A SEMI-ANALYTICAL NUMERICAL METHOD OF DETERMINING STRESS INTENSITY FACTORS FOR MULTIPLE-SITE DAMAGE STRUCTURE

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Abstract

A semi-analytical numerical method of determining stress intensity factors for complex geometrical configurations with multiple sites damage is presented. The accuracy of the method is assessed using single cracked hole configuration for which alternative solutions are available. Finally stress intensity factors are obtained for cracks at a two-cracked-hole in finite-width strip model.

1 General Introduction

Multiple-Site Damage (MSD) is the term used to indicate the development of structural damage in various locations due to stress concentration, which specially means that cracks take place at the fastener holes of aircraft structural rivet line. The presence of the cracks may weaken the structure and lead to a reduction in its operational life. Therefore, it is necessary to predict the behavior of the cracks under service conditions so as to estimate the structural safety and reliability, which strongly depends upon the precise evaluation of the stress intensity factors (SIF) at the crack tips that govern the damage development.

With single finite element method, the computational effort is insurmountable to perform the SIF calculation when a little more complex MSD configuration is considered. The use of weighting function method is often restricted by the complexity of crack damage configuration. And also the compounding method [1] is difficult to conduct varieties of complicated SIFs computation in computer programming. In this paper, finite element method, compounding method, weighting function and alternating iteration technique are combined and developed into a new semi-analytical numerical method to apply to the computation of the SIFs in the MSD structure.

In section 4, one model of MSD configuration has been taken as case study, and the solved SIFs of a cracked hole strip model have been compared with the SIFs curves in the SIFs Handbook, meanwhile, good accuracy coincidence can be found. A more complex configuration is studied in section 5, where the finite-width strip with a row of cracks from holes with different lengths is subjected to two different load cases.

2 Method

The SIF of a crack in a MSD configuration containing co-linear cracked holes is influenced by the complex force and geometrical boundaries, e.g. holes, other cracks and sheet edges. When the weight function is being used, the two kinds of boundary conditions can be considered separately. Using the method presented, the distribution of stress on the crack surface is calculated through finite element method, and then is used in the single crack's weight function of complex configuration, again through alternating iteration, the SIFs of MSD configuration can be solved.

2.1 Weight function and finite element method

Under complex load circumstance the weight function method [2] provides powerful and simple means for calculating SIF. For any given stress distribution $\sigma(x)$ in the uncracked component the SIF results in, see Fig. 1,

$$K = f\sigma\sqrt{\pi a W} \tag{1}$$

where σ is a normalizing stress which can be chosen freely, f is the non-dimensional SIF and written in form

$$f = \int_0^a \frac{\sigma(x)}{\sigma}\frac{m(x,a)}{\sqrt{\pi a}}dx, \qquad a = \frac{A}{W} \qquad x = \frac{X}{W} \tag{2}$$

$$m(x,a) = \frac{1}{\sqrt{2\pi a}}\sum_{i=1}^{4}\beta_i(a)(1-\frac{x}{a})^{i-\frac{3}{2}} \tag{3}$$

here, eq(3) is the expression of the weight function for edge crack. For a given configuration with a crack, if the reference loading $\sigma_r(x)$ and the corresponding reference SIF $f_r(a)$ are known, the reference displacement $u_r(x,a)$ can be determined based on some natural conditions, such as

- crack tip relation
- self-consistency
- zero curvature at the crack mouth.

Using the relationship between $u_r(x,a)$ and $m(x,a)$, the $\beta_i(a)$ -function can be obtained.

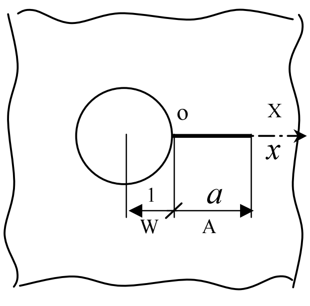


Fig.1 Single crack at the edge of a hole

For complicated configuration with complex force boundaries, the analytical solution of the $\sigma(x)$ is always difficult to get. So the finite element method is used here to calculate the stress $\sigma(x)$. By use of 8-node isoparametric elements, not only the stress of the Gauss integration point on the crack line can be obtained directly from interpolation, but also a high accuracy can be kept. Combining eq (2)(3) with the finite element calculating, the expression of f is written in a discrete form

