AN APPROACH TO MODELLING AND PREDICTING IMPACT DAMAGE IN COMPOSITE STRUCTURES

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Abstract

Composite materials have a number of properties that make them attractive for use in aerospace applications. While composites offer excellent in-plane performance, they are susceptible to damage due to severe out-of-plane loads, such as in the case of localised impacts. The resulting damage exists in the form of potentially critical matrix cracks, fibre failure and delamination. A numerical tool for predicting this damage in a representative structure would be beneficial in limiting impact testing, at all structural levels, and supporting design and maintenance of composite structures.

A modelling technique has been developed which uses single layer shell elements to predict both the in-plane damage and delamination. This technique has been implemented into a parametric damage assessment tool to permit the rapid assessment of impact damage tolerance in the design and maintenance of composite, aerospace structures. The commercial explicit finite element (FE) code, LS-Dyna, and parametric FE pre-processor, Ansys, were used. Preliminary validation of this approach using experimental results available in the literature has demonstrated the effectiveness of the modelling technique.

1 Introduction

Over the years, the performance of composite materials in secondary aircraft/aerospace structures has shown superiority over metals. Currently, there is increasing interest to use composites for primary structures for higher weight savings and potential cost reduction. For this reason, a stringer-stiffened panel, with the potential to be used in aircraft fuselage structures, is considered in this research.

A major concern in the use of fibre-reinforced composites is their susceptibility to damage resulting from the effects of impact loads. Damage due to the low-velocity impact from accidents such as dropped tools or rough handling during maintenance may be undetectable by visual inspection but has the potential to alter the local composite stiffness and strength considerably. Typically, impact velocity of less than 5 m/s and mass less than 5 kg are considered under low-velocity impact. Tests conducted on coupons have shown that the damage can lead to reductions of up to 50% in the compressive properties [1-2]. Tests conducted on stringer-stiffened panels show a reduction in compressive strength as high as 20% [3].

The aerospace industry has used laboratory drop tests on coupon size specimens to explore the nature of impact damage and to seek ways of minimising it. However, for a given material the coupon test may be a poor guide to the performance of a real structure [4]. The ability of a large flexible structure to store more energy elastically plays a crucial role for a material whose fibres and resin are much more brittle than aluminium alloys. Hence, the dynamic response of an impacting mass will be quite different if a panel is supported by stiffeners, spars, ribs, frames and so on.

One way of establishing the response or damage of composite structures due to impact loads is through an intense structural testing program. This option is very time consuming and costly for fully representative panels. A numerical model of a realistic structure would be beneficial in order to limit the impact tests at all structural levels and support design and maintenance of composite structures.

Impact events involve contact between the impactor and the target. The response of the composite structure under impact is dynamic in nature and depends upon the progressive failure of the laminate. Numerical modelling of such a phenomenon is therefore complex and requires progressive degradation to simulate loss in stiffness. Explicit finite element codes with contact and failure definitions for composites offer the means of analysing damage in composite structures due to impact load.

2 Impact Damage on Composites

Based upon experimental results, low-energy impact results in four major damage modes, *viz.* contact damage, delamination, matrix failure and fibre failure. The exact sequence of events is difficult to ascertain because of the large number of parameters involved and small duration of the phenomenon. The general description of the sequence of events is illustrated in Fig. 1 showing the occurrence of damage in four stages with each stage absorbing different amounts of impact energy [4-5]:

a) Localised Hertzian contact damage dependent upon the magnitude of the impact force.
b) Internal delamination due to the transverse shear stresses (or strain).
c) Matrix and fibre failure due to the compressive bending strains on the impact face.
d) Matrix fracture (or fibre breakage) caused by tensile bending strains on the lower face precipitating delaminations in the layer adjacent to the outer ply.

