RE-ENTRY MOTION OF AN AXIALSYMMETRIC VEHICLE AND ITS ANALYSIS BASED ON FLIGHT SIMULATION

A. Guidi
Delft University of Technology, Delft, The Netherlands

Keywords: *re-entry, stability, hypersonic*

Abstract

This work started during the Stability Analysis of the Delft Aerospace Re-entry Test demonstrator (DART) which is a small axisymmetric ballistic re-entry vehicle. The dynamic stability evaluation of an axisymmetric re-entry vehicle is especially concerned on the behavior of its angle of attack during the flight through the atmosphere. The variation in the angle of attack is essential for prediction of the trajectory of the vehicle and for heating requirement of the structure of the vehicle. The concept of the total angle of attack and the windward meridian plane are introduced. The position of the center of pressure can be a crucial point in the static stability of the vehicle. Although the simpleness of an axisymmetric shape, the re-entry of such a vehicle is characterized by several complex phenomenologies that were analyzed with the aid of the flight simulator.

1 Introduction

Delft Aerospace of Delft University of Technology is proposing to build a small reusable ballistic re-entry vehicle together with other European partners.

An aircraft in flight is constantly subjected to forces that disturb it from its flight path. How the aircraft reacts to such a disturbance from its flight attitude depends on its stability characteristics. During the initial design phase, a strong interaction between different disciplines is demanded for an effective design. The vehicle equations of motion are a unifying framework for studying many important aspects of a design and for evaluating problems that arise during the analysis phase of the design. From these equations a 'State Space Model' can be built to simulate all the aspects of the design. The software implementation of the equations of motion is the 'Flight Simulator'. In the severity of this flight, stability assumes a new important role that is connected to the prediction of the heat load distribution and to its minimization. Selection of the shape of a re-entry vehicle is usually driven by the need for high aerodynamic drag, low aerothermal heating, and sufficient aerodynamic stability. The flight of DART from the re-entry into atmosphere to the touch-down must be aerodynamically stable, without tumbling, within the wide range from hypersonic to subsonic. Therefore, the shape was established on the premise that it should be a simple combination of spherical part and conical part, so that the aerodynamics could be figured out by simple investigation for such a wide flight velocity range. To minimize complexity, other critical choices in the design of DART were made. Passive flight control and a ballistic trajectory were chosen (in order to contain aerodynamic heating a semi-ballistic trajectory can also be considered). Spin-up of the vehicle is required in order to maintain its trajectory attitude and to eliminate deviations from the desired initial attitude and errors caused by misalignment of the center of gravity. DART requires sufficient aerodynamic stability in order to overcome the spin stiffness and minimize any angle of attack variation throughout all speed regimes, maintaining a controlled attitude until parachute deployment.

Two configurations were investigated. Thereinafter we will refer to them as REVolution configuration (Fig.1) and VOLNA configuration (Fig. 2).

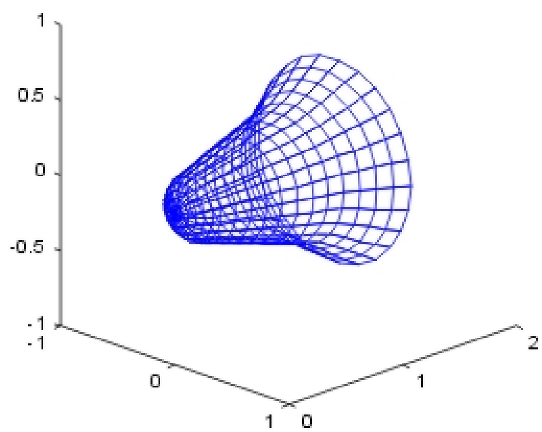

Fig. 1 REVolution configuration, R_N=0.250 m, R_B=0.670 m, L=1.360 m $\vartheta_1=15^o$, $\vartheta_2=30^o$

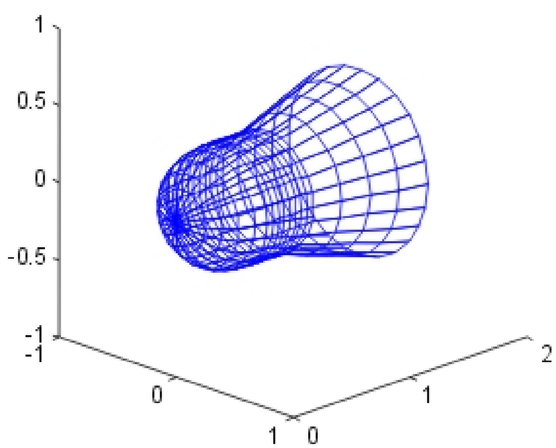

Fig. 2 VOLNA configuration, R_N=0.392 m, R_B=0.6 m, L=1.2 m $\vartheta_1=0^o$, $\vartheta_2=15^o$

