FATIGUE LIFE PREDICTION OF 3-D PROBLEMS BY DAMAGE MECHANICS WITH SPECTRUM LOADING

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Abstract

The importance of applied load on the structure members cannot be denied in fatigue life prediction. A basic problem in fatigue research is the prediction of lifetime of a structural part when the amplitude of cyclic load varies in prescribed fashion with the number of cycles. This paper presents the prediction of fatigue lives for 3-D problems in elasto-plastic range with variable load amplitude. The effect of loading sequence on the fatigue life of a structure member and the validity of Miner's rule are studied. The computational method is derived according to damage mechanics, theory of plasticity and finite element analysis (FEA). A two-block cyclic loading is considered with high-low and low-high load sequence. To consider the effect of the high stress level beyond the yielding point of material in one load block, deformation theory and iteration method are applied to the stress analysis of the first load cycle. The damage evolution under given stress and damage fields for each block loading is determined by damage evolution equation. Furthermore, an additional loading method is introduced to perform the stress analysis with given damage field to avoid the reassembling of the stiffness matrix of structure member. Damage increment at critical element is considered as step length instead of load cycle increment. Comprehensive computer programs are developed to consider both elastic and elasto-plastic cases.

1 INTRODUCTION

Structures are rarely stressed repeatedly at a single load level, and the failure of a structure is, therefore, the result of fatigue damage accumulation caused by a multiplicity of loading cycles having different amplitudes and frequencies. As it is known, the prediction of fatigue life under spectrum loading has been the subject of extensive research.

A simplest and well-known model of fatigue damage accumulation is proposed by Palmgren [1] and Miner [2] known as Miner's Rule. According to the rule, the failure in a multi-stage loading is defined by

$$\sum_i \frac{n_i}{Ni} = 1$$

where, n_i is the number of load cycles at load level σ_i, and N_i the total fatigue life at the same load level in constant amplitude.

According to Miner's rule, the damage produced by σ_i for n_i cycles is defined as n_i/N_i and the individual damages are additive and independent of loading sequence. This simplest approach does not, in general, comply with reality. It is shown from experimental research that the sequence of loading can significantly affect the lifetime of a specimen. In multi-stage constant amplitude loading, the left side of above equation, which is sometimes termed Miner's index, is usually different from unity and is dependent on loading sequence.

A simple case of two block cyclic loading is considered with the stress level in high-low and low-high sequence, as shown in Fig-1 and 2. The amplitude of loading in each respective block is kept constant but different from each other. Both elastic and elasto-plastic cases caused by load blocks are investigated to see the effect of the combination and sequence of different load amplitudes.

Fatigue life properties of the structure member under constant amplitude loading are needed as a baseline data [5]. The effects of high-load cycles on fatigue life at subsequent low-load cycles are determined by comparing the computational results from the variable-amplitude analysis with the baseline data.

Damage mechanics-finite element-additional load method for the prediction of crack initiation life with damage increment as step length is introduced [4]. Reassembling of stiffness matrix during material degradation is eliminated by use of this approach, hence the CPU time is drastically reduced. Comprehensive computer programs are developed to cater for both elastic and elasto-plastic cases.

2 INITIAL ELASTO-PLASTIC ANALYSIS

During the first cycle of repeated loading, if the stress level at the critical point(s) of structure member is beyond the yielding point, the critical zone becomes plastic.

2.1 Governing Equations

The general governing equations from solid mechanics are given as follows:

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}) \quad (1)$$

$$\sigma_{ji,j} + B_i = 0 \quad (2)$$

$$\sigma_{ij} l_j = p_i \ \text{ on } \ S_p, \quad u_i = \bar{u}_i \ \text{ on } \ S_u \quad (3)$$

2.2 Constitutive Relations and Plastic Correction

The deviatoric strain and stress components can be given as

$$e_{ij} = \varepsilon_{ij} - \delta_{ij}\frac{1}{3}\varepsilon_{kk}, \quad s_{ij} = \sigma_{ij} - \delta_{ij}\frac{1}{3}\sigma_{kk} \quad (4)$$

where, $\frac{1}{3}\sigma_{kk}$ and ε_{kk} are the hydrostatic stress and volume strain, respectively, δ_{ij} is the kronecker delta, s_{ij} and e_{ij} are the deviatoric stress and strain components, respectively.

The relationship between deviatoric stress and strain components can be given by the deformation theory of Hencky as follows:

$$s_{ij} = \frac{2\sigma_e}{3\varepsilon_e} e_{ij} \quad (5)$$

where, σ_e and ε_e are the equivalent stress and strain, respectively.

The relationship between hydrostatic stress and volume strain can be derived from Hooke's Law as follows:

$$\sigma_{kk} = 3K\varepsilon_{kk} \quad (6)$$

where, K is the bulk modulus in terms of Lame's constants.

