A METHOD OF ESTIMATING AIRPLANE TURBULENCE USING WIND DATA

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Abstract
Using the nonlinear Boussinesq equations, the analysis of the relations between the wind and the turbulence kinetic energy (TKE) development is made, and a method of estimating the airplane turbulence using the wind data is obtained.

1 Introduction
Turbulence is a phenomenon that often occurs in flight. A light turbulence can make the crew and passengers uncomfortable, even injured. In a severe turbulence, the airplane can be thrown up and down for many times in a minute and the change of the altitude can be up to several hundreds meters, so as to make the airplane out of control. When the turbulence is extremely severe, the overload element (namely overload) it produces can result in the disaggregation of the airplane, and this will be a great threat to the safety of flight. Therefore, the turbulence has long been attached great importance by the aviation world. Although in the past people have paid a lot of effort on that, there are still not sufficient means of predicting. According to aerodynamics, the cause of turbulence is the flight in which the airplane enters the air turbulence area (namely the airplane air turbulence, the space scale is generally within $10^1 \sim 10^3$) [1] whose space scale are similar to the airplane. The present essay, using the two-dimensional nonlinear Boussinesq equations, analyses the relations between the wind and the TKE(turbulence kinetic energy) development. And thus a method of estimating the airplane turbulence using wind data is obtained. It can account for the airplane turbulence better in practice.

2 The relations between the wind and the turbulence kinetic energy(TKE) development, and a method of estimating the airplane turbulence
Since the airplane turbulence is a phenomenon of relatively smaller scale, and the Coriolis force does not affect the kinetic energy. Therefore, the Boussinesq equations of ageostrophic motion effect can be used as follows[2]:

$$\begin{cases} \dfrac{d\vec{V}}{dt} = -\dfrac{1}{\rho_s}\nabla p' + \dfrac{\theta'}{\theta_s} g \\ \nabla \cdot \vec{V} = 0 \\ \dfrac{d\theta'}{dt} + \theta_s N w = 0 \end{cases} \tag{1}$$

Thereinto $N=\frac{1}{\theta_s}\frac{\partial\theta_s}{\partial Z}$, θ_s is the average potential temperature, p' and θ' respectively are the pressure disturbance and the potential temperature disturbance, $\vec{V}$ and w are the general value of speed. The others are the common meteorological symbol.

For the purpose of the easy and convenient research as well as not losing universalism, two-dimensional of XZ - coordinate system is the only thing to be considered, therefore:

$$\begin{cases}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_S}\frac{\partial p'}{\partial x} & (2a)\\ \frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_S}\frac{\partial p'}{\partial z}+\frac{\theta'}{\theta_S}g & (2b)\\ \frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 & (2c)\\ \frac{\partial\theta'}{\partial t}+u\frac{\partial\theta'}{\partial x}+w\frac{\partial\theta'}{\partial z}+\theta_S Nw=0 & (2d)\end{cases}$$

u and w in the formula above are respectively the wind speed along the direction of X and Z. (2a)× u +(2b)× w is:

$$\frac{1}{2}\frac{\partial}{\partial t}(u^2+w^2)=-\frac{1}{2}u\frac{\partial}{\partial x}(u^2+w^2)-\frac{1}{2}w\frac{\partial}{\partial z}(u^2+w^2)-\frac{1}{\rho_S}(u\frac{\partial p'}{\partial x}+w\frac{\partial p'}{\partial z})+\frac{\theta'}{\theta_S}gw$$

According to the formula above, the accumulation of the kinetic energy is decided by the level advection of the kinetic energy on the horizontal and vertical direction, and the power made by the pressure gradient force and the gravity. Since the item of the power made by the pressure gradient force has little effect, it can be omitted[1]. That is:

$$\frac{1}{2}\frac{\partial}{\partial t}(u^2+w^2)=-\frac{1}{2}u\frac{\partial}{\partial x}(u^2+w^2)-\frac{1}{2}w\frac{\partial}{\partial z}(u^2+w^2)+\frac{\theta'}{\theta_S}gw \quad (3)$$