2 Static and Dynamic Stability

An aircraft may be *inherently stable,* that is, stable due to features incorporated in the design, but may become *unstable* due to changes in the position of the center of gravity caused by consumption of fuel, improper disposition of the disposable load, and so fuort.

In order to achieve a highly accurate prediction of the landing site, and a high "comfort" for the on-board instrumentation DART has to be statically and dynamically stable.

2.1 Static Stability

"The stability characteristics of an aircraft has is roots in the aerodynamics of the vehicle", this because the response of the aircraft at the forces acting on its surface during the flight is given by its "aerodynamic coefficients". But more important for the stability analysis are the derivatives of these aerodynamic coefficients. The derivatives express in fact the behavior of the coefficients (and consequently of the vehicle) at variation of flight condition. Variation of flight condition cause variations in the value of the aerodynamic coefficients. "*An aircraft is statically stable in a given steady flight condition, if a change in angle of attack or in angle of side slip, causes a change of the aerodynamic moment about the center of gravity, i.e. a change in C_m or a change in C_n respectively. Which tend to rotate the aircraft back to its original attitude relative to the airflow.*"

Both the two shapes of DART respects this condition and are therefore statically stable.

A statically stable vehicle always assumes 'Trim' condition during the flight. From Fig. 3 both the configuration reach an 'α_{TRIM}' after a certain dumping time .

2.2 Center of Gravity and Center of Pressure Location

However, static stability is strictly connected to the location of the center of gravity. The coefficients of moment are all referred to a fix point, in this case coincident with the center of gravity of the vehicle, therefore they are not the only sources of moment on the vehicle. A component deriving from the aerodynamic normal forces should be added and this component depends on the location of the center of pressure through:

$$\frac{dC_m}{d\alpha} = \frac{dC_{m_{c.g.}}}{d\alpha} + \left(x_{c.g.} - x_{c.p.}\right)\frac{dC_N}{d\alpha} .$$

The position of the center of pressure can be a crucial point in the static stability of the vehicle. When the location of the center of pressure falls between the center of gravity and the nose of the vehicle static instability may occur during the flight. As first notation the location of center of gravity and center of mass were assumed coincident. This assumption is not restrictive for a re-entry vehicle. For a good stability of a blunted body the center of gravity (c.g.) location has to be closes enough to the nose. At hypersonic continuum condition, blunt shapes exhibits acceptable stability even for a c.g. position behind the maximum diameter location of the vehicle. Conversely at supersonic speed a c.g. not enough close to the nose can results in a dynamic instability that induces oscillatory motions. At transonic speed this motion is further amplified and in subsonic regime the strength of the dynamic instability can overcome the static stability of the shape and causing the vehicle's oscillations to diverge into tumbling motion. Therefore the c.g. requirement for a re-entry vehicle whose entry profile includes subsonic flight is driven by the subsonic dynamic stability of the vehicle. Designers sometimes avoid this further study in the subsonic flight regime by deploying parachutes at supersonic speeds.

