

# HYBRID TWO-LEVEL GENETIC OPTIMIZATION ALGORITHM WITH DIFFERENT FIDELITY MODELS FOR AERODYNAMIC DESIGN PROBLEMS

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## Abstract

*A new hybrid genetic algorithm for solving optimization problems in multidimensional space has been developed. The idea behind the method is to simultaneously use the numerical models of different fidelity. The bulk of computations is based on low-fidelity and low time-consuming model, while the small amount of computations are based on a high-fidelity model for refining the behavior of the objective function. As a result, the computational accuracy of the developed algorithm corresponds to higher accuracy model, and total computational time is reduced by several times as compared to a conventional genetic algorithm. Basic features of the developed algorithm are described and several examples of applying this algorithm to transonic aircraft aerodynamic configuration design are presented in the paper.*

## 1 Introduction

The models of different complexity can be employed in numerical simulation of objects or phenomena under study. These may be quite dissimilar approaches or different levels of approximations in the framework of the single approach. An example of dissimilar approaches in aerodynamics is using the panel methods and Navier-Stokes methods for low speed flow analysis. The different approximation levels can be exemplified by use of coarse and fine grids in the framework of the same method. As a rule, going to a more complicated model entails a significant increase in amount of computations. The optimization methods,

which are presently the most powerful design tool, often require hundred or even thousands of direct system computations. When performing such computations with the use of a high-fidelity model, the computation time becomes unacceptably large. On the other hand, the use of low-fidelity models does not allow one a true optimum to be determined. Therefore the creation of a hybrid algorithm capable of operating alternatively with the models of different complexity is a logical solution. With the computational process well organized, it is hoped that the computer time may be considerably reduced as compared to the high-level optimization while retaining accuracy of high-fidelity model.

Taylor series approximation in the vicinity of a base point is the most simple and widely used method of hybrid representation of objective functions and constraints [1]. Unfortunately, the accuracy of such an approximation drastically decreases with the distance from the base point. A well-accepted approach is to construct a response surface (RS) approximation, in particular quadratic RS describing the behavior of functions under study [2-4]. However, the amount of computations required for constructing an approximation surface increases quickly (as  $\frac{1}{2}(N+1)(N+2)$  in case of quadratic approximation) as problem dimensionality increases. Alternative ideas about constructing hybrid algorithms can be met in the literature. For example, proposed in [5,14] is the local-global approximation implying the matching of the values of both the functions and their derivatives, obtained in various models. The idea of accumulated approximation that refines a rough model with increasing available

information is employed in [6,7]. The authors of ref. 3 propose to approximate not the objective function itself but the difference between the values corresponding to the accurate and rough models.

In the present work the hybrid optimization procedure is developed as applied to the genetic algorithm (GA). The genetic algorithms recently acquired general popularity [8-11] in view of their simplicity, robustness, convenience in parallelizing the solution process and capability of finding global extremum. The main advantage of the genetic approach is that the gradients are not required in the search process. Hence, the method is not susceptible to the pitfalls of gradient-like techniques. The payment for these advantages of the GA is large computer time, which hinders the use of complex numerical models, thus making important all sorts of attempts to accelerate the computation process. The details of the algorithm and computational test examples are presented below.

## 2 Problem formulation and description of the optimization algorithm

Let there be an N-dimensional space of the design variables  $x_i$ ,  $i=1,N$ , where an extremum of the objective function  $F(\mathbf{X})$  is being sought. We make no assumptions regarding the behavior of the objective function in particular it may be even discontinuous. It is assumed that there are two direct methods for computing the values of the objective function in the N-dimensional space: the accurate  $F(\mathbf{X})$  and approximate  $G(\mathbf{X})$  ones. Denote the difference between the objective function values  $F(\mathbf{X})$  and  $G(\mathbf{X})$  by  $\Delta(\mathbf{X})$ . The difference  $\Delta(\mathbf{X})$  is assumed to be a continuous and sufficiently smooth function. Let us take a piecewise linear approximation for  $\Delta(\mathbf{X})$  representation:

$$\tilde{\Delta}(\mathbf{X}) = \Delta(\mathbf{X}^k) + \mathbf{C}^T \cdot (\mathbf{X} - \mathbf{X}^k) \quad (1)$$

where  $\mathbf{X}^k$  is the nearest point ("node") where both the accurate  $F(\mathbf{X}^k)$  and approximate  $G(\mathbf{X}^k)$

values of the objective function are known;  $\mathbf{C}$  is the sought vector of linear form coefficients. To determine the coefficients  $c_i$ , we use the procedure of interpolation over the neighboring "nodes".