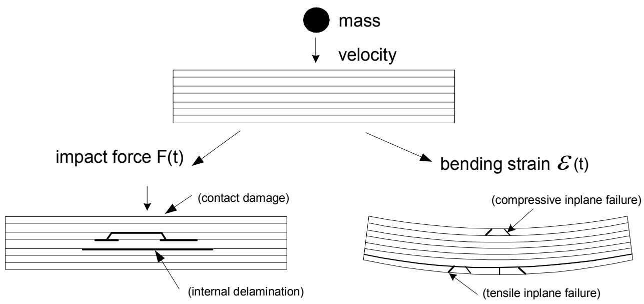

Fig. 1. Impact phenomenon

The proportions of the different damage modes are controlled by a variety of parameters [4], mainly impactor conditions, stacking sequence, material properties, and structural geometry.

For low velocity impacts, the delamination and matrix damage area is proportional to the impact energy. Tougher materials show higher impact resistance than brittle systems. The amount of matrix cracking and delamination area decrease with an increase in toughness. Delamination typically occurs between plies of different orientation, and increases in size with thickness and mismatch angle of the plies (typically on 0°/90° or 45°/-45° interface) [4]. Matrix cracking occurs within blocked plies. For low velocity impacts, the delamination area increases towards the back surface of the laminate and is the largest in the furthest interface from the impact surface [4,6]. In general, internal delaminations are peanut shaped and elongated in the fibre directions. Also, delamination growth in thin laminates occurs in a conical region by growth of delamination towards the lower face of the laminate.

Impact damage results in a local stiffness reduction. Such stiffness reductions affect delamination growth by reducing the buckling loads and causing stress concentrations, which may promote in-plane "notch type" failure. Experimental studies show that the fibre failure in the impact damage zone causes significant local reductions of the tensile and compressive stiffness [7]. Such stiffness losses cause stress concentrations, which in tension may be comparable to the effect of a hole or slit. A damage assessment tool should capture these phenomena as much as possible.

3 Damage Assessment Tool Requirements

A complete impact damage assessment tool should provide information on the structural response and the formation of damage during impact and its effect on the strength and the stability of the structure. The first step, which may be termed "impact damage resistance", involves the prediction of impact response and damage such as fibre/matrix failure or delamination. The second step, termed "damage tolerance", involves the determination of structural response and the amount of damage growth, under service loads. The flowchart for an integrated tool is shown in Fig. 2.

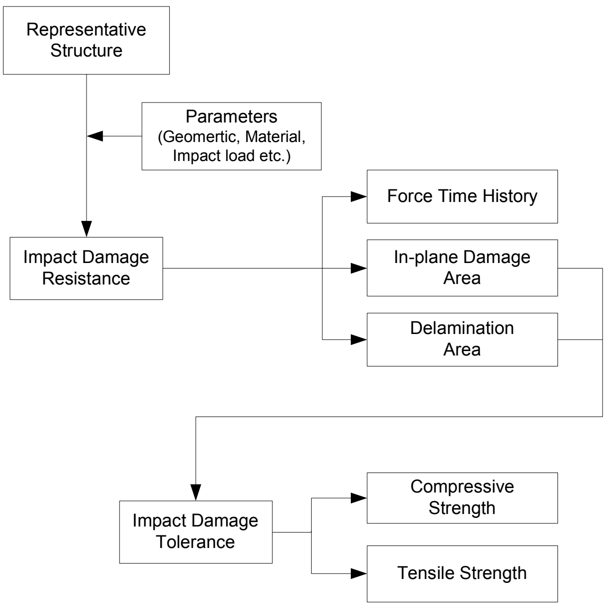

Fig. 2. Structure of damage tool

4 Failure Criteria for Composites

4.1 In-plane Failure

In-plane failures are related to damage in fibres and matrices due to tensile, compressive or shear loads. The existing failure theories for composite plies fall into two categories: those that treat all failure modes together, and those that treat each failure mode independently [8].