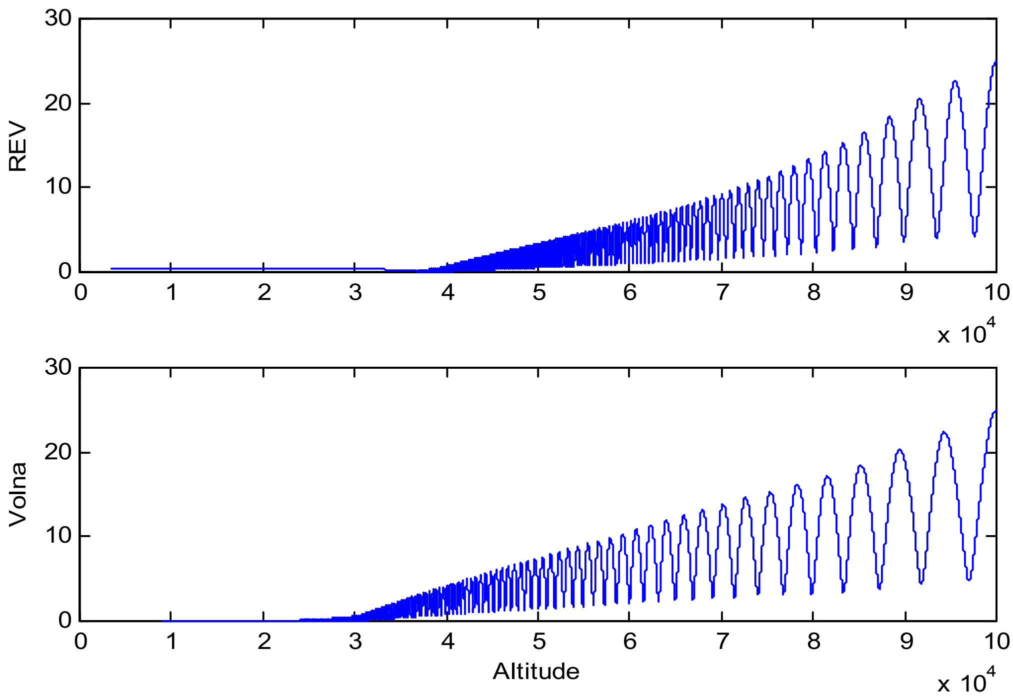

Fig. 3: Total angle of attack (in degrees)

A variation of mass in the nose part of the vehicle result in a backward motion of the c.g. with destabilizing effect on the vehicle. In turn with the decreasing of the Mach number, especially in the supersonic flight, the vehicle experiences a forward movement of the center of pressure. The center of pressure is identified for our purposes as that longitudinal station at which the components of the aerodynamic moment vector normal to the vehicular plane of symmetry is zero. The center of pressure would move longitudinally with changes in the angle of attack. This movement of the center of pressure with angle of attack means that either the vehicle could operate at only one trimmed angle of attack or the center of gravity would have to be moved to change the trim angle of attack. The importance of the simultaneous movement of this two point is clear because the moment due to the aerodynamic forces is proportional to the arm (Xc.g. – Xc.p.) and will be a stabilizing moment if the c.g. is located between the c.p. and the nose, destabilizing in the opposite case, for this last characteristic is also often defined as the 'static margin'. A backward motion of the c.g., and a forward motion of the c.p., can give arise destabilizing condition.

2.3 Dynamic Stability and Angle of Attack convergence.

The dynamic stability evaluation of an axisymmetric re-entry vehicle is especially concerned on the behavior of its angle of attack during the flight through the atmosphere. The variation in the angle of attack is essential for prediction of the trajectory of the vehicle and for heating requirement of the structure of the vehicle.

2.3.1 Total angle of attack

In an axial-symmetric body, operating at combined attitudes of pitch and yaw, the wind vector approaches the body at some equivalent wind plane at an angle of attack that is a combination of the two angles of pitch and yaw. The equivalent plane will be between the XZ and YZ planes of the body axis and will be called windward-meridian plane. To calculate these equivalent angle and plane two different ways can be followed: Geometrically or Analytically. In the first method some considerations on the geometry of an axial-symmetric body, in a flow with the wind vector inclined with the axis of symmetry, are made. The axis of symmetry of the body is inclined at an angle α in the XZ plane and angle β in the XY plane.

Using trigonometry the following relations are derived. The equivalent angle of attack is equal to $\varepsilon = \arccos(\cos\alpha \cos\beta)$ and the plane of action is located

at an angle: $\phi' = \arctan\left(\dfrac{\tan\beta}{\sin\alpha}\right)$ from the Y axes.