For the introduction of the TKE (turbulence kinetic energy), presumed:

$$u=\overline{u}+u' \;;\quad w=\overline{w}+w' \quad (4)$$

Introduce (4) into (3) formula, thus:

$$\begin{aligned}&\underline{\frac{1}{2}\frac{\partial}{\partial t}}(\overline{u}^2+2\overline{u}u'+u'^2+\underline{\overline{w}^2}+2\overline{w}w'+w'^2)\\ &\underline{=-\frac{1}{2}\overline{u}\frac{\partial}{\partial x}}(\overline{u}^2+2\overline{u}u'+u'^2+\underline{\overline{w}^2}+2\overline{w}w'+w'^2)\\ &-\frac{1}{2}u'\frac{\partial}{\partial x}(\overline{u}^2+2\overline{u}u'+u'^2+\underline{\overline{w}^2}+2\overline{w}w'+w'^2)\\ &\underline{-\frac{1}{2}\overline{w}\frac{\partial}{\partial z}}(\overline{u}^2+2\overline{u}u'+u'^2+\underline{\overline{w}^2}+2\overline{w}w'+w'^2)\\ &-\frac{1}{2}w'\frac{\partial}{\partial z}(\overline{u}^2+2\overline{u}u'+u'^2+\underline{\overline{w}^2}+2\overline{w}w'+w'^2)\\ &\underline{+\frac{\theta'}{\theta_S}g(\overline{w}}+w')\end{aligned}$$

The part of the real line in the formula above is the average kinetic energy balance, therefore:

$$\begin{aligned}&\frac{1}{2}\frac{\partial}{\partial t}(2\overline{u}u'+2\overline{w}w'+u'^2+w'^2)\\ &=-\frac{1}{2}\overline{u}\frac{\partial}{\partial x}(2\overline{u}u'+2\overline{w}w'+u'^2+w'^2)\\ &-\frac{1}{2}u'\frac{\partial}{\partial x}(\overline{u}^2+2\overline{u}u'+u'^2+\overline{w}^2+2\overline{w}w'+w'^2)\\ &-\frac{1}{2}\overline{w}\frac{\partial}{\partial z}(2\overline{u}u'+2\overline{w}w'+u'^2+w'^2)\\ &-\frac{1}{2}w'\frac{\partial}{\partial z}(\overline{u}^2+2\overline{u}u'+u'^2+\overline{w}^2+2\overline{w}w'+w'^2)\\ &+\frac{\theta'}{\theta_S}gw'\end{aligned}$$

Average the formula above and omit the small items above the third order power and settle the formula, and the following is deduced:

$$\frac{1}{2}\frac{\partial}{\partial t}(\overline{u'^2}+\overline{w'^2}) = -\frac{1}{2}\bar{u}\frac{\partial}{\partial x}(\overline{u'^2}+\overline{w'^2}) -\frac{1}{2}\bar{w}\frac{\partial}{\partial z}(\overline{u'^2}+\overline{w'^2}) - \overline{u'^2}\frac{\partial\bar{u}}{\partial x} - \overline{w'^2}\frac{\partial\bar{w}}{\partial z} - \overline{w'u'}(\frac{\partial\bar{u}}{\partial z}+\frac{\partial\bar{w}}{\partial x}) - \bar{u}\overline{u'\frac{\partial u'}{\partial x}} - \bar{w}\overline{w'\frac{\partial w'}{\partial z}} - \bar{w}\overline{u'\frac{\partial w'}{\partial x}} - \bar{u}\overline{w'\frac{\partial u'}{\partial z}} + \frac{g}{\theta_S}\overline{w'\theta'} \tag{5}$$