$$\tilde{\Delta}(\mathbf{X}^j) = \Delta(\mathbf{X}^k) + \mathbf{C}^T \cdot (\mathbf{X}^j - \mathbf{X}^k), \quad j=1,2,\dots,N \quad (2)$$

For this system of linear algebraic equations to be well-conditioned, it is necessary that the system of vectors  $\mathbf{X}^j - \mathbf{X}^k$  be linearly independent. Used in the present work is a genetic algorithm with a binary representation of the variables, that is, the space of design variables is discrete. With the space partition being discrete, the neighboring nodes often form a dependent vector system. Hence, to construct an independent system we use the well-known Gramm-Schmidt orthogonalization process. If in the process of the orthogonalization a certain vector  $\mathbf{X}^j - \mathbf{X}^k$  turns out to be a linear combination of the preceding ones (that is, the orthogonal projection approaches zero), then this node is excluded from consideration and the next one is taken. Notice, that the Gramm-Schmidt orthogonalization process is equivalent to the reduction of a matrix to a lower triangular form and the linear form coefficients in a new orthogonal basis are determined immediately. For the verified values  $N \leq 100$ , the construction of the linear form presents no problems and takes less than 0.1 sec of the Pentium III 1000 CPU. For the problems of greater dimensionality it is likely best to use a genetic algorithm with the real representation of the variables. In the case of strongly oscillating function  $\Delta(\mathbf{X})$  it seems to be reasonable to determine the linear coefficients by the least square method.

Similar to conventional genetic algorithm [8], basic operators of the hybrid GA are selection, crossover and mutation. The general optimization algorithm is as follows:

- selection of the design variables  $x_i$ ,  $i=1,\dots,N$  and their variation ranges;
- random formation of  $N_{INIT} \sim 4 \cdot N$  initial vectors ("nodes") for which the value of  $F$ ,  $G$  and  $\Delta$  are calculated;

- random formation of the initial population of  $NPSIZE=2 \div 8 \cdot N$  vectors (individuals).

Next, the following actions are executed for each of the generations:

Selection step:

1. Calculation of the approximate objective function  $G$  for every individual.
2. Calculation of a linear interpolant  $\tilde{\Delta}$  for every individual.
3. Assignment of a fitness proportional to the value  $(G+\tilde{\Delta})^s$  (where  $s=1 \div 4$ ) to every individual. Selection of  $NADDB \sim 0.1 \cdot NPSIZE$  additional nodes among best candidates for which the accurate values of the function  $F$  are calculated.

Crossover step:

1. The probability of access to crossover proportional to the fitness value is assigned for each individual. Such a selection is usually referred to as a “roulette” method.
2. Crossover of the randomly selected pairs of individuals. Herein a standard one-point binary crossover is adopted. For this purpose first the normalized  $X$  values are transformed into a binary code and “chromosome” ribbons are formed for every individual. The ribbon length is proportional to the number of varying parameters and  $L$  being the number of bits adopted for each variable. The  $L$  is commonly equal to 4-8, that is, the range of parameters variation is divided into  $2^L - 1 = 15 \div 255$  segments. Then, the breakpoint is determined in a random manner and the ribbon portions from the “parents” are pasted together in a crossover way.
3. A portion of individuals ( $NELIT \sim 0.1 \cdot NPSIZE$ ) corresponding to the nodes which are the best at this point, find their way to the next generation without changes (elite strategy).

