

**Bend/Twist Coupling Effects for Unsymmetrical & Unbalanced Laminate Used
in Aircraft Wing Structure Analysis
(Part II)**

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1. Introduction

It is an interesting task to use un-symmetry and un-balance laminated composite to development wing structure. For high-speed aircraft aero-elastic issue become more important and more critical for wing design. The aerodynamic efficiency for some sweep wing is only 60% in high speed. The banding/twist coupling characteristic of un-symmetry and un-balance laminated composite plate can be used in wing structure design to reduce the structure weight. This research work is mainly to solve the problems in wing development using this type structure. The research work is including:

- a) To solve the deflection in manufactory cure process.
- b) The analysis tools for development for engineering application
- c) The application analysis for a wing structure

The research work is including crate analytical methods and some tests to evaluation the analysis results. Finally try to use analysis for aero-elastic tailoring in a wing structure. It is on the way to obtain the all solution.

2. Deflection analytic method for un-symmetry & un-balance laminated plate

When the un-symmetry and un-balance laminated composite material panel in cure process from 180C⁰ to the room temperature, the panel is going to deflection in very seriously. It is necessary to create a non-linear method to analysis the deflection of un-symmetry & un-balance laminated plate. Because, there is a big disagreement between test result and the linear analytical result in the thermal loading.

The research work started from a plated panel, a non-linear analysis method, which considered the effect of Transverse Shearing of laminates, is established. The strain-displacement relations used in analysis are presented as follows.

The strain of the plat has been decrypted

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \\
\varepsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \\
\varepsilon_{xy} &= \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \bullet \frac{\partial w}{\partial y} \right) \\
\gamma_{xz} &= \frac{\partial u^z}{\partial z} + \frac{\partial w}{\partial y} = \phi_x + \frac{\partial w}{\partial x}; \\
\gamma_{yz} &= \frac{\partial v^z}{\partial z} + \frac{\partial w}{\partial y} = \phi_y + \frac{\partial w}{\partial y};
\end{aligned} \tag{1}$$

Where consider the Transverse Shearing of laminates

$$\begin{aligned}
u^z &= u + z\phi_x \\
v^z &= v + z\phi_y
\end{aligned} \tag{2}$$

The strain energy can be expressed in the following form

$$\begin{aligned}
W &= \int_{-l_x/2}^{l_x/2} \int_{-l_y/2}^{l_y/2} \int_{-l_z/2}^{l_z/2} \left(\{\varepsilon\}^T \bullet [Q] \bullet \{\varepsilon\} / 2 - [Q] \bullet \{\alpha\} \bullet \Delta T \right) dx dy xz + \int_{-l_x/2}^{l_x/2} \int_{-l_y/2}^{l_y/2} \left(\{\gamma\}^T \bullet [C] \bullet \{\gamma\} / 2 \right) dx dy \\
&= \int_{-l_x/2}^{l_x/2} \int_{-l_y/2}^{l_y/2} \int_{-l_z/2}^{l_z/2} \psi \left(a_1, a_2, b_1, b_2, a, b, c, d, Q_{ij}, \alpha_x, \alpha_y, \Delta T, x, y, z \right) dx dy xz + \\
&\int_{-l_x/2}^{l_x/2} \int_{-l_y/2}^{l_y/2} \Pi(a_1, a_2, b_1, b_2, a, b, c, d, Q_{ij}, x, y, z, m_1, m_2, n_1, n_2) dx dy
\end{aligned} \tag{4}$$

$$W = W_1 + W_2$$

W_1 ----strain energy with temperature loads

W_2 ---- transverse shearing strain energy

Based on the variations of strain energy $\frac{\partial W}{\partial a} = 0$; $\frac{\partial W}{\partial b} = 0$, there are 11 equations to be