$$f = \frac{1}{\sqrt{2\pi a}\sigma} \times \sum_{k=1}^{N}\frac{l_k}{2}\{\sum_{j}^{3}\sigma_{k,j}\alpha_j[\sum_{i=1}^{4}\beta_i(a)(1-\frac{x_{k,j}}{a})^{i-\frac{3}{2}}]\} \tag{4}$$

where N - the total number of the elements on the crack line, l_k - the side length of the k th element on the crack line, $\sigma_{k,j}$ - the stress of the j th Gauss integration point on the k th element, α_j - the weight coefficient corresponding to Gauss points, and $x_{k,j}$ - the coordinate of the j th Gauss integration point on the k th element.

The SIFs of the single crack configurations subjected to complex loading can be solved by weight function method mentioned above, then the results will be used in the alternating iteration process to determine the SIFs of MSD configuration.

2.2 Alternating iteration process

Alternating technique was originally developed for co-linear cracks in infinite sheet. Together with the weight function and finite element calculating, the method is used here for cracks at the edge of holes in finite sheet. The theory and process of the alternating iteration technique is described as shown in Fig.2. If we want to get the result of Fig.2(a) from Fig.2(b), the residual stress N_{A1} at the location of surface A in otherwise imaginary uncracked body must be eliminated by loading the reverse stress of N_{A1} on surface A, Fig.2(c). Then the reverse stress of N_{B1}, which arises at the surface B of the uncracked body from the loading of the $-N_{A1}$, should be loaded on crack surface B to erase the residual stress, Fig.2(d). Again the residual stress N_{A2} will arise on the surface A ···. Reiterate the steps described above until the residual stress vanishes. But in a practical calculation, the calculation can stop as the absolute value of the residual stress goes in an admissible stress. The alternating process finally yields

$$K_A^{(a)} = K_A^{(c)} + \cdots; \qquad K_B^{(a)} = K_B^{(b)} + K_B^{(d)} + \cdots \tag{5}$$

By alternating technique the SIFs of MSD configuration can be converted into the summation of single crack's SIF, and the weight function was used to calculate the SIF of each single crack.

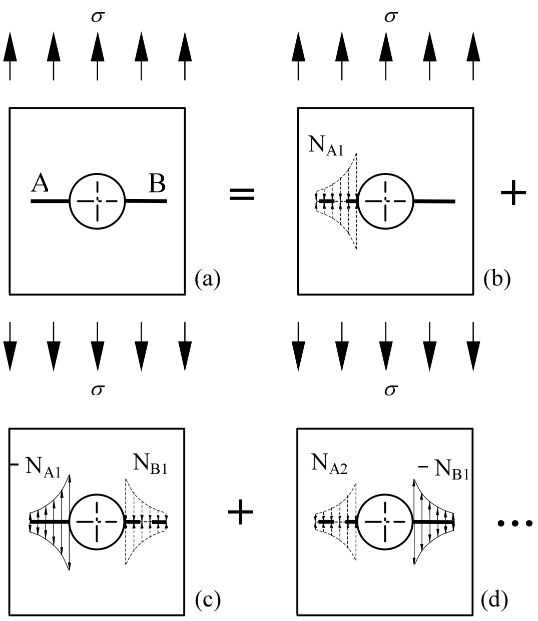


Fig.2 Alternating technique for two cracks at a hole edge in a finite sheet

2 Determination of reference SIF

The foregoing derivation of weight functions is based on the knowledge of a reference load case and the corresponding reference SIF $f_r(a)$.

Remote uniform tension is often taken as the reference load case for which accurate stress intensity factors are easy to get with the compounding method. For the MSD configurations considered here, the SIF of single crack is influenced by the three factors of the cracked hole, the holes nearby and the boundaries of the sheet. So the non-dimensional reference SIF is

$$f_r = f_0 \times f_h \times f_{oh} \times f_w \tag{6}$$

f_0 -the non-dimensional SIF of center crack in a infinite sheet, equal to 1;

f_h -the modified factor of the cracked hole;

f_{oh} -the modified factor of the hole nearby;

f_w -the modified factor of the boundaries of the sheet.