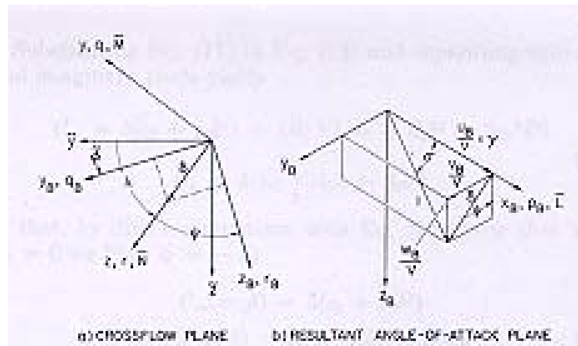

Fig. 4 Total angle of attack

Disadvantage of this method is that in the analysis it is assumed that both of the angles are small because in the modeling of the equations the arc of the angle is assumed to be its chord. This solution can be considered accurate up to an angle of about 15 degrees. In the study of the stability of Dart this assumption can be made considering that it is intended to design a ballistic re-entry configuration. However, in order to achieve more accurate results, and considering that in the angle of attack convergence analysis will be useful start simulation for angle also bigger then 15 degree, this problem can be carried out analytically. This also considering that maybe in the high atmosphere some wide oscillations can be present. It is of importance during the analysis not to confuse the source of error of a too restrictive approximation with the true oscillation of the vehicle. This means that with a series of transformations between different reference frames, that the resulting final reference frame in which there acts only one angle of attack is determined. The concept is that a rotation from a reference frame to an other can be obtained with several angular rotation and with a different order, applying it to a transformation from the body frame to a body frame two rotation matrix can be obtained. The first use the angle of attack and the angle of side slip

$$T_{B\leftarrow W}(\alpha,\beta) = R_y(\alpha)\cdot R(\beta)$$

A second transformation use angle the "total angle of attack" and the windward-meridian plane rotation. The problem it is solved simply in an equality between matrix.

With this different method the value of the angle of attack is unvaried from the geometrical value. What is slightly different is the inclination of the windward-meridian plane.

$$\phi' = a\sin\frac{\cos\alpha\cdot\sin\beta}{\sqrt{1-(\cos\alpha\cdot\cos\beta)^2}}$$

Finally, the force acting in this new plane is calculated and then projected in the X and Y directions.

Following the behavior of the wind-ward meridian plane, its rotation and its rotation rate, is carried out from the output of the flight simulator. Its rotation rate is an important parameter because can generate asymmetry in the wear down of the material of the vehicle and consequence of general asymmetry in the dynamic motion of the vehicle.

2.3.2 Angle of attack convergence

In order to have a dynamically stable vehicle its angle of attack has to converge. A lot of study in literature can be found on the angle of attack convergence behavior of axisymmetric vehicles. Most of them with spin condition. The importance of the spin is analyzed in the next section.

For what concerns the study of the dynamic stability during a re-entry flight the aid of a flight simulator is strongly requested. It is in fact practically impossible to execute analytically a complete non-linear analysis of the attitude of a re-entry flight.

The REV shape is characterized by a longer dumping time and a smaller amplitude in the oscillations. However it also experience a higher frequency of oscillation. A simulation at high spin rate was executed in the REVolution shape and, as expected, a very high spin rate induces a smaller oscillation due to the stiffness of the spin. However, the convergence is lightly delayed.

Most of the study treats the case of constant roll rate. However, because of the susceptibility of re-entry vehicles to roll rate excursions it is necessary to investigate the influence of roll acceleration on the angle of attack convergence and on the windward meridian rotation rate.

During atmospheric re-entry a spinning body with a trim angle of attack configuration, such as the once inducible by aerodynamic configuration asymmetries, encounters a trim amplification which is strongly dependent on the magnitude of the ratio of the spin frequency to the body natural or critical frequency. In the vicinity of equal spin and critical frequencies, the body angle of attack can become very large and result in a significant perturbation in the subsequent trajectory history. Although flight simulators are generally used to predict angle of attack divergence in the region of transient amplification, a simplified analytical description of the phenomenology is interesting.

Several studies have shown that spin rate changes arise primarily from coupling between lateral mass asymmetries and steady trim forces.

The resultant angle of attack is described by two time varying damped vectors, R_1 and R_2, rotating at different rates, $\omega_0 - \Delta\omega$ and $\omega_0 + \Delta\omega$, respectively, about the trim component α_{TRIM}. For positive spin rate p, R_2 always rotates faster than R_1 in a counterclockwise direction, whereas R_1 changes its direction of rotation depending on the relative magnitude of ω_0 and $\Delta\omega$. The α_{TRIM} orientation and magnitude vary with proximity to resonance $\omega = p\left(1-\frac{I_x}{I}\right)$.

With a c.g. displaced from the geometric (or aerodynamic) centerline, a roll torque is generated and its magnitude and direction depend on the orientation and magnitude of the total angle of attack. α_{TRIM} variate in magnitude and orientation with proximity to resonance p/ω.

