives a framework in building a system model by employing the Robust μ approach which can involve the modeling error into the nominal model via the uncertainty operators.

Model validation approach uses experimental data to adjust the uncertainty operators, not the model parameters, so that a realistic model, compared with the real system, can be obtained.

The uncertainty level determine the level of conservatism of the model obtained. Too many uncertainties will give a too conservative system, which reflects a unrealistic system, and lack of uncertainty will give a model which is biased from its real system. The level of conservatism will affect the result of any analysis performed on the system.

The approach of this robust μ framework, can be extended to develop aeroelastic analysis method, like the determination of the flutter margin.

In figure 4 and 5, it can be seen that there are differences between experimental data and the data generated from an analytical model. These differences are caused by the existence of unmodelled uncertainties and the negligence of

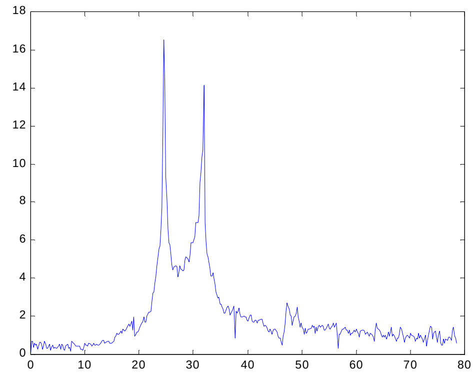

Fig. 4. The FRF of the wing system (U=3 m/s)

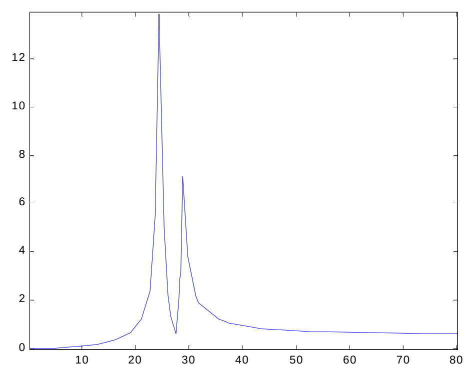

Fig. 5. Bode Diagram (magnitude) of the nominal analytical model (U=3 m/s)

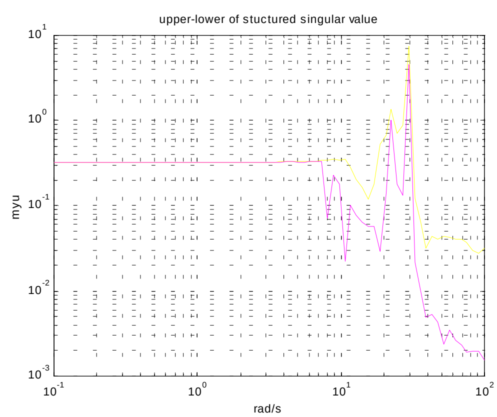

Fig. 6. Structured Singular Value Plot

structural damping parameters in the analytical model.

The structural damping are neglected since the curve fitting technique used to extract the structural dynamics parameters from the data can not give an accurate value of damping parameters. This limitation will affect the determination of flutter margin of the model. A further study is needed to investigate this aspect.

Figure 6 shows that the structured singular value (SSV) analysis at flow speed 7 m/s gives a lower-myu value that is bigger than one, which means that at this flow speed an instability condition (fluter) occurs. It can also be figured out that the flutter frequency is 4.7 Hz.

As comparation, the experimental results give flutter speed of 7.5 m/s at the frequency of 4.5 Hz., and the calculation of analytical model using P-method gives flutter speed of 8 m/s and the flutter frequecy of 4.6 Hz.

These results shows that analytical P-method predicts a higher flutter speed than the experimental one. If the structural damping is considered in the analytical model, the predicted speed will get higher and this means an increasing of the difference with experimental result.

From safety point of view, this result can not be accepted, since it gives a higher flutter speed than that of the real system (experimental result).

Alternatively, the SSV approach gives a better result, either the result is closer to or lower than the experimental result, which means it can cope the safety aspect.

In further study, some improvement in modeling will be carried out, which involve structural damping in the model. An improvement in determining the weighting matrices w_r and w_i will also be explored by observing the modal parameters data in a range of flow speed value.

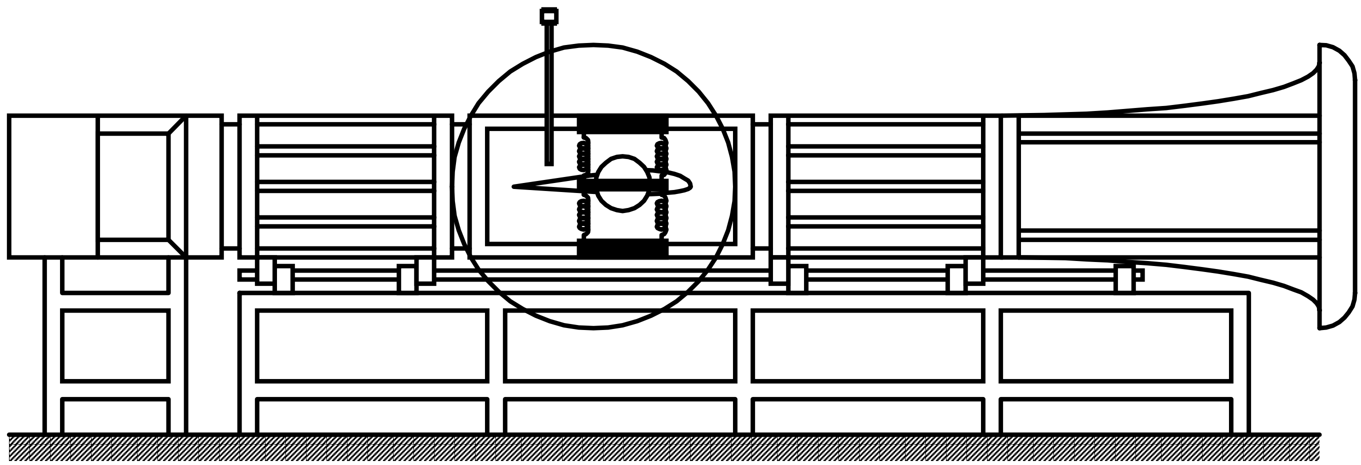

Fig.7. Open-Loop Wind-tunnel used in the test

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