In eq.(6), when the holes aligns with the same line, each nearby hole has its modified factor f_{oh}. If the space between two holes is greater than or equal to triple times of the hole-radius, it is enough only to consider the effects of two holes neighbored to each cracked hole, and the error is less than 3%.

- The calculation of f_h

  The effect of the cracked hole on the SIF of its edge crack, see Fig.1, takes the form of [3]

$$f_h = [1 + \frac{1}{2x^2 + 1.93x + 0.539} + \frac{1}{2(1+x)}] \times [\sqrt{\frac{2+x}{2+2x}} \times (1 + \frac{0.2x}{(1+x)^3})] \tag{7}$$

  when the infinite sheet subject to the remote uniform tension perpendicular to crack line is considered.

- The calculation of f_{oh}

  As the crack has touched the hole, the original crack length should be modified into [4]

$$a' = a f_h^2 \tag{8}$$

The modified factors of the hole nearest to the crack [5], shown as Fig.3, are

$$f_{ohA} = \sum_{n=0}^{25} c_n(\mu)\lambda^n$$
$$f_{ohB} = \sum_{n=0}^{25} c_n(\mu)(-\lambda)^n \qquad (9)$$
$$\mu = R/b \qquad \lambda = a'/b$$

The coefficients c_n in the last expression can be obtained from the fitting of the curves that are presented in SIFs handbooks.

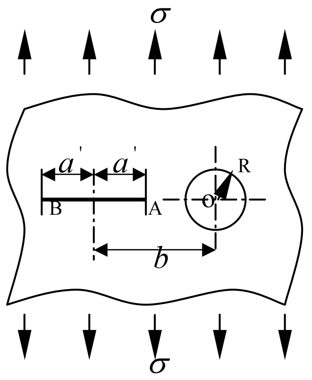


Fig.3 A crack approaching a hole

- The calculation of f_w

The effects of the edges of the sheet, Fig 4, can be written as [6]

$$f_{wA} = 1 + \sum_{n=2}^{19}\sum_{m=0}^{10} d_{mn}\varepsilon^m\lambda^n$$
$$f_{wB} = 1 + \sum_{n=2}^{19}\sum_{m=0}^{10} d_{mn}\varepsilon^m(-\lambda)^n \qquad (10)$$
$$\varepsilon = e/b \qquad \lambda = a'/b_1$$

where the coefficients d_{mn} come from parametric fittings and the data resource can be acquired from published documents.

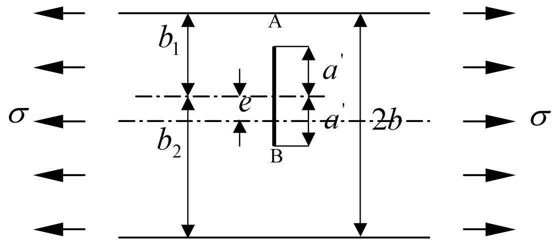
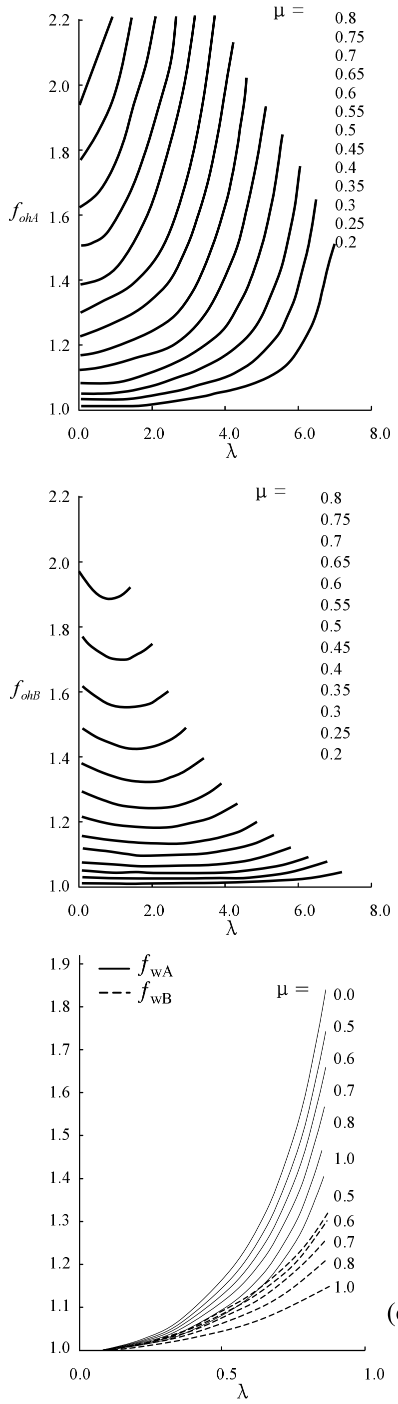