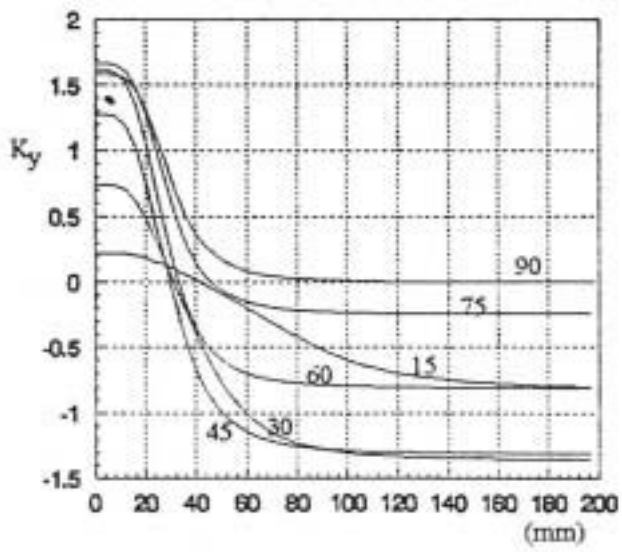
created. The unknown coefficients included will be determined by using numerical solution.

It is shown that the influence of out-of-plane shear deformation of laminates is depended on the laminates angels and the size (thickness) of the plate.

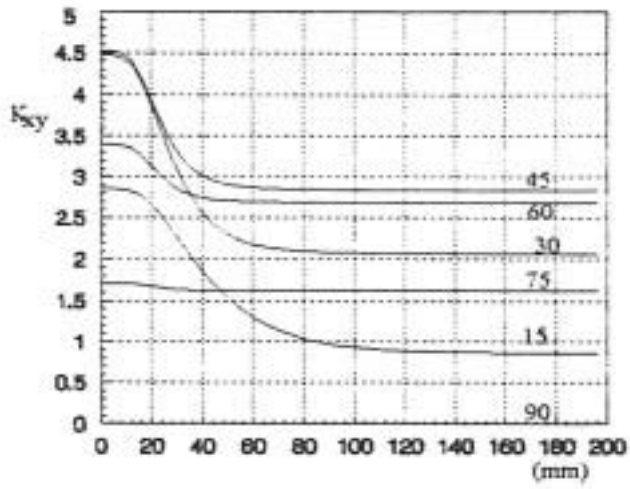
$$K_X = \frac{\partial^2 W}{\partial X^2} \quad K_Y = \frac{\partial^2 W}{\partial Y^2} \quad K_{XY} = \frac{\partial^2 W}{\partial XY}$$

The analysis results of the plate $[0^0/90^0/90^0/\theta^0]$ are given in the fig1.

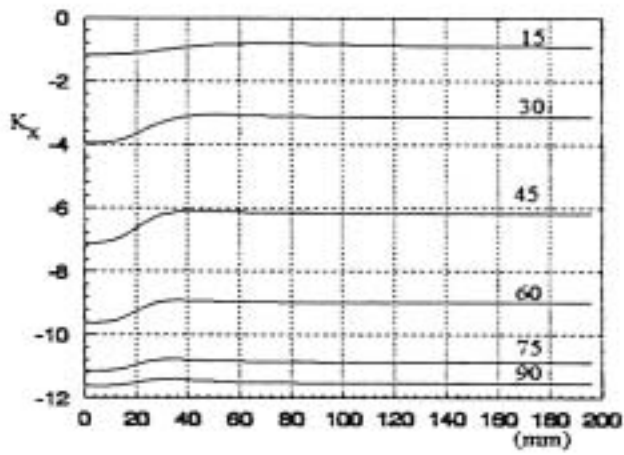
The fig1 (a), (b), (c) is show the relationship between the deflections K_X , K_Y , K_{XY} and the T300/5208 material plate size, laminate angel θ^0 , panel thickness.



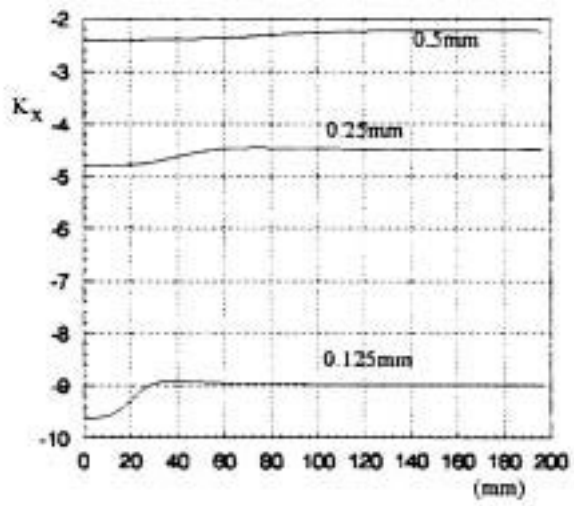
(a)