Eqn-4 can be re-written after substitution of eqn-5 and 6 as follows:

$$\sigma_{ij} = \frac{2\sigma_e}{3\varepsilon_e} e_{ij} + \delta_{ij} K\varepsilon_{kk} \quad (7)$$

where, $\frac{\sigma_e}{\varepsilon_e} = E(1-w)(1-D)$ or

$$w = \frac{E\varepsilon_e(1-D) - \sigma_e}{E\varepsilon_e(1-D)} \quad (8)$$

In which w can be described as the relative reduction of young's mudulus due to plastic effect and D is the initial damage before the first cycle of loading, if any. It is assumed that the relationship between σ_e and ε_e is the same as that in uni-axial stress case.

Eqn-7 can finally be written in the following form

$$\sigma_{ij} = \sigma_{ij}^{(e)} - \sigma_{ij}^{(P,D)} \quad (9)$$

In above equation, $\sigma_{ij}^{(e)}$ is the linear stress components and $\sigma_{ij}^{(P,D)}$ is the stress reduction components caused by plasticity and damage, given as follows:

$$\sigma_{ij}{}^{(e)} = \frac{2}{3} E e_{ij} + \delta_{ij} K \varepsilon_{kk} \ , \ \ \sigma_{ij}{}^{(P,D)} = \frac{2}{3} E e_{ij} \left(w + D - wD \right) \tag{10}$$

For the case of linear hardening, w can be expressed as

$$w = (1-\xi)\left(1 - \frac{\varepsilon_p}{\varepsilon_e}\right) \tag{11}$$

where, ξ can be determined by the elasto-plastic curve shown in Fig-3, as follows:

$$\xi = \frac{\sigma_u - \sigma_s}{E(\varepsilon_u - \varepsilon_s)} \tag{12}$$

In which, σ_u, σ_s and $\varepsilon_u, \varepsilon_s$ are the ultimate and yielding stresses and strains, respectively.

2.3 Elasto-Plastic Finite Element Analysis

According to above elasto-plastic constitutive relation, the stress components of an element can be expressed as:

$$\{\sigma\}_e = \{\sigma\}_e^{(e)} - \{\sigma\}_e^{(P,D)} = [E]_e [B]_e \{\delta\}_e - \{\sigma\}_e^{(P,D)} \tag{13}$$

in which, $[E]_e$ is the matrix of element material stiffness,$[B]_e$ the matrix of strain-displacement transformation and $\{\delta\}_e$ the vector of element nodal point displacements.

By applying the principle of virtual work at element level, we will get

$$[K]_e \{\delta\}_e = \{F\}_e + \{F\}_e^{(P,D)} \tag{14}$$

where, $[K]_e = \int_{V_e} [B]_e^T [E]_e [B]_e \, dv$ - the element stiffness matrix

$\{F\}_e = \int_{V_e} [N]^T \{P_v\} dv + \int_{S_e} [N]^T \{P_s\} ds$ - the conventional element nodal force vector

$\{F\}_e^{(P,D)} = \int_{V_e} [B]_e^T \{\sigma\}_e^{(P,D)} dv$ - the additional nodal force vector to consider the relative reduction of stiffness due to plastic effect and initial damage, if any.

Expanding each matrix and vector in eqn-14 at structure level, the governing system of non-linear equations is assembled as follows:

$$[K]\{\delta\} = \{F\} + \{F\}^{(P,D)} \tag{15}$$

where, $[K]$ is the matrix of global stiffness of 3-D sound body, $\{\delta\}$ the vector of global nodal point displacements, $\{F\}$ the overall nodal force vector caused by original loading and $\{F\}^{(P,D)}$ the additional nodal force vector at structure level to consider the effect of plasticity and initial damage state.

To solve eqn-15, an additional load-iteration method is introduced.

2.4 Iteration Method Solution

In the first step of iteration it is assumed that the material is linearly elastic, that is

$$w_1 = 0, \quad \sigma_{ij\ 1}^{(P,D)} = \frac{2}{3} E e_{ij\ 0} D_0 \tag{16}$$

where, D_0 represents the initial damage state before iteration process.

In the second step, if the stress level is above the yield limit then

$$w_2 = (1-\xi)\left(1 - \frac{\varepsilon_p}{\varepsilon_{e1}}\right), \quad \sigma_{ij\ 2}^{(P,D)} = \frac{2}{3} E e_{ij\,1} \left(w_2 + D_1 - w_2 D_1 \right) \tag{17}$$

Similarly, for the nth step

$$w_n = (1-\xi)\left(1 - \frac{\varepsilon_p}{\varepsilon_{e\,n-1}}\right) \text{ and }$$

$$\sigma_{ij\ n}^{(P,D)} = \frac{2}{3} E e_{ij\,(n-1)} \left(w_n + D_{(n-1)} - w_n D_{(n-1)} \right) \tag{18}$$

The system of equations at structure level for the nth step has the form

$$[K]\{\delta\}_n = \{F\} + \{F\}_n^{(P,D)} \tag{19}$$

The iteration process will be paused when the relative error in the $\varepsilon_{e\ n}$ and $\varepsilon_{e\ n\text{-}1}$ at critical element is less then 1%. The solution of iteration method is shown in Fig-4. A Computer program is developed for the elasto-plastic finite element analysis by means of the iteration method to determine the initial elasto-plastic equivalent stress and the modified stress ratio, which will be used in the damage analysis later.