Fig.4 A crack in a finite width strip

Through the polynomial parametric fittings of the SIF modified factors, the computational process of compounding method is formulized and facilitates the implementation of computer programming. Compared with the results listed in the reference [7], the fitting errors of c_n are within less than 1%, and within 0.1% for d_n , shown as Fig.5.



(a)The coefficient f_{ohA}

(b)The coefficient f_{ohB}

(c)The coefficients f_{wA} and f_{wB}

Fig.5 The modified curves from data fitting

4 Tested MSD configuration

In order to test the method, a typical MSD configuration is supposed as case study. Consider two diametrically opposed radial cracks emanating from the edge of a circular hole in a rectangular plate of finite width with the aspect ratio H/w≥2. The crack lengths are a_1 and a_2 respectively, and remote uniform tension is applied to the both infinite ends. Fig.6 shows the f-values for varied crack lengths calculated by fitting in this paper. The tendencies and values of the curves coincide very well with those presented in the SIF handbook [7].

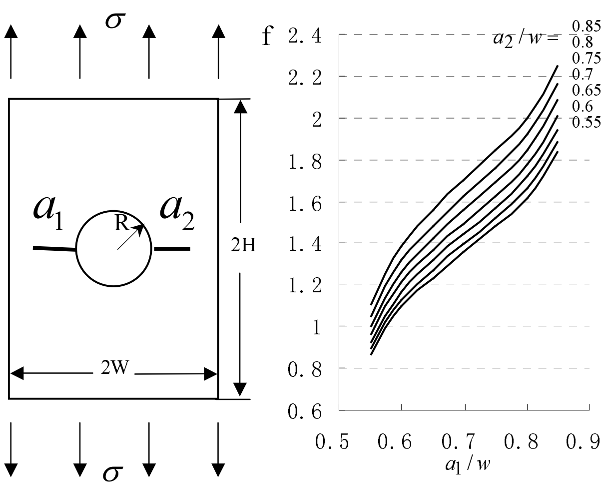


Fig.6 SIFs of the left model (H≥2W, R/W=1/2), $f = K_I / \sigma\sqrt{\pi(a_1 + R)}$

5 Double cracked holes in finite width strip

A finite width sheet , Fig. 7, has two holes with one or two cracks in it. Denote the radius of the hole by r, the crack length by a, the space between the holes by s, and the space between the hole and the sheet edge by e.

5.1 Uniform tension

The sheet is subjected to uniform tension stress σ at the both remote ends. Keep the sheet width, and decrease s from 12r to 4r along with e increase from 3r to 8r. The calculations to non-dimensional SIFs for one or two crack(s) with different lengths have been performed, and the f-a curves for different s and e are presented

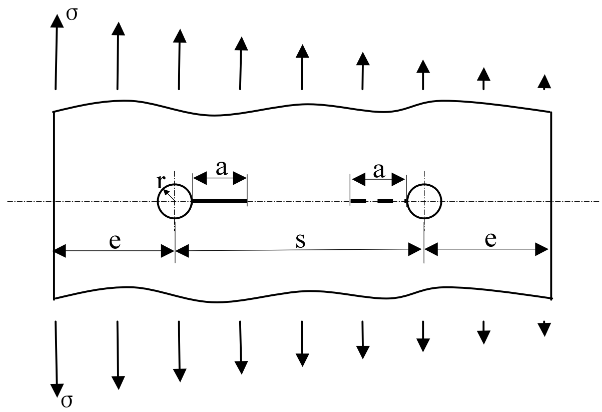


Fig.7 A MSD configuration

in Fig.8 ($f = K_I / \sigma\sqrt{\pi a}$).