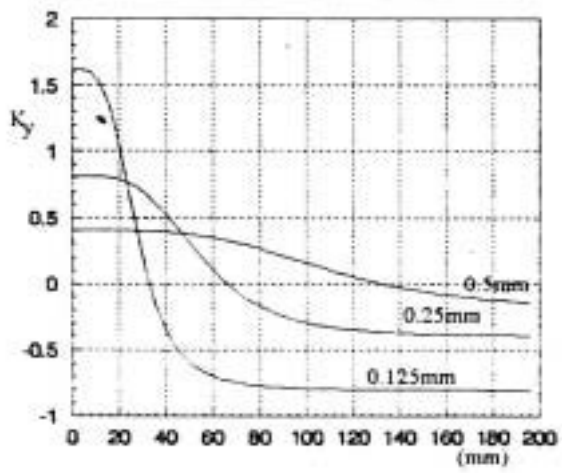
(b)



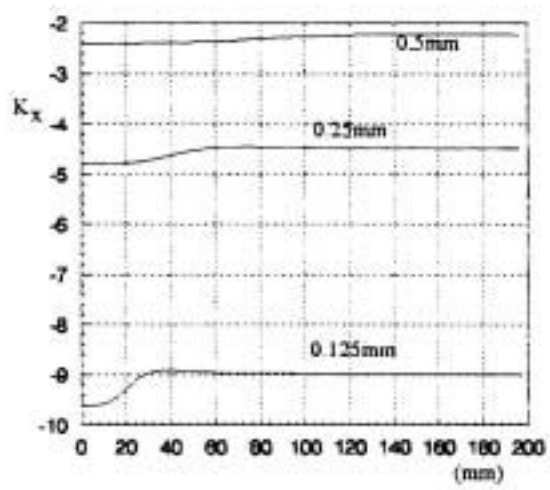
(c)



(d)



(e)



(f)