2.5 Additional Loads for Stress Analysis During Damage Evolution

Since the stiffness of structure member reduces with every increment of damage, the concept of damage-induced additional loads is introduced in order to avoid recurrent calculation of the stiffness matrix during damage evolution. It is known that after the initial loading, the stress and strain components will vary in elastic range under subsequent loading, with the consideration of the damage coupling effects. Then, we have

$$\tilde{\sigma}_{ij} = C_{ijkl}(1-D)\tilde{\varepsilon}_{kl} \qquad (20)$$

where, $\tilde{\varepsilon}_{ij}$ and $\tilde{\sigma}_{ij}$ are the range of variation of strain and stress components, respectively.

According to eqn-20, we have

$$\tilde{\sigma}_{ij} = \tilde{\sigma}^E_{ij} + \tilde{\sigma}^D_{ij} \qquad (21)$$

where, $\tilde{\sigma}^E_{ij}$ can be defined as the range of variation of elastic stress components, i.e.,

$$\tilde{\sigma}^E_{ij} = C_{ijkl}\ \tilde{\varepsilon}_{kl} \qquad (22)$$

and $\tilde{\sigma}^D_{ij}$ the range of variation of damage-induced additional stress components, i.e.,

$$\tilde{\sigma}^D_{ij} = -D\ C_{ijkl}\ \tilde{\varepsilon}_{kl} \qquad (23)$$

By substituting these equations into the equilibrium equations and boundary conditions, we get:

$$\tilde{\sigma}^E_{ji,j} + \tilde{B}_i + \tilde{B}^D_i = 0 \qquad \text{in } v \qquad (24)$$

$$\tilde{\sigma}^E_{ij}\, l_j = \tilde{p}_i + \tilde{p}^D_i \qquad \text{on } S_p \qquad (25)$$

where, $\tilde{B}^D_i = \tilde{\sigma}^D_{ji,j}$ is the range of variation of damage-induced additional body force components and $\tilde{p}^D_i = -\tilde{\sigma}^D_{ij}\, l_j$ is the range of variation of damage-induced additional surface traction components

3 FINITE ELEMENT ANALYSIS FOR GIVEN DAMAGE FIELD

Additional load-finite element method [3] is introduced in the analysis of 3-D stress field for given damage field. By means of damage coupled constitutive relation, we have

$$\{\tilde{\sigma}\}_e = (1-D_e)[E]_e\{\tilde{\varepsilon}\}_e = (1-D_e)[E]_e[B]_e\{\tilde{\delta}\}_e \qquad (26)$$

where $\{\tilde{\sigma}\}_e$, $\{\tilde{\varepsilon}\}_e$ and $\{\tilde{\delta}\}_e$ are the vectors for the range of variation of stress, strain and displacement components of an element, respectively, D_e is the damage of the element.

By applying the principle of virtual work at element level, we can obtain

$$[K]_e\{\tilde{\delta}\}_e = \{\tilde{F}\}_e + \{\tilde{F}\}^D_e \qquad (27)$$

where, $\{\tilde{F}\}_e$ is the vector for the range of variation of conventional nodal forces and $\{\tilde{F}\}^D_e$ the vector for the range of variation of additional nodal forces given by

$$\{\tilde{F}\}^D_e = D_e[K]_e\{\tilde{\delta}\}_e \qquad (28)$$

Expanding each matrix and vector in eqn-27 to the order of global level, the governing system of equations is assembled as follows

$$[K]\{\tilde{\delta}\} = \{\tilde{F}\} + \{\tilde{F}\}^D \qquad (29)$$

where, $\{\tilde{F}\}$ and $\{\tilde{F}\}^D$ are the vectors for the range of variation of conventional nodal forces and additional nodal forces at the structure level, respectively. Obviously, with the application of additional load method, the analysis of displacement and stress fields for a damaged body can be carried out as that for sound body. 3-D mesh arrangement is also optimized by convergence tests. In the finite element analysis the nodal point displacements are obtained by using 8-node brick elements. Then each brick element is divided into 5 four-node tetrahedral elements for further damage analysis. After obtaining the nodal point displacements for each brick element, the stress field and additional load vector can be calculated by using eqn-26 and 28 from tetrahedral elements.

4 EQUATIONS OF DAMAGE EVOLUTION

Lemaitre and Chaboche are amongst those researchers who introduced equations of damage evolution during 1970s. The equation of damage evolution can be expressed as follows:

$$\frac{dD}{dN} = \gamma(R)\left\{\frac{\tilde{\sigma}_e}{1-D}\right\}^m \left\{\frac{1}{1-D}\right\}^q \qquad (30)$$

where, N means the number of load cycles, γ, m and q represent the material behaviors which can be determined by means of conventional fatigue testing, and $\tilde{\sigma}_e$ is the range of variation of equivalent stress given by

$$\tilde{\sigma}_e = \sigma_e(1-R) \qquad (31)$$

with R being the stress ratio in one load cycle.