Mutation step:

1. The generation of “children” obtained as a result of the crossover is subjected to an

additional mutation procedure with a probability of  $P_m \sim 0.005$ . For this purpose “1” is changed to “0” or vice versa in some of randomly chosen cells. The mutation step is necessary to prevent the population from degeneration, that is, from the optimization process sticking in a local optimum. At initial optimization stages the probability of mutation is increased with the aim of checking the entire space for the presence of the regions with high values of the objective function. To curtail the search region the  $P_m$  is decreased as the global optimum is approached.

Overall, the hybrid GA differs little from the conventional GA. In the next section their performance and required computer execution time will be compared on the basis of some test problems.

### 3 Computational results

To assess the effectiveness of the hybrid optimization algorithm, we first consider a demonstration problem. The exact objective function is expressed in the following analytical form

$$F = I + \frac{\sqrt{\prod_{k=1}^N |X_k| \cdot |\text{Sin}(X_k)|}}{(3.5\pi)^N}$$

The design variables change within limits  $0 < x_i < 4\pi$ . For binary representation, the 8-bit subdivision is chosen, i.e., the entire range is divided into 255 intervals. Let us assume that the difference between the exact  $F$  and the approximate  $G$  solutions is the following long-wave sinusoidal function:

$$\Delta = \frac{\sum_{k=1}^N \text{Sin}\left(\frac{1}{2}(X_k - \frac{1}{2}\pi)\right)}{5N}$$

The shapes of the function  $F$  and  $\Delta$  in the case of two variables are shown in fig.1. Let us set the population dimension to be  $NPSIZE=4 \cdot N$ , and

the number of nodes being added at each generation equal to  $NADDB=0.1 \cdot NPSIZE$ . The probability of mutation is taken to be constant:  $Pm=0.003$ .

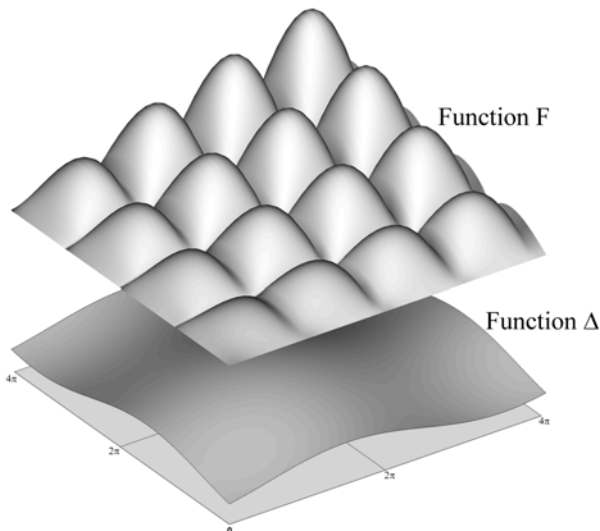


Fig. 1

The test computations were performed for the cases  $N=10$  and  $N=50$ . The conventional and hybrid GAs were compared with respect to convergence. Fig.2 shows the convergence histories for 100 different trials thus illustrating the statistical nature of the evolution algorithm. The horizontal line corresponds to the absolute maximum of the function  $F$ , obtained analytically. Considered further are values

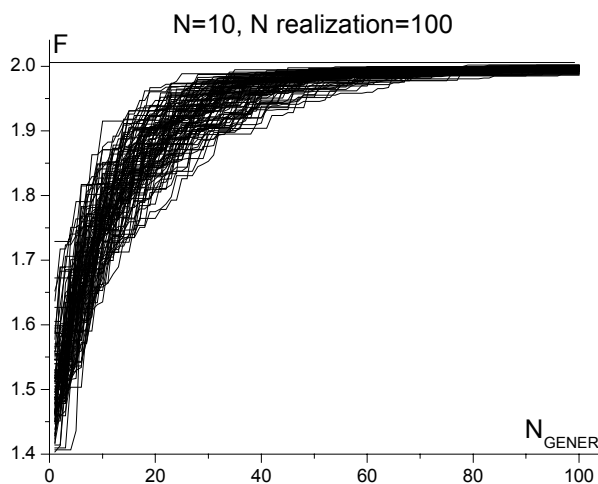


Fig. 2 Single evolution histories

already averaged. At  $N=10$ , the result of 100 realizations were averaged; at  $N=50$ , the

averaging was performed over 10 realizations. Figure 3 demonstrates convergence for the test cases. It can be seen that the hybrid GA is inferior to the conventional one with the equal sizes of population  $NPSIZE$ , but has the same effectiveness for doubled population. In last case, the number of computations using the exact model is less by a factor of five for the hybrid method, while the net gain in the computer times depends on the relative costs of the high- and low-fidelity calculations.