From continuity equations

$$\frac{\partial\bar{u}}{\partial x}+\frac{\partial\bar{w}}{\partial z}=0 \quad\Rightarrow\quad \frac{\partial\bar{u}}{\partial x}=-\frac{\partial\bar{w}}{\partial z} \tag{6a}$$

$$\overline{\frac{\partial u'}{\partial x}}+\overline{\frac{\partial w'}{\partial z}}=0 \Rightarrow \overline{\frac{\partial u'}{\partial x}}=-\overline{\frac{\partial w'}{\partial z}} \tag{6b}$$

Introduce (6a) and (6b) into （5）formula, and deduce:

$$\frac{1}{2}\frac{\partial}{\partial t}(\overline{u'^2}+\overline{w'^2}) = -\frac{1}{2}\bar{u}\frac{\partial}{\partial x}(\overline{u'^2}+\overline{w'^2}) -\frac{1}{2}\bar{w}\frac{\partial}{\partial z}(\overline{u'^2}+\overline{w'^2}) - \frac{\partial\bar{u}}{\partial x}(\overline{u'^2}-\overline{w'^2}) - \overline{w'u'}(\frac{\partial\bar{u}}{\partial z}+\frac{\partial\bar{w}}{\partial x}) - \bar{u}\overline{u'\frac{\partial u'}{\partial x}} + \bar{w}\overline{w'\frac{\partial u'}{\partial x}} - \bar{w}\overline{u'\frac{\partial w'}{\partial x}} - \bar{u}\overline{w'\frac{\partial u'}{\partial z}} + \overline{\frac{\theta'}{\theta_S}w'g} \tag{7}$$

The formula(2d)× θ' deduced:

$$\frac{\partial}{\partial t}(\frac{1}{2}\theta'^2)+u\theta'\frac{\partial\theta'}{\partial x}+w\theta'\frac{\partial\theta'}{\partial z}+\theta_S\theta' Nw=0 \tag{8}$$

From continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \quad\Rightarrow\quad \theta'^2\frac{\partial u}{\partial x}+\theta'^2\frac{\partial w}{\partial z}=0 \tag{9}$$

The formula(8)+(9) deduced:

$$\frac{\partial}{\partial t}(\frac{1}{2}\theta'^2)+\theta'[\frac{\partial(\theta' u)}{\partial x}+\frac{\partial(\theta' w)}{\partial z}]+\theta_S\theta' Nw=0 \tag{10}$$

The item of the potential temperature fluctuations flux divergence in the (10) formula has little effect, thus can be omitted[1], and introduce $w=\bar{w}+w'$ into (10) formula and deduced $\frac{\partial}{\partial t}(\frac{1}{2}\theta'^2)+\theta_S\theta' N(\bar{w}+w')=0$, average it and deduce afterwards

$$\frac{\partial}{\partial t}(\frac{1}{2}\overline{\theta'^2})=-\theta_S\theta N\overline{w'\theta'} \tag{11}$$

After the integral of (7) and (11) formula on the S surface:

$$\begin{cases}\frac{\partial}{\partial t}\iint_S\frac{1}{2}(\overline{u'^2}+\overline{w'^2})ds=-\frac{\bar{u}}{2}\iint_S\frac{\partial}{\partial x}(\overline{u'^2}+\overline{w'^2})ds-\frac{\bar{w}}{2}\iint_S\frac{\partial}{\partial z}(\overline{u'^2}+\overline{w'^2})ds-\frac{\partial\bar{u}}{\partial x}\iint_S(\overline{u'^2}-\overline{w'^2})ds \\ -(\frac{\partial\bar{u}}{\partial z}+\frac{\partial\bar{w}}{\partial x})\iint_S\overline{w'u'}ds-\bar{u}\iint_S(\overline{u'\frac{\partial u'}{\partial x}})ds+\bar{w}\iint_S(\overline{w'\frac{\partial u'}{\partial x}})ds-\bar{w}\iint_S(\overline{u'\frac{\partial w'}{\partial x}})ds-\bar{u}\iint_S(\overline{w'\frac{\partial u'}{\partial z}})ds \\ +\frac{g}{\theta_S}\iint_S\overline{w'\theta'}ds \qquad (7a) \\ \frac{\partial}{\partial t}\iint_S\frac{1}{2}\overline{\theta'^2}ds=-\alpha\iint_S\overline{w'\theta'}ds \qquad (11a)\end{cases}$$