Eqn–30 can also be written in the form of equivalent stress as follows:

$$\frac{dD}{dN} = \alpha\left\{\frac{\sigma_e}{1-D}\right\}^m \left\{\frac{1}{1-D}\right\}^q \qquad (32)$$

where, α can be expressed as follows,

$$\alpha = \beta(1-R)^n \qquad (33)$$

here, β, n, m and q are material dependent constants which can be estimated with the help of *S-N* curves of material for at least two stress ratio cases.

5 DAMAGE FIELD ANALYSIS AND LIFE PREDICTION

For the physically nonlinear problems caused by stress-damage coupling, the increment of damage in a critical element is taken as the step length to analyze the crack initiation and crack propagation for the structure member under repeated loading. In every stage of crack initiation and crack propagation, the current critical element is selected through the assessment of damage evolution rate dD/dN for all elements. For a given increment of damage ΔD^* of current critical element, the relevant increment in load cycles and the damage increments of other elements are given by

$$\Delta N = \frac{\Delta D^*}{\beta\left(\tilde{\sigma}_e^*\right)^m}\left(1-D^*\right)^{m+q}, \quad \Delta D = \left(\frac{\tilde{\sigma}_e}{\tilde{\sigma}_e^*}\right)^m \left(\frac{1-D^*}{1-D}\right)^{m+q} \Delta D^* \qquad (34)$$

where, "*" represents the field quantities for current critical element.

Then, the current damage field will be expressed by $D^* + \Delta D^*$ and $D + \Delta D$ for the current critical element and other ones, respectively. The corresponding field quantities u_i, ε_{ij} and σ_{ij} $(i, j = 1,2,3)$ can be obtained by the finite element-additional loading method. The above two processes will be proceeded alternatively and paused when the damage in the current critical element reaches 1.0. The number of load cycles for the complete failure of the first critical element represents the crack initiation life. The total number of load cycles during crack initiation is, then, given by the following expression,

$$N = \sum_{r=1}^{M} N_r \qquad (35)$$

where, M is the number of steps during crack initiation, and N_r the number of load cycles of the r^{th} step of crack initiation.

6 FATIGUE ANALYSIS UNDER SPECTRUM LOADING

For simplicity, the load spectrum consists of only one period with two blocks of cyclic loading at different stress level. The load amplitude remains constant within each block but different from one block to another. To investigate the validity of Miner's Rule and the effect of load sequence, all possible load cases have been considered, including Elasto-Elastic, Elasto-Plastic, Plasto-Elastic and Plasto-Plastic load cases.

6.1 Elasto-Elastic Load Case

In this case the stress level in each of the blocks is below the yield limit. The load spectrum is shown in Fig-1. The stress-strain relation for this case is shown in Fig-5. The number of load cycles N_1 for the first block is a given quantity as follows:

$$N_1 = kN_f \qquad (36)$$

where, N_f is the crack initiation life under constant amplitude loading and k (<1) a known quantity to define the number of load cycles of block-1.

6.1.1 Fatigue Damage Analysis for 1st Block

The damage analysis for block-1 is the same as that of constant amplitude fatigue analysis. The additional load-finite element formulation for block-1 can be given in the form of the range of variation as follows

$$[K]\{\tilde{\delta}_1\} = \{\tilde{F}_1\} + \{\tilde{F}_1\}^D \qquad (37)$$

where, $\{\tilde{F}_1\}$ and $\{\tilde{F}_1\}^D$ are the ranges of variation of conventional and additional loads for block-1, respectively.

The damage evolution equation for block-1 has the form

$$\left(\frac{dD}{dN}\right)_1 = \beta\left(\frac{\tilde{\sigma}_{e,1}}{1-D}\right)^m\left(\frac{1}{1-D}\right)^q \qquad (38)$$

The n[th] increment of load cycles corresponding to the damage increment ΔD^* in block-1 can be given as

$$\Delta N_{n,1} = \frac{\Delta D^*}{\beta\left(\tilde{\sigma}^*_{e,1}\right)^m_{(n-1)}}\left(1-D^*\right)^{m+q} \qquad (39)$$

Summation of the increments of load cycles, finally, will reach the given number of load cycles, N_1, as follows:

$$\sum_n \Delta N_{n,1} = N_1 \quad \text{with} \quad D^* = D^*_1 \qquad (40)$$

where, D^*_1 is the damage of critical element at the end of block-1. It must be mentioned that in each step of damage increment the corresponding field quantities u_i, ε_{ij}, σ_{ij} and D can be obtained by the finite element method with additional loads.

6.1.2 Fatigue Damage Analysis for 2[nd] Block

When the number of load cycles is equal to N_1 the load amplitude is changed from $\tilde{\sigma}_1$ to $\tilde{\sigma}_2$ (Fig-5). In order to calculate the residual life due to the loading of block-2, the additional load-finite element formulation for block-2 has the form,

$$[K]\{\tilde{\delta}_2\} = \{\tilde{F}_2\} + \{\tilde{F}_2\}^D \qquad (41)$$

where, $\{\tilde{F}_2\}^D$ is the range of variation of additional loads caused by the damage of all elements.