With increasing in p/ω, α_{TRIM} rotates counterclockwise, to a $\psi = -90°$ at resonance and grows to a peak magnitude α_T limited only by the aerodynamic damping factor. A further increase in p/ω above resonance rotates and decreases α_{TRIM} to 0. It should be noted that large α_{TRIM} is maintained about resonance. With changing in the environment, only the magnitude of the amplification $\alpha_T/\ \alpha_{ST}$ is affected by the changing in the damping factor, the orientation is only slightly affected.

Thus, if significant undamping occurs due to unsteady aerodynamic and ablation effects, the trim exhibits a growth similar to the usual dynamic motion instability. The motion of a spinning re-entry body, is analogous to the motion of a spinning top or gyroscope under the action of gravity in which the static moment $\omega^2 I\sin\theta$, which tends to decrease θ for a statically stable missile, is equivalent to the top turning moment $-Wl\sin\theta$, which tend to increase θ. Such a motion is described in literature in terms of the classical Euler angle coordinates. A great contribution of D.H. Platus in this field was the application and development of the classical Euler coordinate system to describe modern ballistic re-entry vehicle motion. Certain aspects of re-entry vehicle motion are conveniently described in the terms of Euler angles. One of the examples is the roll-rate behavior that was one of the “big surprises” in the development of the current high performance ballistic missiles. Some vehicles spun into resonance (in which the roll rate is approximately equal to the vehicle natural frequency) with catastrophic effects of large angle of attack amplification.

One Euler angle is for practical purposes, the precession angle of the lift vector in space. The coupled behavior of this Euler angle with the total angle of attack Euler angle, analogous to the nutation angle of a spinning top or gyro determines cross-range dispersion associated with lift non-averaging. The vehicle motion during re-entry, described in terms of time-dependent Euler angle $[\psi(t),\phi(t),\theta(t)]$, is assumed to be of the form:

$$[\psi(t),\phi(t),\theta(t)]=[\psi_{qs}(t),\phi_{qs}(t),\theta_{qs}(t)]+[\psi_+(t),\phi_+(t),\theta_+(t)]$$

where $[\psi_{qs}(t),\phi_{qs}(t),\theta_{qs}(t)]$ represents a quasi-steady component that varies relatively slowly with time (of the order of the dynamic pressure) and $[\psi_+(t),\phi_+(t),\theta_+(t)]$ represents an oscillation of higher frequency about the average (quasi-steady) values.

Analytical results indicate that for roll rates that are not excessively supercritical, the angle of attack convergence depends only on the re-entry rate and is independent on the roll acceleration.

3 Coning Motion and Initial Re-entry condition

Before re-entry, the vehicle is in a state of moment-free motion, and the relation between the various angular rates is uniquely determined. In general the vehicle has an arbitrary motion around the center of gravity compounded by the sum of 4 simple motions. Three of these are well known and are an initial roll rate p_0, pitch rate q_0, yaw rate r_0. The fourth motion the “coning motion” is a steady precession with a precession rate Ω and a cone half-angle α This last motion should be further analyzed. From experimental investigation (Tobak M., 1969) vortexes induced by this motion take asymmetrical disposition with respect to the angle of attack plane during coning motion and become great source of side moment.

The coning axis is, in general, inclined with respect to the flight path, and the velocity vector may lie inside the coning circle or outside the coning circle. Limiting cases are exoatmospheric motion, where the vehicle is at an angle of attack with no coning or is coning symmetrically about the flight path. The exoatmospheric roll rate, precession rate, and cone half-angle are related by the expression:

$$\Omega = \frac{I_x p_0}{I\cos\alpha}.$$

It is interesting to notice that the Volna Shape experiences a precession smaller than the REVolution shape and with a smaller value of the variation of the rate. Another interesting note is that increasing the value of the spin rate, increases also the variation of the spin acceleration. This will be interesting in the study of the Lunar motion that is strongly dependent from that rate.

After entering the atmosphere, the statically stable vehicle is subjected to an aerodynamic pitch moment that tends to align the vehicle with the flight path. However, because of the angular momentum comprised of the roll and the precession rates, which, on the average, is directed along the coning axis, the pitch moment is resisted by gyroscopic forces that induce a precession of the angular momentum vector about the average flight path. A projection of the path described by the nose of the vehicle on a plane perpendicular to the flight path would be as shown in a and b when the path is viewed along the direction of the flight.