Thereinto $\alpha=N\theta_S={\partial\theta_S}/{\partial z}$

The field of the disturbance advection is represented as stream functions

$$u'=\frac{\partial\psi}{\partial z} \quad ; \quad w'=-\frac{\partial\psi}{\partial x} \tag{12}$$

Presumed in the time interval discussed, the structure of the turbulence remains unchanged while the oscillation length can be changed, we can presume that

$$\psi=A(t)\phi(x,z) \quad ; \quad \theta'=B(t)\Theta(x,z) \tag{13}$$

Therefore

$$u'=A\frac{\partial\phi}{\partial z} \;\; ; \;\; w'=-A\frac{\partial\phi}{\partial x} \tag{14}$$

Introduce (13) and (14) formula into (7a) and (11a) formula, and deduced

$$\begin{cases}\frac{1}{2}\frac{\partial}{\partial t}\iint_S A^2[(\frac{\partial\phi}{\partial z})^2+(\frac{\partial\phi}{\partial x})^2]ds=\iint_S-\frac{1}{2}\bar{u}\frac{\partial}{\partial x}[A^2(\frac{\partial\phi}{\partial z})^2+A^2(\frac{\partial\phi}{\partial x})^2]ds \\ -\frac{1}{2}\iint_S\bar{w}\frac{\partial}{\partial z}[A^2(\frac{\partial\phi}{\partial z})^2+A^2(\frac{\partial\phi}{\partial x})^2]ds-\frac{\partial\bar{u}}{\partial x}\iint_S A^2[(\frac{\partial\phi}{\partial z})^2-(\frac{\partial\phi}{\partial x})^2]ds+(\frac{\partial\bar{u}}{\partial z}+\frac{\partial\bar{w}}{\partial x})\iint_S A^2\frac{\partial\phi}{\partial z}\frac{\partial\phi}{\partial x}ds \\ -\iint_S A^2\frac{\partial}{\partial x}(\frac{\partial\phi}{\partial z})(\bar{u}\frac{\partial\phi}{\partial z}+\bar{w}\frac{\partial\phi}{\partial x})ds+\iint_S A^2\bar{w}\frac{\partial\phi}{\partial z}\frac{\partial}{\partial x}(\frac{\partial\phi}{\partial x})ds+\iint_S A^2\bar{u}\frac{\partial\phi}{\partial x}\frac{\partial}{\partial z}(\frac{\partial\phi}{\partial z})ds-\iint_S\frac{g}{\theta_S}AB\Theta\frac{\partial\phi}{\partial x}ds \qquad (7b) \\ \frac{\partial}{\partial t}\iint_S\frac{1}{2}B^2\Theta^2ds=\alpha B\iint_S A\Theta\frac{\partial\phi}{\partial x}ds \qquad (11b)\end{cases}$$

While $V=\frac{\partial\phi}{\partial z}\,;W=-\frac{\partial\phi}{\partial x}$ (15) [2]

V and W are the values of the actual wind speed at any time unit.