The damage evolution equation for block-2 has the form

$$\left(\frac{dD}{dN}\right)_2 = \beta\left(\frac{\tilde{\sigma}_{e,2}}{1-D}\right)^m\left(\frac{1}{1-D}\right)^q \qquad (42)$$

The n[th] increment of load cycles corresponding to ΔD^* in block-2 can be given as

$$\Delta N_{n,2} = \frac{\Delta D^*}{\beta\left(\tilde{\sigma}^*_{e,2}\right)^m_{(n-1)}}\left(1-D^*\right)^{m+q} \qquad (43)$$

The above process will be accomplished, when D^*=1. Then, the summation of the increments of load cycles will give the residual fatigue life as follows:

$$\sum_n \Delta N_{n,2} = N_2 \quad \text{with} \quad D^*=1 \qquad (44)$$

6.1.3 Total Fatigue Life

After determining the number of load cycles for each block, the fatigue life of structure member will be

$$N_{cr} = N_1 + N_2 \qquad (45)$$

The Miner's Rule index can be defined at this stage as follows:

$$\sum_i \frac{n_i}{N_{i,cr}} = \frac{N_1}{N_{1,cr}} + \frac{N_2}{N_{2,cr}} \qquad (46)$$

where, $N_{1,cr}$ and $N_{2,cr}$ are the fatigue lives from constant load amplitude fatigue analysis for respective loads. If the summation in Eqn-46 closes to unity, then it is shown that Miner's Rule is available.

Similar analysis can be carried out when the sequence of applied loads is reversed, to show the availability of Miner's Rule and the effect of load sequence on total fatigue life. The load spectrum for reverse sequence is shown in Fig-2.

6.2 Elasto-Plastic Case

In this case pure elastic deformation occurs in block-1 and plastic deformation takes place only at critical zone in block-2. It is assumed that the form of the load spectrum is similar to Fig-1. Fatigue damage analysis for block-1 in this case is same as that in elasto-elastic case.

6.2.1 Fatigue Damage Analysis for Block-2

At first, an additional load-iteration method is introduced to give the plastic correction for block-2 in order to determine the initial elasto-plastic maximum equivalent stress and modified stress ratio. According to eqn-19, the formulation in finite element analysis has the form

$$[K]\{\tilde{\delta}_2\} = \{\tilde{F}_2\} + \{\tilde{F}_2\}^{(P,D)} \qquad (47)$$

where, $\{\tilde{F}_2\}$ is the vector for the variation of conventional loading corresponding to $\Delta P = P_{2,\max} - P_{1,\min}$ (Fig-1), and $\{\tilde{F}_2\}^{(P,D)}$ is the vector for the variation of additional loads to consider the material degradation after the first block and the effect of plasticity.

After the completion of above iteration process, the variation of stresses, $\Delta\sigma^{(1,2)}$, corresponding to ΔP is obtained. Then, the range of variation of equivalent stress, $\tilde{\sigma}_{e,2}$, corresponding to $\tilde{P} = P_{2,\max} - P_{2,\min}$ can be determined by means of the finite element analysis with additional loads in order to consider the damage caused by block-1. Therefore, the modified stress ratio will be obtained as follows:

$$\sigma_{2,\max} = \sigma_{1,\min} + \Delta\sigma^{(1,2)} \qquad (48)$$

$$\sigma_{2,\min} = \sigma_{2,\max} - \tilde{\sigma}_{e,2} \qquad (49)$$

and $$R_2 = \frac{\sigma_{2,\min}}{\sigma_{2,\max}} \qquad (50)$$

Above range of variation of equivalent stress, $\tilde{\sigma}_{e,2}$, and modified stress ratio, R_2, obtained will be applied in eqn-42 to determine the initial damage evolution ratio, $(dD/dN)_2$, for block-2. According to eqn-41 to eqn-43, the increment of load cycles and the corresponding displacement, stress and damage fields will be calculated for each step of damage increment. Eventually, when $D^*=1$, the residual fatigue life will be obtained by use of eqn-44. The stress-strain relation for the elasto-plastic case is shown in Fig-6.

6.3 Plasto-Elastic Case

In this case, block-1 will cause plastic deformation and block-2, the elastic one in a structure member. The load spectrum is shown in Fig-2. Additional load-iteration method is introduced to give the plastic correction for block-1. The formulation in finite element analysis has the following form

$$[K]\{\tilde{\delta}_1\} = \{\tilde{F}_1\} + \{\tilde{F}_1\}^P \qquad (51)$$

where, $\{\tilde{F}_1\}^P$ is the vector for the range of variation of additional loads to consider the effect of plasticity.