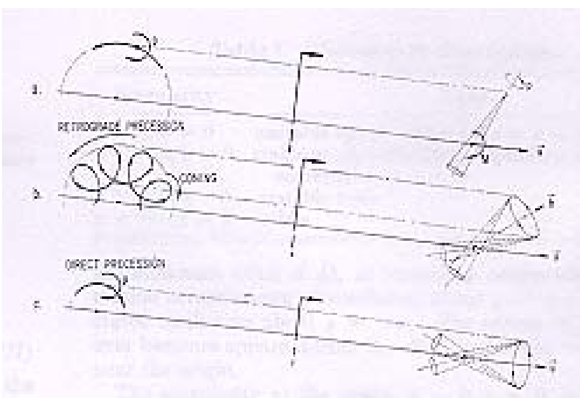

Fig. 5 Precession mode

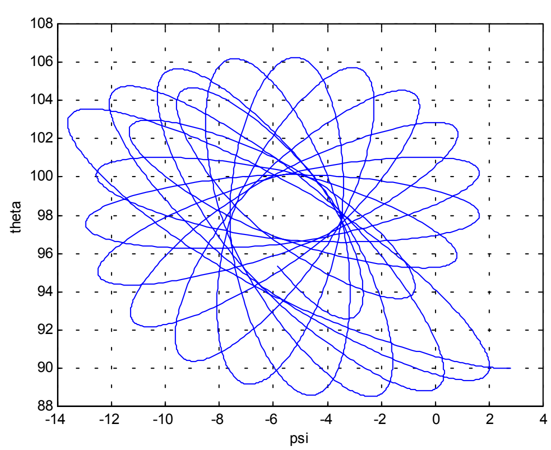

Fig. 6 Projection of the path of the Nose.(REVolution shape between sec 35-50) - Tridimensional projection of the nose

The residual coning motion, in the same direction as the angular momentum vector (clockwise), has been called nutation (Figure 5 c) by Nicolaides (Nicolaides, J.D. 1953), and the retrograde precession in the opposite direction has been simply called precession (Figure 5 a). In general the two motion exists simultaneously. The Euler angles were defined with respect to an inertial frame of reference that moves along the flight path, so that $\dot{\psi}$ represents the precession about the velocity vector.

4 Impact dispersion due to Lift Nonaveraging, Lunar Motion

Assuming that eventual asymmetries relative to the vehicle are fixed in time, the Lift force derived in the trim configuration, and necessary to balance the torque caused by asymmetry, is also fixed. Consequently there is an unbalanced force acting on the vehicle in a constant direction and generating an acceleration away from the zero-lift flight path. In order to average out this lifting effects most of the statically stable mis siles and re-entry vehicles are provided with a nominal roll rate.

The response frequency of the vehicle to angle of attack is in general much greater then the roll rate, at list for altitude where the variation in roll rate are determinant for impact prediction. As consequence the resulting angle of attack does not converge and the vehicle assumes, in contrast, an initial, or re-entry, angle of attack. When the initial angle of attack is damped, and the vehicle assumes the α_{TRIM} , the vehicle response in pitch and yaw will prevent the roll rate from rotating the vehicle meridian plane contains α_{TRIM} , out of the wind (with respect to the body reference frame). It means that the windward meridian plane is always the meridian that contains α_{TRIM} , just as it would be if the vehicle were not rolling. However, the vehicle is rolling and the consequence is that the vehicle-fixed lift force is rotating about the mean flight path at the rate of the roll, and the resulting vehicle acceleration is similarly distributed. The result is a constant lift at constant roll that generates a spiral trajectory about the mean flight path without significant deviation from the zero lift trajectory. However, if the roll varies, the lift is not distributed for an equal amount of time in all directions about the zero-lift flight path and deviation occurs. The previously described motion of a re-entry vehicle rolling with a vehicle-fixed windward meridian is called "Lunar motion" (Crenshaw J.P. 1971).

Two main kinds of Lunar motion are possible: with constant roll rate and with variable roll rate. Of major importance on the re-entry vehicle impact is the second. The first is a very particular case but has been reported in order to better understand the more general case of a roll rate variation.