Fig 1 The relationship of the deflections K_X , K_Y , K_{XY} and the plate size

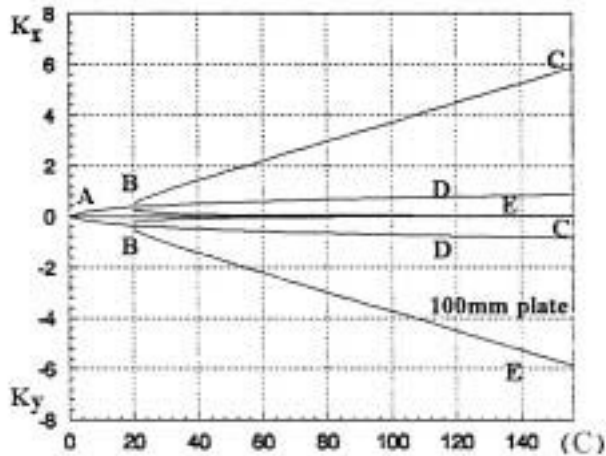


Fig. 2 The relationship of the deflections K_X , K_Y , and the temperature

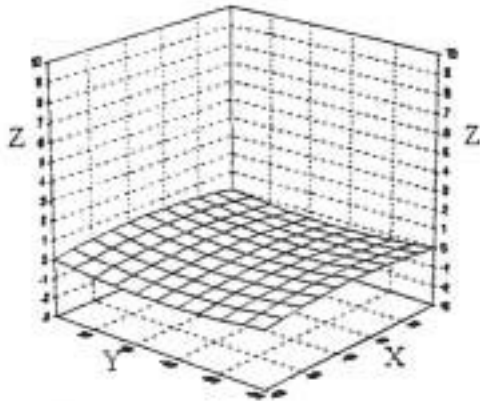


Fig. 3 The 3D view of the plate deflection

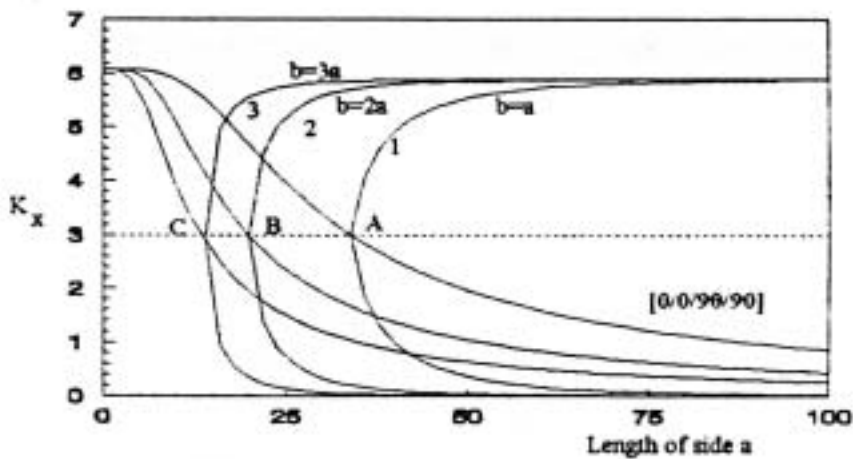


Fig. 4 The relationship between deflection K_X and the plate sizes

The analysis given the following conclusion:

- Compare the linear and non-linear analytical results with testing results. It is shown that it is necessary to use non-linear analytical method for investigating the thermal deflection of un-symmetry & un-balance laminated plate.
- The K_X , K_Y , K_{XY} are decreased when the size and thickness of the panel are increased.

- c) The K_X, K_Y are increased when the temperature of the panel is increased.
- d) Based on non-linear analytical method, the panel has two deflection models in some situation (in Fig. 4). It is difficulty to overcome these type thermal deflections by using change the die. The best way to use un-symmetry and un-balance laminated composite is in a symmetrical wing structure.

To conform the correction of the analysis results some test works have been done. One test is ‘**Test Part 1**’—Considering a laminated plate of $300 \times 300 \text{mm}^2$ with stacking sequence: $[0/45/90/45/0/-45/90/-45/0]$, the thickness for each layer is 0.12mm and the temperature change is $\Delta T = 160^\circ \text{C}$. The material characteristics of the laminate are $E_{11} = 135000 \text{MPa}$, $E_{22} = E_{33} = 940 \text{MPa}$, $\nu_{12} = 0.28$, $\nu_{13} = \nu_{23} = 0.28$, $G_{12} = 5000 \text{MPa}$, $G_{13} = G_{23} = 5000 \text{MPa}$, and $\alpha_1 = 0.1 \times 10^{-6}$, $\alpha_2 = 0.25 \times 10^{-4}$, $\alpha_3 = 0.25 \times 10^{-4}$. The plate is supported at 1,2,3 points (Fig 1). The testing and analytic results are given in following table.

Tab 1 The test result and the linear and un-linear results

	Linear analytical result	Nonlinear analytical result	Test result
Maximum deflection(point4)	143.4	57	56
Error (analytical/test)	256%	1.8 %	0

From the results it is shown that there is big difference between linear and non-linear solutions, and the non-linear solution is far more close to the test results under thermal deflected condition.

3. FEM method for un-symmetry and un-balance laminated plate

The FEM method and program is a necessary approach for investigating un-symmetry and un-balance laminated composite structure for aircraft development and for further development optimization functure. A new composite element was developed by extending a mixed interpolation method of tensorial components. The new element has been integrated into a general FEM program APOLANS for analyzing the un-symmetry and un-balance laminated composite structure. The new element with 4-nodes mixed interpolation of tensorial components is given as following:

The displacements can be interpolated by the 4 element notes:

$$u^i = \sum_{k=1}^4 N_k u_k^i + \sum_{k=1}^4 N_k \frac{t_k}{2} \xi (-\alpha_k \nu_{2i} + \beta_k \nu_{2i}) \quad (7)$$

Where u^i is the displacements u, v, w, α, β ; $i=1, 2, \dots, 5$

t_k is the laminate thickness at k points; $k=1, 2, \dots, 4$

ν_{ij} is the function as $\nu_{ij} = 0$ when $j \neq i$; $\nu_{ij} = 1$ when $j = i$. $J=1, 2$