After the completion of iteration process, the maximum equivalent stress, $\tilde{\sigma}_{e,1}$, and the modified stress ratio, R_1, obtained will be applied in eqn-38 to 40 as an initial data to determine the fatigue life, N_1, for block-1. When the number of load cycles arrives at N_1, the load amplitude is changed. The formulation in finite element analysis will have the form of eqn-41, in which, $\{\tilde{F}_2\}$ is then, the vector for the variation of conventional loading corresponding to $\Delta P = P_{2,\max} - P_{1,\min}$ (Fig-2). The maximum equivalent stress, and modified stress ratio obtained will be applied to determine the residual fatigue life N_2 due to block-2, according to eqns-41 to 44, until $D^*=1$. The stress-strain relation for the plasto-elastic case is shown in Fig-7.

6.4 Plasto-Plastic Case

In this case plastic deformation will take place in both the blocks. After the application of load in block-1, there is going to be a plastic deformation and material will undergo a permanent set. The formulation in finite element analysis will be different depending upon the load sequence. If the load in block-1 is lower than in block-2, then new yield and ultimate limits are required to be calculated for the iteration approach applied to block-2. Whereas, there will be no change in these limits if block-1 loading is higher than 2.

6.4.1 Low-High Sequence

Additional load-iteration method is applied to block-1 for plastic correction. The load spectrum is shown in Fig-1. The finite element analysis will be performed according to eqn-51, then eqns-37 to 40. After the application of block-1, further plastic deformation will take place due to the higher load level of block-2. Then, to perform the analysis of plasticity, new yield and ultimate limits for stress and strain are calculated by transformation of $\sigma - \varepsilon$ co-ordinate system to $\Delta\sigma - \Delta\varepsilon$ co-ordinate system (Fig-8) as follows:

$$\left.\begin{aligned} &\Delta\sigma_s = \sigma_{1,\max} - \sigma_{1,\min}\ , \ \ \Delta\varepsilon_s = {\Delta\sigma_s}/{E} \\ &\Delta\sigma_u = \sigma_u - \sigma_{1,\min}\ , \ \Delta\varepsilon_u = \varepsilon_u - \varepsilon_{1,s} + \Delta\varepsilon_s \end{aligned}\right\} \quad (52)$$

where, $\varepsilon_{1,s}$ is the new yielding strain of the material damage caused by block-1.

Additional load-iteration method, with above yield and ultimate limits, is applied to give plastic correction to block-2 in order to determine the next initial maximum stress and modified stress ratio. The formulation in finite element analysis has the form of eqn-51. The residual fatigue life N_2 will then be calculated in a similar process as that for block-2 in elasto-plastic case. The stress-strain relation for low-high sequence is shown in Fig-8.

6.4.2 High-Low Sequence

In this case, additional load-iteration method is applied for plastic correction to block-1 only. The load spectrum is shown in Fig-2. The formulation in finite element analysis has the form of eqn-51, then eqns-37 to 40. After the application of load cycles N_1 for block-1, the load amplitude is changed and the material will follow linearly elastic behavior. The formulation in finite element analysis for block-2 will then be governed by eqn-41 and residual fatigue life N_2 will be then determined by eqns-43 and 44. The stress-strain relation for high-low sequence is shown in Fig-9.

7 NUMERICAL RESULTS AND ANALYSIS

Comprehensive computer programs are developed based on the above-mentioned theory to perform the additional load-iteration method, linearly elastic FEA, elasto-plastic FEA, damage mechanics-linearly FEA with additional loads, and damage mechanics-elasto-plastic FEA with additional loads. With the help of these programs the fatigue lives for above different cases are predicted. Before the application of above approaches for the investigation of spectrum loading, they are applied to predict the fatigue lives under constant load amplitude for verification. The numerical results are in good agreement with the experimental data as shown in Table-1. Variable load amplitudes in two steps are applied to specimens made of LC4CS aluminum alloy and 30CrMnSiNi2A steel alloy with stress concentration factors (K_t) of 2.0 and 3.0, and stress ratio R=0.5. The computational results are shown in Fig-10 to show the availability of Miner's Rule for different cases and the analysis for each case is given below:

7.1 Elasto-Elastic Case

Fatigue life prediction analysis for specimen made of LC4CS aluminum alloy, with K_t = 3.0 and R = 0.5 is carried out at constant load amplitude and two step variable load amplitude. The load levels in both blocks are considered to be elastic. It is found that the Miner's Rule is available for this type of loading. The reversal of loading sequence does not affect the total fatigue life of the specimen.

7.2 Elasto-Plastic Case

Fatigue life prediction for the same specimens as mentioned above is performed when block-1 undergoes elastic deformation and block-2, plastic deformation. From computational results it is found that Miner's Rule can be satisfied approximately only in those cases when the difference in the load amplitudes of both block-1 and 2 is small. If the difference is larger, then Miner's Rule is not available. It is shown that load sequence has a significant effect on the total fatigue life of structure member when the difference in load amplitudes of blocks is greater. The fatigue life of the specimens is extended significantly when a low one follows the high level load. This phenomenon suggests that exposure to medium cycle fatigue (MCF) prior to high cycle fatigue (HCF) may effect HCF life. This numerical result has also been verified by some experimental work performed in [6].