The first category attempts to encompass all modes of failure in a single expression. Both Tsai-Hill and Tsai-Wu criteria belongs to this category. Using Tsai-Hill, ply failure occurs when Eqn. 1 is satisfied.

$$\frac{\sigma_{11}^2}{X^2} - \frac{\sigma_{11}\sigma_{22}}{X^2} + \frac{\sigma_{22}^2}{Y^2} + \frac{\tau_{12}^2}{S^2} \geq 1 \tag{1}$$

Where, σ_{11}, σ_{22} and τ_{12} are the respective stresses in longitudinal, transverse and shear directions and X, Y and S are the longitudinal, transverse and shear strengths, respectively. The second category separates the modes into various components, such as fibre tension and compression, matrix tension and compression, and shear. The original Maximum Stress (or) Strain, Hashin [8] and Chang Chang [10] Criteria, which belong to this category, propose five, four and three failure modes, respectively. Fig. 3 and Eqns 2 to 5 represent failure modes for the 2D-Hashin criterion.

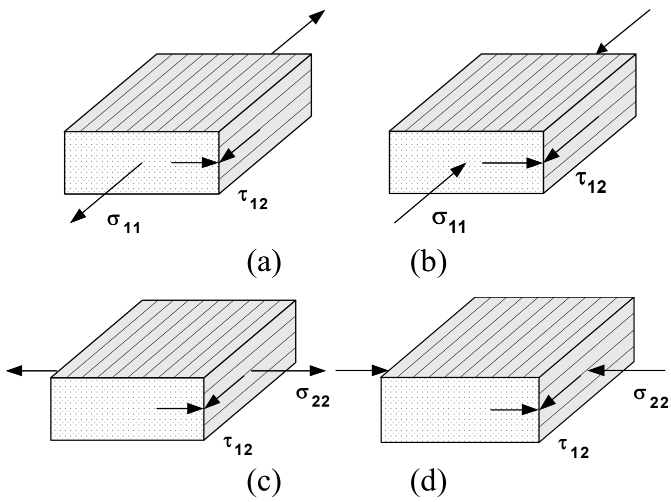

Fig. 3. Four in-plane failure modes
a) tensile fibre, b) compressive fibre
c) tensile matrix, and d) compressive matrix

Tensile fibre mode (σ_{11}>0):

$$\left(\frac{\sigma_{11}}{X_T}\right)^2 + \left(\frac{\tau_{12}}{S_{12}}\right)^2 = 1 \qquad (2)$$

Compressive fibre mode (σ_{11}<0):

$$\frac{|\sigma_{11}|}{X_C} = 1 \qquad (3)$$

Tensile matrix mode ($\sigma_{22} > 0$):

$$\left(\frac{\sigma_{22}}{Y_T}\right)^2 + \left(\frac{\tau_{12}}{S_{12}}\right)^2 = 1 \qquad (4)$$

Compressive matrix mode:

$$\left(\frac{\sigma_{22}}{2S_{23}}\right)^2 + \left[\left(\frac{Y_C}{2S_{23}}\right)^2 - 1\right]\frac{\sigma_{22}}{Y_C} + \left(\frac{\tau_{12}}{S_{12}}\right)^2 = 1 \qquad (5)$$

The subscripts '$_T$' and '$_C$' refer to tension and compression, respectively. The subscripts '$_{12}$' and '$_{23}$' refer to respective shear directions.

The Maximum Stress or Strain Criteria are simple and provide a direct way to predict failure of composites. However, there is no interaction between the stresses/strains acting on the plies and they under-predict the strength in the presence of combined actions of in-plane stresses [11]. The Hashin and Chang Chang failure theories on the other hand include interactions and are more reliable. Several other theories, which are the derivatives of these two theories, have also been proposed. Details and sources for other failure criteria can be found in Reference [11].

4.2 Delamination Failure

Delamination is a significant part of impact damage in composite structures, yet the mechanism of failure itself is not fully understood. Several methods have been proposed to predict the initiation and growth of delaminations. These are discussed in this section.

Fracture mechanics based delamination failure criteria are very frequently used. In this technique, the total strain energy release rate, G, the Mode I component due to interlaminar tension, G_I, the Mode II component due to interlaminar sliding shear, G_{II}, and the Mode III component, G_{III}, due to interlaminar scissoring shear, are calculated using the virtual crack closure technique [12].

The Bending Strain Energy Density (BSED) model proposed by Tang et al. [13] for predicting impact delamination is based upon the failure mechanism for a simply supported laminated composite beam under pure bending. The model concentrates on the qualitative geometric descriptions. The normal stress term does not appear in the expression and hence can be conveniently used in two dimensional finite element analyses.