The function $N_k(\xi, \eta)$ is given as follows:

$$N_k(\xi, \eta) = \frac{1}{4} (1 + \xi_k \xi)(1 + \eta_k \eta)$$

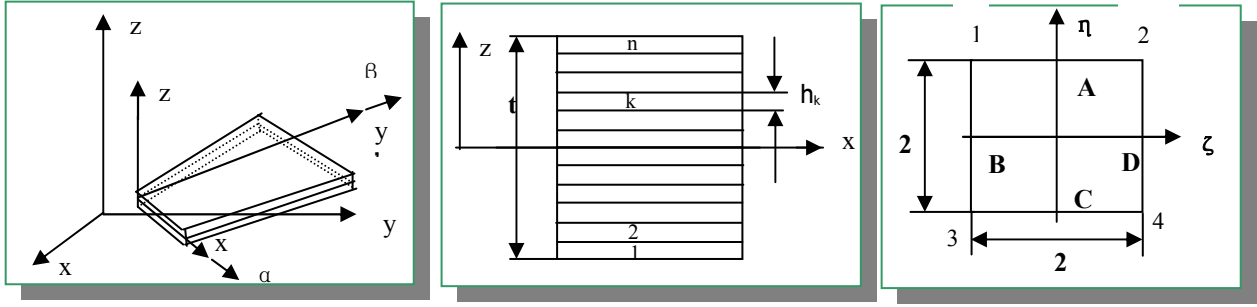


Fig .5 Coordinate system for shell element in APLANS

The banding and membrane strain of the panel is given by equation (7), but the transversl shearing strain of the tensional components is interpolated by the following form to avoid element ‘lock’.

$$\begin{aligned} \varepsilon_{\xi\xi} &= \frac{1}{2}(1+\eta)\varepsilon^A_{\xi\xi} + \frac{1}{2}(1-\eta)\varepsilon^C_{\xi\xi} \\ \varepsilon_{\xi\xi} &= \frac{1}{2}(1+\xi)\varepsilon^D_{\xi\xi} + \frac{1}{2}(1-\xi)\varepsilon^B_{\xi\xi} \end{aligned} \quad (8)$$

The $\varepsilon^A, \varepsilon^B, \varepsilon^C, \varepsilon^D$ is the shearing strain tensorial components at the points A, B, C, D and ε^A is

$$\varepsilon^A = 1/4[1/4(u_1-u_2)(t_1V_1^1+t_2V_1^2) + 1/4(v_1-v_2)(t_1V_2^1+t_2V_2^2) + 1/4(w_1-w_2)(t_1V_3^1+t_2V_3^2)]$$

The descriptions of parameters $\varepsilon^B, \varepsilon^C, \varepsilon^D$ are similar to above.

The stiffness matrix K^e of the element is:

$$K^e = \sum_{k=1}^n \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 B^T DB |J| \frac{h_k}{t} d\xi d\eta d\zeta \quad (9)$$

For the dis-continue laminated plate the K^e can be formed:

$$K^e = \sum_{k=1}^n \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 B^T DB |J| \frac{h_k}{t} d\xi d\eta d\zeta \quad (10)$$

We use the ‘‘Test Part 1’’ as example to analyze the function of new APOLANS. The test and the analysis results are show in following table.

Table 1. The error between test and the analysis results (for Test part 1)

	Test results	Analytic results	FEM results
Maxim deflection	56mm	57mm	54.5mm
Error	0	1.8 %	- 2.7 %