7.3 Plasto-Elastic Case

Fatigue life prediction for the same specimens is performed when plastic deformation takes place in block-1 and elastic one in block-2. The exposure of MCF prior to HCF extends the HCF life by approximately two times. With the increase in the difference of load amplitudes the extension factor will increase and vice versa.

7.3.1 Plasto-Plastic Case

Fatigue life prediction under variable load amplitude for the specimen made from 30CrMnSiNi2A steel alloy with K_t =2.0, 3.0 and R=0.5 and specimen made from LC4CS aluminum alloy with K_t =3.0 and R=0.5, is taken as numerical examples. Plastic deformation takes place in both the blocks. It is observed that when the difference between two load levels is larger, the availability of Miner's rule becomes remote and the change of load sequence affects the total fatigue life significantly. Moreover, MCF pre-straining prior to HCF may also extend the fatigue life of specimen in plasto-plastic case.

8 CONCLUSIONS

1. Linear Damage Accumulation Rule (Miner's rule) is available in the elasto-elastic cases of two-block loading, when the loading in both blocks is below the yield limit. It may be available in some of elasto-plastic, plasto-elastic and plasto-plastic cases when the difference between load amplitudes of spectrum loading is small or the loadings are close to the yield limit of the material.
2. There is a direct effect of loading sequence on fatigue life if the difference between the variable load levels is larger in the cases of elasto-plastic, plasto-elastic and plasto-plastic variable amplitude loads.
3. Exposure of MCF prior to HCF loading may extend HCF life significantly. Which shows that fatigue life of a structure member can be extended by a suitable choice of MCF pre-straining.
4. Use of additional loading method instead of the damage mechanics-finite element method of variable stiffness and damage increment at critical element as step length instead of the increment of loading cycles is more flexible and reduces the computational time significantly.

9 REFERENCES

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Specimen	K_t	Stress (Mpa)	Computed Fatigue Life (log_{10})	Experimental Fatigue Life (log_{10})
LC4CS	3.0	196	4.578	4.715
		157	4.995	5.148
		118	5.368	5.493
		88	6.232	6.0
		70	7.069	7.0
30CrMnSiNi2A	2.0	1400	4.649	4.663
		1200	4.938	4.914
30CrMnSiNi2A	3.0	999	4.659	4.672
		924	4.818	4.903
		849	4.988	5.041

Table-1, Comparison of computational and experimental fatigue lives for constant load amplitude