Although, energy based theories are physically more accurate, damage prediction using stress criteria is still valid considering the short duration for contact events. The Brewer and Lagace criterion for delamination initiation is given by Eqn. 6. The criterion takes into account the normal stresses and shear stresses. The subscript '$_{33}$' in Eqn. 6 refers to normal direction.

$$\left(\frac{\sigma_{33}}{Z_T}\right)^2 + \left(\frac{\sigma_{23}}{S_{23}}\right)^2 + \left(\frac{\sigma_{31}}{S_{31}}\right)^2 = 1 \qquad (6)$$

Zhang [4] used stress-based relations to predict delamination given by Eqns 7 and 8.

$$\sqrt{\left(\tau_{13}^2 + \tau_{23}^2\right)} \geq \text{Inter- laminar shear strength} \qquad (7)$$

$$\sigma_{peel} > Z_T \qquad (8)$$

5 Finite Element Techniques

5.1 Contact Modelling

The impact interface is treated as a contact problem in explicit finite element codes. Interfaces are defined by listing in arbitrary order all triangular and quadrilateral segments that comprise each side of the interface. One side of the interface is designated as the slave surface, and the other is designed as the master surface. Slave nodes are constrained to slide on the master surface after impact and to remain on the master surface until a tensile force develops between the node and the surface.

To estimate the tensile force developed, normal interface springs are placed between all penetrating nodes and the contact surface. The interface force developed, F_i, is then given by:

$$F_i = -e \cdot k_k \quad (9)$$

where e is the penetration depth and k_k is the interface stiffness modulus computed internally for the element in which it resides. The interface stiffness is chosen to be approximately the same order of magnitude as the stiffness of the interface element normal to the interface. Apart from defining the contact between the impactor and the structure, it is also used to define connections between layers of elements.

5.2 Modelling In-plane Failure

The in-plane failure theories described in Section 4.1 can be applied in explicit finite element codes to predict the onset of failure. Once the failure has initiated, the response of the structure is based upon the degradation of the properties, which in turn is determined by the type of failure that has occurred. Table 1 shows the default settings in LS-Dyna for the modes of failure and the associated properties that are degraded [14-15].

Table 1 Degradation rules in LS-Dyna [14-15]

Material Constant	Failure Mode			
	Fibre		Matrix	
	Tension	Comp	Tension	Comp
E_{11}	X	X		
E_{22}	X		X	X
G_{12}	X		X	X
ν_{12}	X	X	X	X

In order to avoid numerical instabilities or shock waves from developing in the analysis following failure, the properties affected by failure cannot be reduced to zero in one time interval. The properties must be decayed over a number of time steps, to avoid such instabilities. This decay is also required from a physical point of view, since in the analysis, the structure is discretised into finite elements. Failure, which in reality occurs at a point and propagates, is modelled as the failure of an entire element. Hence numerical failure is directly related to the element size, and therefore parameters that control the rate of decay must also relate to the FE mesh used.

5.3 Delamination Modelling

The delamination criterion is not as easily applied as the in-plane failure criterion. While the actual energy absorbed by delaminations is usually low compared with other modes of failure, delaminations do have a strong influence on failure progression. A laminate that has split into a number of sub-laminates, each of which has significantly lower bending stiffness than the original laminate, is likely to fail under dynamic loading in a different manner than the original, intact laminate.

5.3.1 Stacked Solid Elements

An ideal form of delamination modelling would be a system of elements representing a composite laminate, which would separate into individual plies as delamination occurs. Hoof et al. [16] used layers of solid elements connected at each interface by a tied contact definition (Fig. 4). This contact is broken when the failure criterion is satisfied.