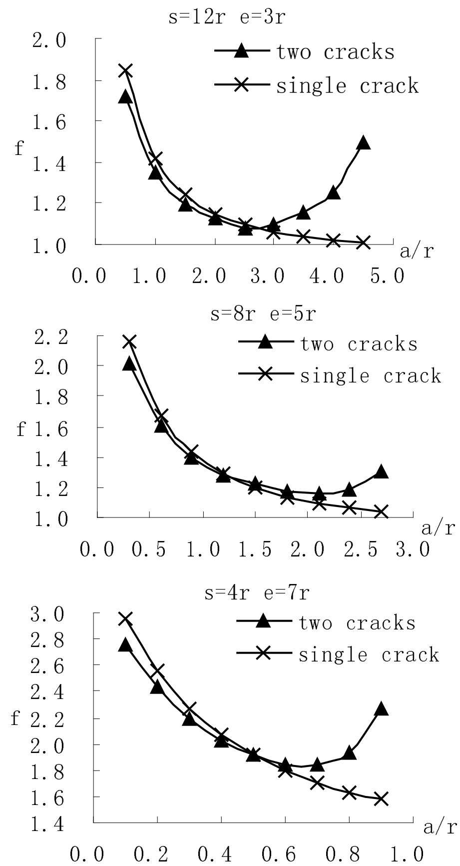


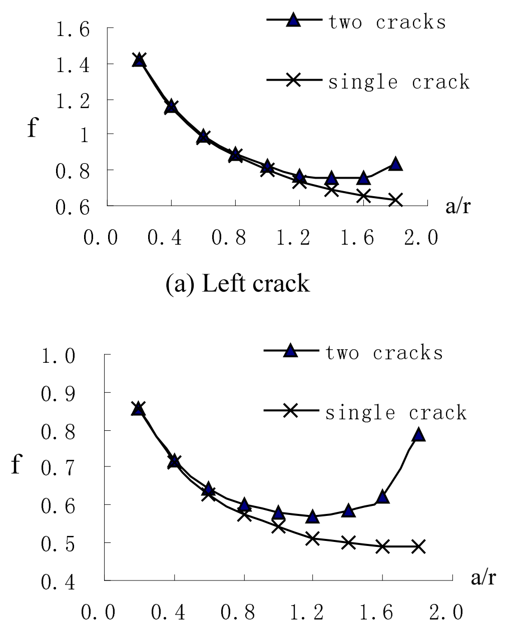
Fig.8 Stress intensity factors for one or two crack(s) in the sheet with double holes

For the constant s, the single crack's SIF is slightly more than those of double cracks at short crack phase. As the increase of crack length, the interference between the cracks becomes stronger, and the SIFs for two cracks are getting greater than those of the single crack strikingly. For the different s, the differences of

f-values for same crack length are small, that is because the effect of the neighbor hole increases with the decrease of s, but the effect of sheet edge decreases with the increase of e, and the two effects weaken each other.

5.2 Triangle stress

In this section, the sheet is subjected to triangle tension stress at the both remote ends, the load case is showed in fig.7. And the f-a curves for s = 6r and e=6r are given in fig.9($f = K_I / \sigma\sqrt{\pi a}$).



(a) Left crack

(b) Right crack

Fig.9 Stress intensity factors for one or two crack(s) in the sheet with double holes

As the triangle tension stress is employed, the SIFs of the left cracks are obviously higher than that of the right cracks for same crack length. And the interference between the cracks becomes stronger with the increase of crack length, so for long crack, the SIFs for two cracks are strikingly greater than those of the single crack for both left and right crack. The results of this new method close very well with the theoretical analysis.

6 Discussion and conclusions

Finite width sheet with co-linear cracked holes is a typical model of often-happened MSD configuration for aging aircraft structures. The new semi-analytical numerical method developed in this paper integrates finite element method, compounding method, weighting function and alternating iteration technique, so that the calculation of the SIFs of MSD configurations with complicated force and geometrical boundaries can be conveniently implemented along with relatively high numerical precision. Compared with the calculation to the SIF by single finite element method, the efficiency and accuracy are both improved by the new method.

The correctness and reliability of the method has been testified by some MSD configurations. And through the tested MSD configuration, it is shown that the veracities of the reference stress and SIF are the major factors that will affect the accuracy of the new method. The new method can be used directly in the residual strength analysis and the damage tolerance evaluation for the thin-walled structures with MSD.

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