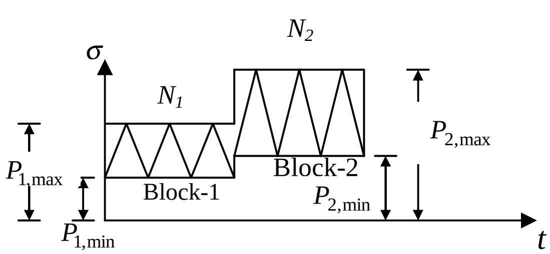

Fig-1, Load spectrum (low-high) case

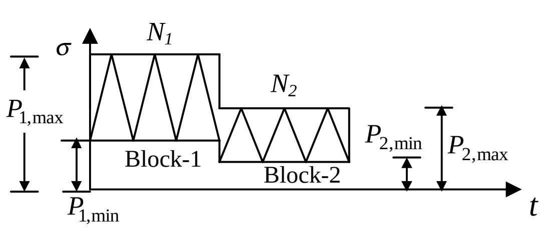

Fig-2, Load spectrum (high-low) case

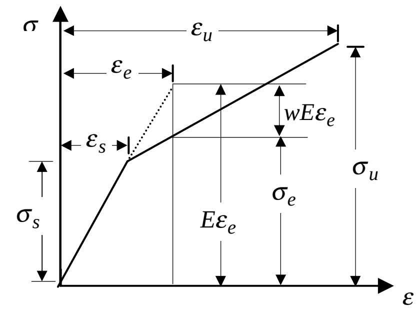

Fig-3, Stress-Strain curve for linearly hardening material

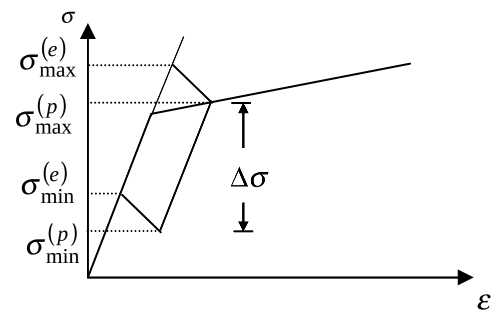

Fig-4, Iteration method

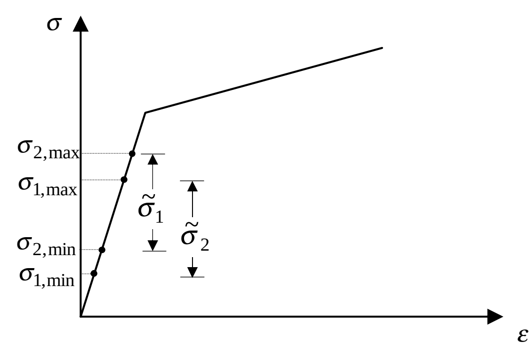

Fig-5, Stress-Strain curve for elasto-elastic case

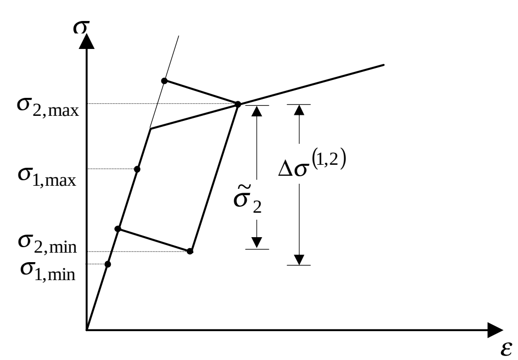

Fig-6, Stress-Strain curve for elasto-plastic case

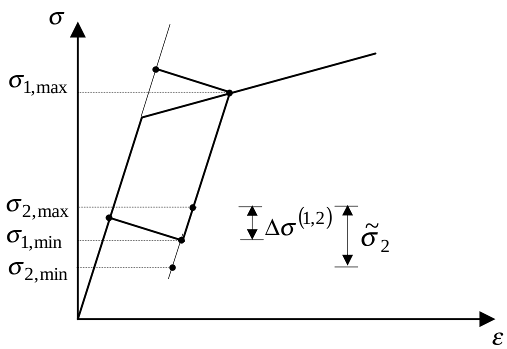

Fig-7, Stress-Strain curve for plasto-elastic case

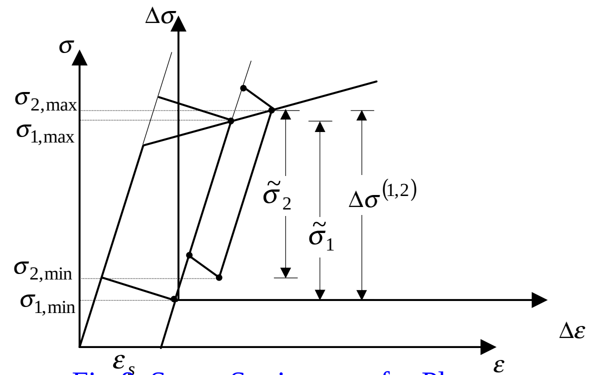

Fig-8, Stress-Strain curve for Plasto-plastic (low-high) case

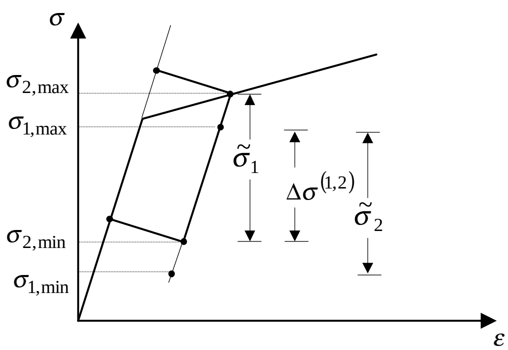

Fig-9, Stress-Strain curve for plasto-plastic (high-low) case

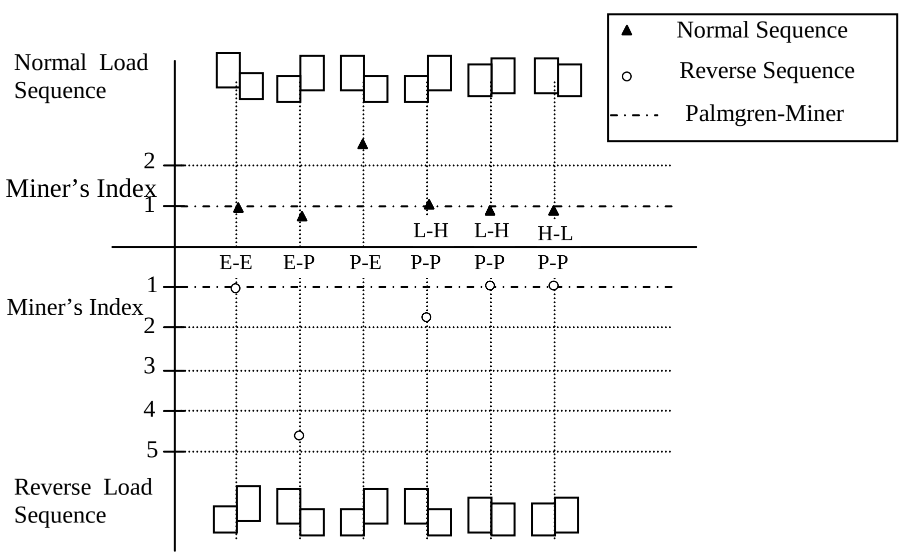

Fig-10, Availability of Miner's Rule for all the cases in normal and reverse sequence loading