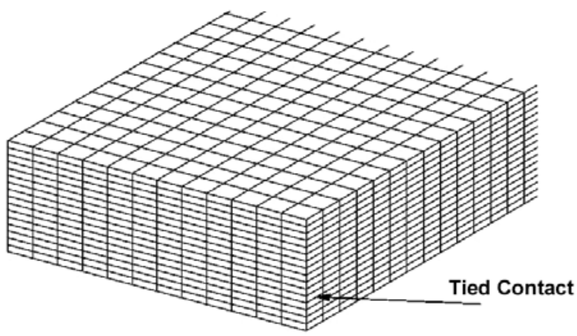

Fig. 4. Solid elements with tied contacts

The time step for explicit finite element analysis depends upon the shortest side length of an element given by the Eqns 9 and 10.

$$\Delta t_{critical} = \frac{L_s}{c} \tag{10}$$

where L_s is the characteristic length and c is the velocity of sound in the material.

$$c = \sqrt{\frac{E}{\rho(1-\nu^2)}} \tag{11}$$

where 'E' is Young's modulus, ρ is density and υ is Poisson's ratio.

Hence, for structures such as fuselage panels and wing skins where the number of layers is in excess of ten, the use of solid elements becomes prohibitive due to high computational costs. For example, an impact zone of 76 mm x 63 mm of a 4 mm thick laminate (32 layers) would easily require about one million brick elements near the impact zone [4].

5.3.2 Stacked Shell Elements

An alternative approach is to use multiple layers of shell elements, in which one shell element is used to model each ply. In this case, a type of one-dimensional element or a contact that can fail under certain predefined conditions ties the layers together. A method based on either nodal forces or strain energy release rate can be implemented to predict the delamination initiation and growth. These options, of course, are much more computationally demanding compared with the conventional approach of modelling composites, where one shell element represents the total ply stack. For this reason, the majority of research work has been limited to predicting delamination in a single selected interface [5-17].

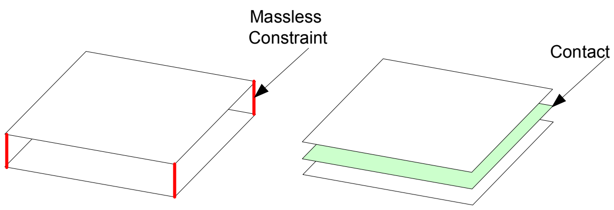

Fig. 5. Stacked shell modelling

The stacked shell technique solves the problem of high computational cost to some extent. Current research with the stacked shell approach has been limited to two layers of shell elements, where delamination is assumed. Modelling and computational time increase significantly by increasing the number layers in the stack. Moreover, the accuracy of the contact surface consistently decreases with an increase in the number of layers. This modelling approach is not ready to be implemented in a structure with a larger number of layers, in which multiple delaminations may occur.

The element formulation for a shell element is such that transverse shear stresses have a parabolic distribution, as shown Fig. 6. In a stacked shell approach, the model will have a number of peaks equal to the number of layers in the stack, instead of having one peak (assuming all plies are of same material and oriented in the same direction). Some explicit finite element codes use averaged transverse shear stress distribution for shell elements. However, this can make the composite models overly stiff and give inaccurate results. Stress based criteria for delaminations are strongly dependent upon the transverse shear stress and hence the approach cannot be used in its current state.

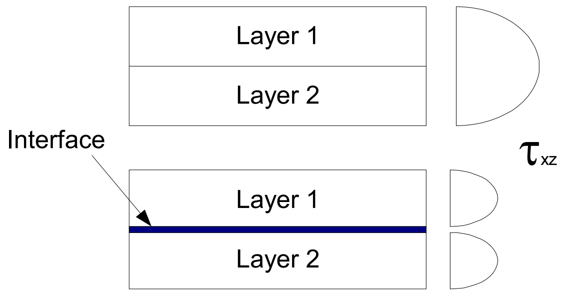

Fig. 6. Transverse shear stress distribution in a normal laminate, and in stacked shell approach

5.3.3 Single Layer Shell Elements

Because of these difficulties, there have been some efforts to predict delamination with a single layer of shell elements. Tang et al. [13] applied the Bending Strain Energy Density method on structure represented by a single layer of shell elements for which the researchers have claimed good correlation with the experimental results. The claimed accuracy of FE techniques without a mechanism for separating delaminated plies is noteworthy. The reason may lie in the fact that impact events occur over a short duration and modelling such phenomena may not be as critical as in other cases such as compression after impact where sublaminate buckling control stiffness, strength and failure.