Introduce (15) into (7b) and (11b) formula and deduced:

$$\begin{cases} \dfrac{dA}{dt} = -(M_1 l_1 + M_2 l_2 + M_3 l_3 - M_4 l_4)A + l_5 B\beta & (7c) \\ \dfrac{dB}{dt} = -l\alpha A & (11c) \end{cases}$$

Thereinto $\alpha = \dfrac{\partial \theta_s}{\partial z}$; $\beta = \dfrac{g}{\theta_s}$; $M_1 = \dfrac{\overline{u}}{2}$;

$M_2 = \dfrac{\partial \overline{u}}{\partial x}$; $M_3 = \dfrac{\overline{w}}{2}$; $M_4 = \dfrac{\partial \overline{u}}{\partial z} + \dfrac{\partial \overline{w}}{\partial x}$; $l = \dfrac{\iint_s W\Theta ds}{\iint_s \Theta^2 ds}$;

$$l_1 = \frac{\iint_s (2W\frac{\partial W}{\partial x} + 2W\frac{\partial V}{\partial z} + 4V\frac{\partial V}{\partial x})ds}{\iint_s (V^2 + W^2)ds};$$

$$l_2 = \frac{\iint_s (V^2 - W^2)ds}{\iint_s (V^2 + W^2)ds};$$

$$l_3 = \frac{\iint_s (2V\frac{\partial V}{\partial z} + 2W\frac{\partial V}{\partial z} - 4W\frac{\partial V}{\partial x})ds}{\iint_s (V^2 + W^2)ds};$$

$$l_4 = \frac{\iint_s WVds}{\iint_s (V^2 + W^2)ds}; \quad l_5 = \frac{\iint_s W\Theta ds}{\iint_s (V^2 + W^2)ds}$$

After the expunction of B in the (7c) and (11c) formula, it is deduced that:

$$\frac{d^2A}{dt^2} + M\frac{dA}{dt} + ll_5 A\alpha\beta = 0 \qquad (16)$$

In the above formula

$M = M_1 l_1 + M_2 l_2 + M_3 l_3 - M_4 l_4$

This ordinary coefficient linear differential equation (16) formula's solution is:

$A = c_1 e^{\sigma_1 t} + c_2 e^{\sigma_2 t}$

The $\sigma_{1,2}$ In the formula above is the growing rate, from the eigenequation of the (16) formula, it is :

$$\sigma_{1,2} = -\frac{M}{2} \pm \sqrt{M^2/4 - ll_5\alpha\beta} \qquad (17)$$

According to the quality of (17) formula, the preconditions of the turbulence growth are as follows:

① The oscillation length of the turbulence grows exponentially,

if: a<0 (static force unstable), or a>0 (static force stable), M < 0 and $ll_5\alpha\beta \le M^2/4$

② The oscillation length of the turbulence grows exponentially and wavelike,

If: a>0 , M<0 and $ll_5\alpha\beta \rangle M^2/4$

According to the summation of ① and ②, under the condition of stable and unstable static force and M< 0, the turbulence will develop and the obvious airplane turbulence will be achieved. That is to say, the estimating condition to produce airplane turbulence is:

$$M_1 l_1 + M_2 l_2 + M_3 l_3 - M_4 l_4 \langle 0 \qquad (18)$$

The actual wind data (V ; W ; $\frac{\partial V}{\partial x}$; $\frac{\partial V}{\partial z}$; $\frac{\partial W}{\partial x}$; $\frac{\partial W}{\partial z}$) in flight can be directly read out in the airplane, and the wind data ($\overline{u}$; $\overline{w}$; $\frac{\partial \overline{u}}{\partial x}$; $\frac{\partial \overline{u}}{\partial z}$; $\frac{\partial \overline{w}}{\partial x}$; $\frac{\partial \overline{w}}{\partial z}$) in a certain area can be provided by the airport weather station. Introduce these wind data into (18) formula and calculate. If the condition of M<0 is met, the possibility of the formation of airplane turbulence can be decided. Therefore, M< 0 is the method of estimating airplane turbulence using wind data.

3 Conclusions

Wind is closely related to the appearance of turbulence and its development. Under a certain condition, wind shear can produce turbulence, and whether the turbulence can be developed is decided by the distribution of wind.

Certain condition of the wind distribution in an area if the turbulence is further developed will cause airplane turbulence in which the airplane goes through. The estimating condition is M<0.

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