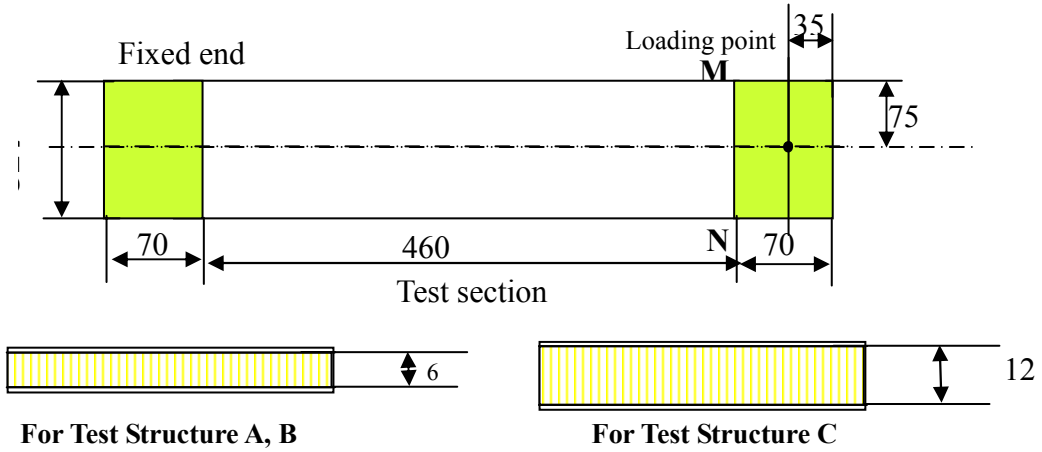
The table shows that the maximum deflection analyzed by newly developed FEM program APOLANS coincides with the results from analytical and testing research.

‘**Test Part 2**’--Some structures using a symmetry honeycombed structure with un-symmetry un-balance composite panels have been tested under tensional and banding loads, and they has been analyzed by the program for checking the APLANS program. The results are shown in Table 2.

Table 2 The parameters of some test honeycomber structures

Test Structure	Laminations	Core Thickness	Loading
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A	$[0_2/honey/0_2/25_2]$	6mm	Tension
B	$[25_2/0_2/honey/0_2/25_2]$	6mm	Bending
C	$[45_2/0_2/honey/0_2/45_2]$	12mm	Bending



For Test Structure A, B

For Test Structure C

Fig. 6 the parameters for test structures a, B, C.

The APOLANS analytical results and the test results are shown in the following Figures.

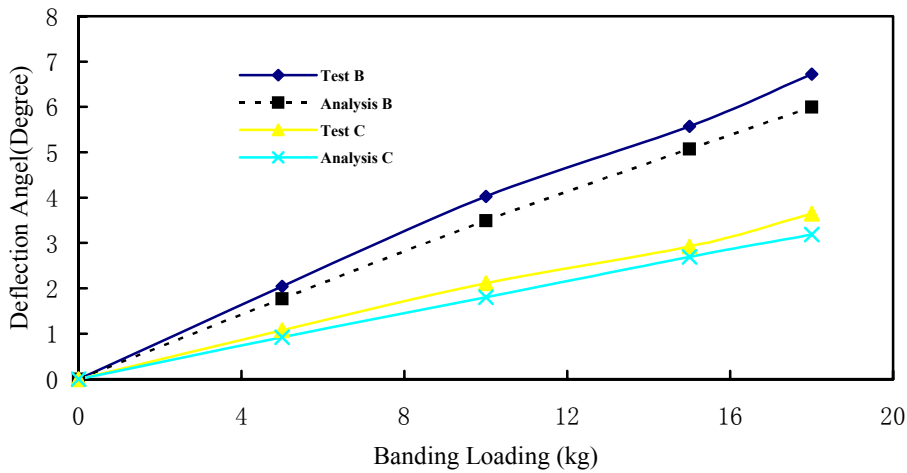


Fig. 7 The analytical deflection results in contrast to the test ones for structure B, C

4. Optimization layer angel analysis for un-symmetry and un-balance lamination

For choose a optimization layer angle lay on a symmetrical structure to realize the right structure deflection. Some analysis has been done and got some results. The analysis is shown that the maximum deflection angle for an un-symmetry and un-balance layer in a panel is depends on the stiffness of the panel Q_{11} , Q_{22} and Q_{66} .