5.4 Residual Stiffness Modelling

For modelling techniques with the ability to separate the individual layers, in-plane damage as well as the delamination can be specified in the FE model before applying loads to predict, the residual compressive strength. However, the computational cost and the volume of data that needs to be processed are very high. For this reason, this procedure may be limited to simple structures with few plies or where delamination occurs at a few selected interfaces.

Alternatively, several researchers have implemented the "soft inclusion" method to study the damage tolerance of the composite structure. This method includes the degradation of the elastic moduli in the impact-damaged region [18]. The stiffness of the damaged area is degraded by the internal damage so that it is lower than that of the rest of the laminate. This technique is also referred to as "inhomogeneity" modelling [19]. This technique can be efficiently applied for all practical structures, however, more research is required for characterising residual stiffness mathematically.

5.5 New Delamination Modelling Technique

The usual method of representing a laminate by a single layer of shell elements is to specify a material property, thickness and orientation represented by through-the-thickness integration points. The number of these integration points is equal to the number of plies in the laminate. After considering the techniques used by other researchers, a new method for modelling delaminations is proposed. As shown in Fig. 7, resin rich interfaces are modelled within a single shell element with extra integration points through the thickness. The resin interfaces are 1% of the standard ply thicknesses. These resin interfaces are modelled by additional integration points equal to the number of interfaces. Delamination is assumed once the allowable transverse shear strain is exceeded in these resin rich interface layers.

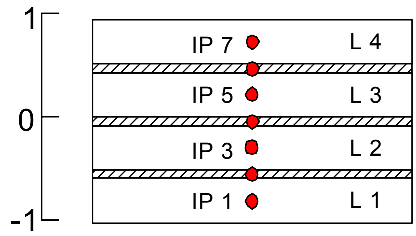

Fig. 7. Through-the-thickness integration points with extra integration points at each interface for delamination prediction

6 Impact Damage Assessment Tool

The modelling technique for the impact damage assessment tool was dictated by the requirement for the quick assessment of a full stringer-stiffened panel. Partial modelling does not capture the actual flexural response of the structure. Furthermore, FE modelling of individual plies is tedious and their analysis is computationally demanding. A parametric impact damage assessment tool has been developed to satisfy the above-mentioned requirements. The features of the modelling tool developed are listed below and the structure of the tool is shown in Fig. 8:

- Parametric model of a representative stringer stiffened panel
- Single layer of shell elements with resin rich interfaces (Fig. 7)
- Modified Hashin failure criterion for in-plane failure (Eqn. 2 to 5)
- Delamination criteria based on transverse shear strain in the resin rich interface
- Mathematically defined rigid body impactor
- Detailed stringer modelling for impact on stringer foot or stringer centreline
- Mechanism to separate the stringer from skin to capture loss in stiffness due to debonding

- Uses commercial codes, Ansys, LS-Dyna and LS-Post
- Uses macros written in C-programming language
- Refined mesh in impact zone
- Parametric nature of the tool allows the modelling of plates or stringer-stiffened panels of any dimension and material.

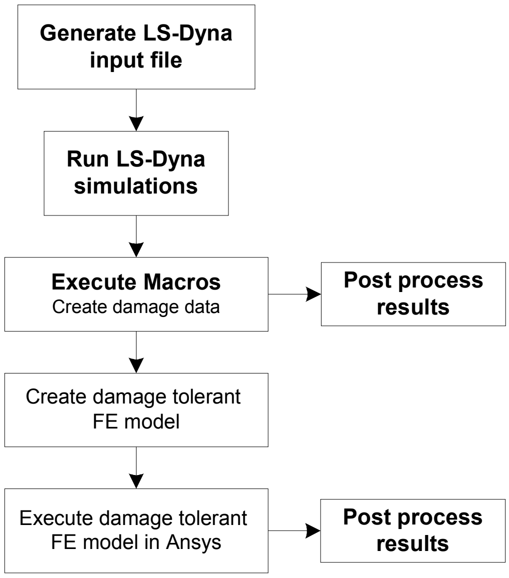

Fig. 8. Damage assessment tool structure

6 Validation for Mid-Bay Impact

For validation, a stringer-stiffened panel tested by Greenhalgh et al. was considered [3]. The structure was a carbon-fibre reinforced panel consisting of a skin and three I-stiffeners. The 4 mm thick skin had the stacking sequence of $(+45°/-45°/0°/90°)_{4S}$. The panels were made of T800/924. Impact on the mid-bay with a 10 mm diameter impactor with 15 J energy was considered.