4.1 Lunar motion at Constant Roll Rate

When the α_{TRIM} is small, after that the oscillatory portion of the angle of attack is dampened to the trim value, the lift and roll can be assumed both constant, and the roll is much less than the response frequency of the vehicle in pitch and yaw but not close to zero. The resulting motion is a rotation of the c.g. about the mean flight path and translation down the main flightpath.

The direction of the lift is toward the mean flightpath and is balanced by the centrifugal force.

The increasing in drag, caused by lift is a negligible fraction of the zero-lift drag for the supposed case with a small α_{TRIM} . Since L is constant, the resulting acceleration is averaged about the mean flight path.

4.2 Lunar motion at a Variable Roll Rate

Define the mean flightpath as place of the instantaneous center of curvature of the motion produced by lift as the vehicle proceeds through the atmosphere. Suppose that although L remain constant the roll variates. Since the lift is no longer averaged in time, there will be a lateral motion of the center of curvature and a motion about the center of curvature as shown in Fig. 7

In Fig. 7 the angles δ_x and δ_y represent the deflections of the mean flight path resulting from lift. The x_b-y_b plane contains the lift vector and the motion about the center of curvature. The velocity V is the velocity of the instantaneous center of curvature as the vehicle proceeds. The magnitude of the tangential velocity is several order bigger then the one of the other components, following that the lift serves only to change the direction of the velocity.

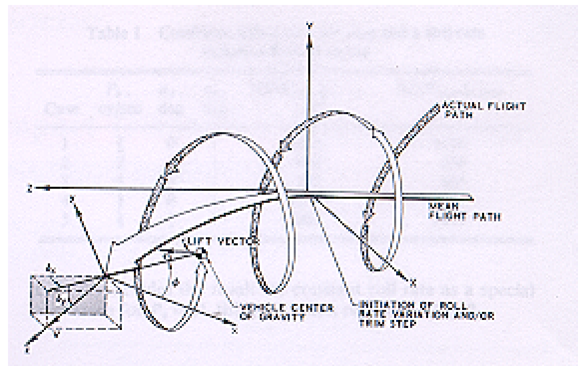

Fig. 7 Re-entry vehicle Lunar motion with lift and roll rate variation

For straight or nearly straight flightpaths, the differential distance at ground level normal to the intended flight path resulting from a differential deflection of the flightpath is approximately equal to the arch length:

$$\Delta_R = d\delta \int_t^{t_{IM}} V dt$$

Where t_{IM} the impact time (Crenshaw J.P. 1971).

References

[1] Andreson, J.D. Jr , *Hypersonic and high temperature gas dynamics*, AIAA 2000

[2] Bertin, J.J. , *Hypersonic aerothermodynamics* AIAA, 1994

[3] Buursink J., Van Baten T.J. et al. "*The Delft Aerospace Re-entry Test demonstrator"* IAF, 2000

[4] Cook, M. V. "*Flight Dynamic Principles*" Arnold, London, 1997

[5] Costa, R.R. , et al , "*Atmospheric Re-entry Modeling and Simulation: Application to a Lifting Body Re-Entry Vehicle"*, AIAA Paper 2000-4086

[6] Nicolaids, J.D. , "*On the Free Flight Motion of Missiles Having Slightly Configurational*

Asymmetry" , Rept. BRL-858, 1953 Ballistic Research Laboratory.

[7] Platus D.H., *"Angle-of-Attack Convergence and Windward-Meridian Rotation Rate of Rolling Re-entry Vehicles"*, AIAA Journal , Vol. 7 NO. 12 pp 2324-2330, New York 1969

[8] Platus D.H. *"Dispersion of Spinning Missiles due to Lift Nonaveraging",* AIAA Journal , VOL. 15 NO 7, pp 909, 915 , Arlington 1976

[9] Platus D.H. , *"Ballistic Re-entry Vehicle Flight Dynamics"* AIAA 82-4019; AIAA Journal vol 5 No 1, pp 4,16, 1982

[10] Prince D.A. and Eircsson L.E. , "*A New Treatment of Roll-Pitch Coupling for Ballistic Re-Entry Vehicles*" , AIAA Journal Vol. 8 NO 9 pp 16081615

[11] Regan, Frank J. , *Dynamics of atmospheric re-entry*, AIAA 1994

[12] Tobak M. et al. "*Aerodynamics of Body of Revolution in Coning Motion"* AIAA Journal VOL. 7 NO 1, pp 95, 99 , New York 1968