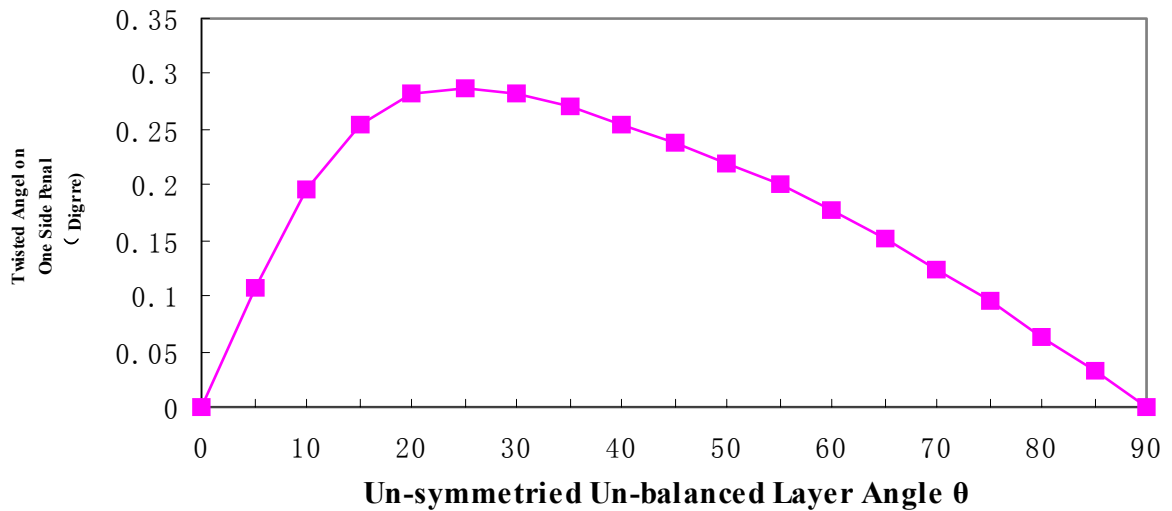


Fig. 8 The twisting angle of a panel corresponding to the ply angle of un-symmetry and un-balanced layer

5. Aero-elastic analysis on un-symmetry and un-balance laminated panel

Aero-elastic analysis for some panel with different angle un-symmetry and un-balance layer has been done. The panels laminated with $0^0/0^0$, $0^0/15^0$, $0^0/22.5^0$, $0^0/25^0$, $0^0/45^0$, $0^0/60^0$, $0^0/75^0$, $0^0/90^0$. The results are in Fig.9.

The conclusion is:

- a) It is a big influence on flutter speed to change the panel layer angel.
- b) In this case, $+\alpha=35^0$ laminated panel has rich high flutter speed.
- c) The flutter speed for panel with $+\alpha$ layer is high than $-\alpha$.

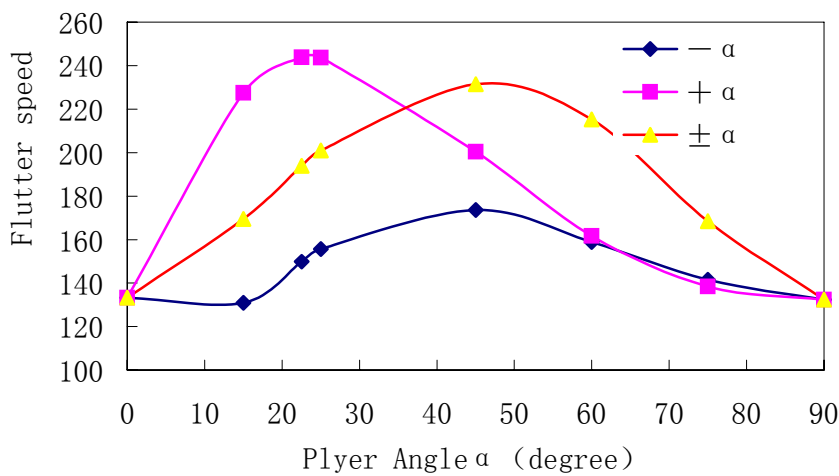


Fig. 9 The flute speed with player angle of the panel

6. Analysis on wing structure with un-symmetry and un-balance laminated skin

The aero-elastic deflection reduces the effective angle of attack in a swiped wing tip and reduces the wing lift in tip part. A wing structure has been analyzed using APOLANS program under the conditions: original lamination and un-symmetry and un-balanced lamination. The results for the wing tip angle of attack has been improved. The wing aerodynamic is also improved.

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