The explicit FE analyses were carried out on multi-processor Hewlett Packard (HP) workstations with a Unix operating system. The analyses were performed with LS-Dyna (version 960) using two processors. The FE model for the problem is shown in Fig. 9. This analysis with a smallest element size of 1.25 mm and 4784 shell elements (using minimal mass scaling) took less than 3 hours to run.

The contact force time history and the damage area were compared with the experimental results. The force-time history obtained using the damage assessment tool is shown in Fig. 10. It can be seen that the contact force history is close in form to the experimental data. Both the maximum contact force and contact duration is predicted within engineering tolerances.

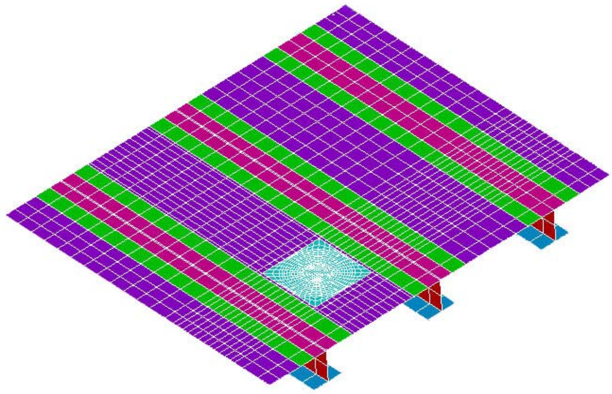

Fig. 9. Finite element model

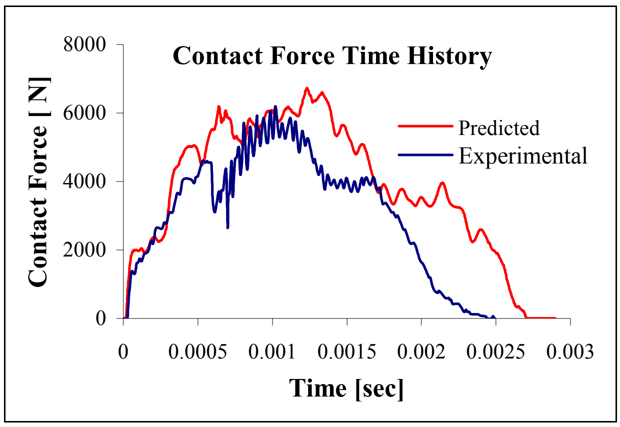

Fig. 10. Contact force time history

For mid-bay impact, the amount of delamination due to shear strain would be minimal, given the panels flexibility. The flexural bending, however, results in considerable matrix cracking on the lower face. Matrix cracking on the lower face usually initiates delamination. Therefore, there should be a close resemblance between the total matrix failure area and the projected delamination area. The predicted matrix cracking region and the experimental delamination region is shown in Fig. 11.

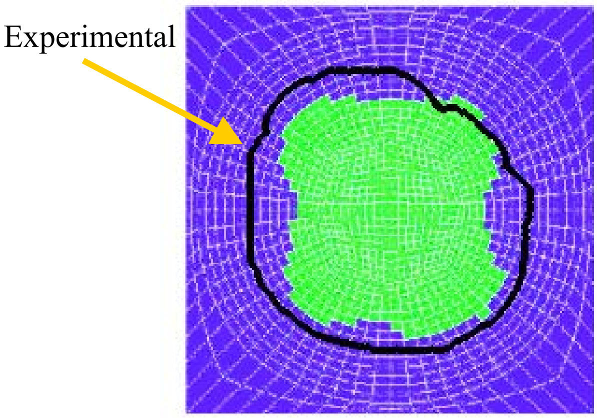

Fig. 11. Matrix failure region comparison

The delamination region was predicted by identifying the failure of the resin rich interface layers. The predicted failure region on the top and bottom interface combined with the experimental results is shown in Fig. 12. In general, the delamination size and shape are predicted very well at both interfaces, demonstrating the effectiveness of the modelling technique.

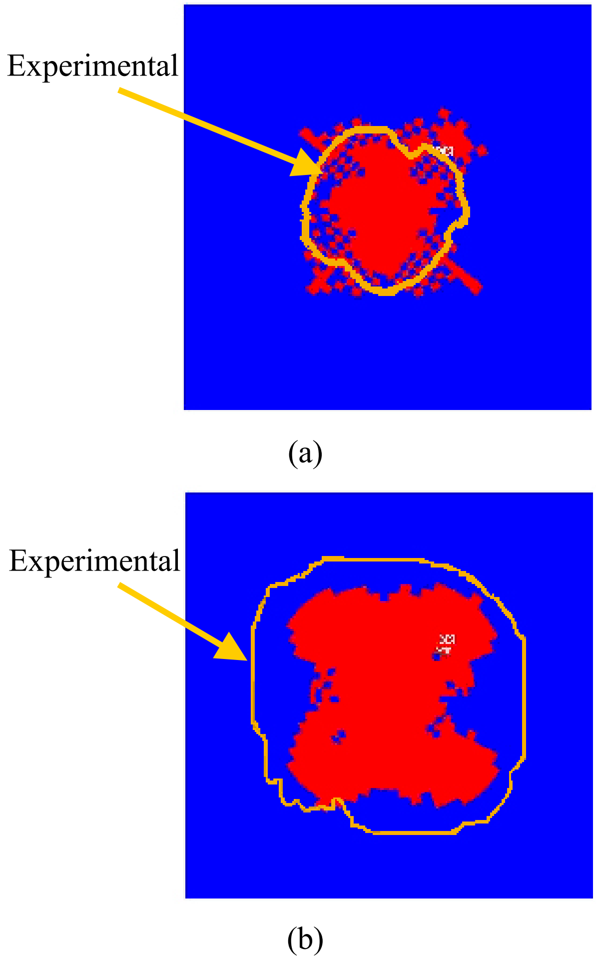

Fig. 12. Delamination area
a) in top interface, b) in bottom interface

7 Conclusion

The review of research on numerical modelling of impact damage showed that the in-plane failure could be effectively incorporated into explicit FE codes. Progressive damage modelling with criteria such as Hashin and Chang Chang are suitable for in-plane failure. Implementing a delamination criterion, however, is more complicated. Modelling with stacked solid, stacked shell elements or a single layer of shell elements has been considered by various researchers. The stacked solid element and stacked shell element techniques are advantageous as they allow the computation of normal stress and hence can be closely related to delamination theory. The stacked solid element approach is computationally very demanding and is therefore unsuitable for the FE modelling of representative stringer-stiffened panels. The stacked shell element approach offers improvements in computational time, but requires considerable modelling time. The stacked shell element approach is best applied when a potential delamination interface can be predicted. Significant inaccuracy and modelling difficulties are expected when multiple delaminations over a large number of laminate interfaces are considered. The single shell approach used in this work, on the other hand, requires minimum modelling time and is free from modelling inaccuracies. This technique, however, suffers from the absence of normal stresses for delamination prediction and cannot account for changes in stiffness resulting from delamination.

Based on this reasoning, the modelling tool using the single shell approach was developed. A simple technique, which predicts delamination area based upon the failure of resin rich interfaces, has been proposed. The explicit FE code, LS-Dyna, and the FE pre-processor Ansys, were used to develop the tool, which is parametric in nature. Validation of the tool was conducted by comparing the predicted results with the experimental values presented in Ref. [3]. The predicted force-time response was within 16% of the experimental results. The predicted damage area was within 20% of the experimental results. These results have established the value of this computationally efficient technique for impact damage prediction.

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