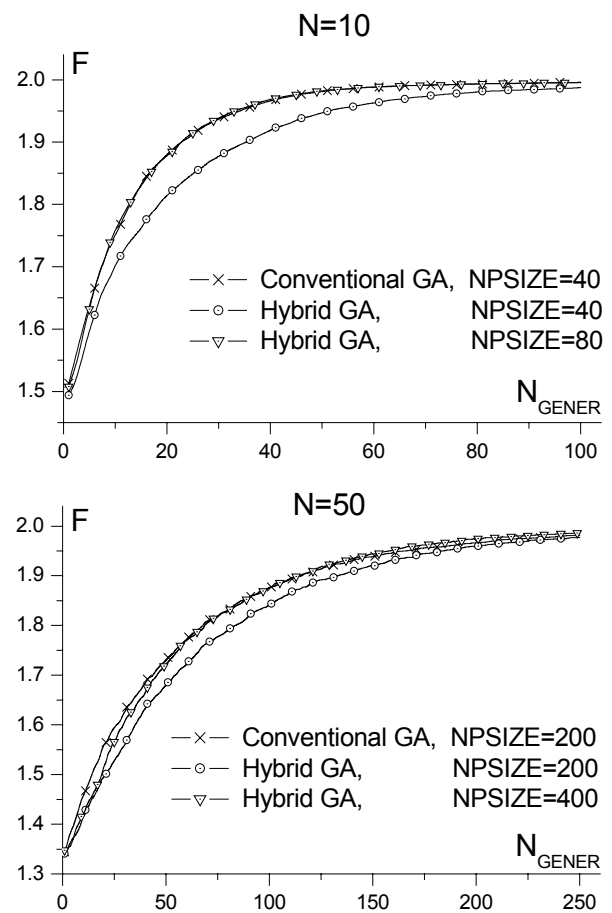


Fig. 3 Comparison of conventional and hybrid GAs

The second example is a practical aerodynamic design of the wing of an advanced short/medium-haul passenger aircraft (fig.4). As a high-fidelity model, the results obtained with the BLWF full-potential code [12] on a fine (third) mesh with regard to viscosity were used. The computer time for a single run for the wing-body configuration is about 30 sec on a PC Pentium III-1000. As a low-fidelity model

computations were used with the same code on the second mesh (run time 7 sec). The original wing with a sweep angle of  $\chi_{1/4}=25^{\circ}$  was developed earlier using inverse and optimization methods [13] for lesser  $M_{cruise}=0.78$ . Comparison of pressure distributions over the original wing obtained on the different meshes is shown in fig.5. The solid and dashed lines correspond to viscous and inviscid calculations on the second mesh respectively, while the crosses present computational results for the third mesh with viscosity taken into account. It can be seen that all results are sufficiently close, i.e. the basic

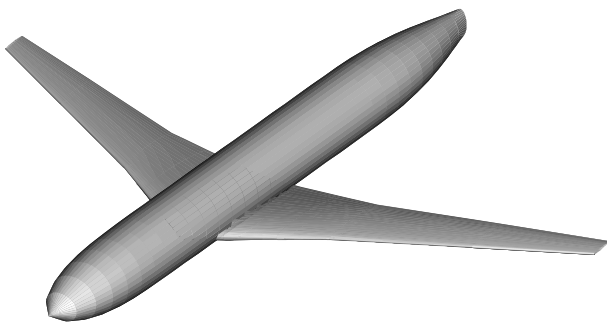


Fig. 4

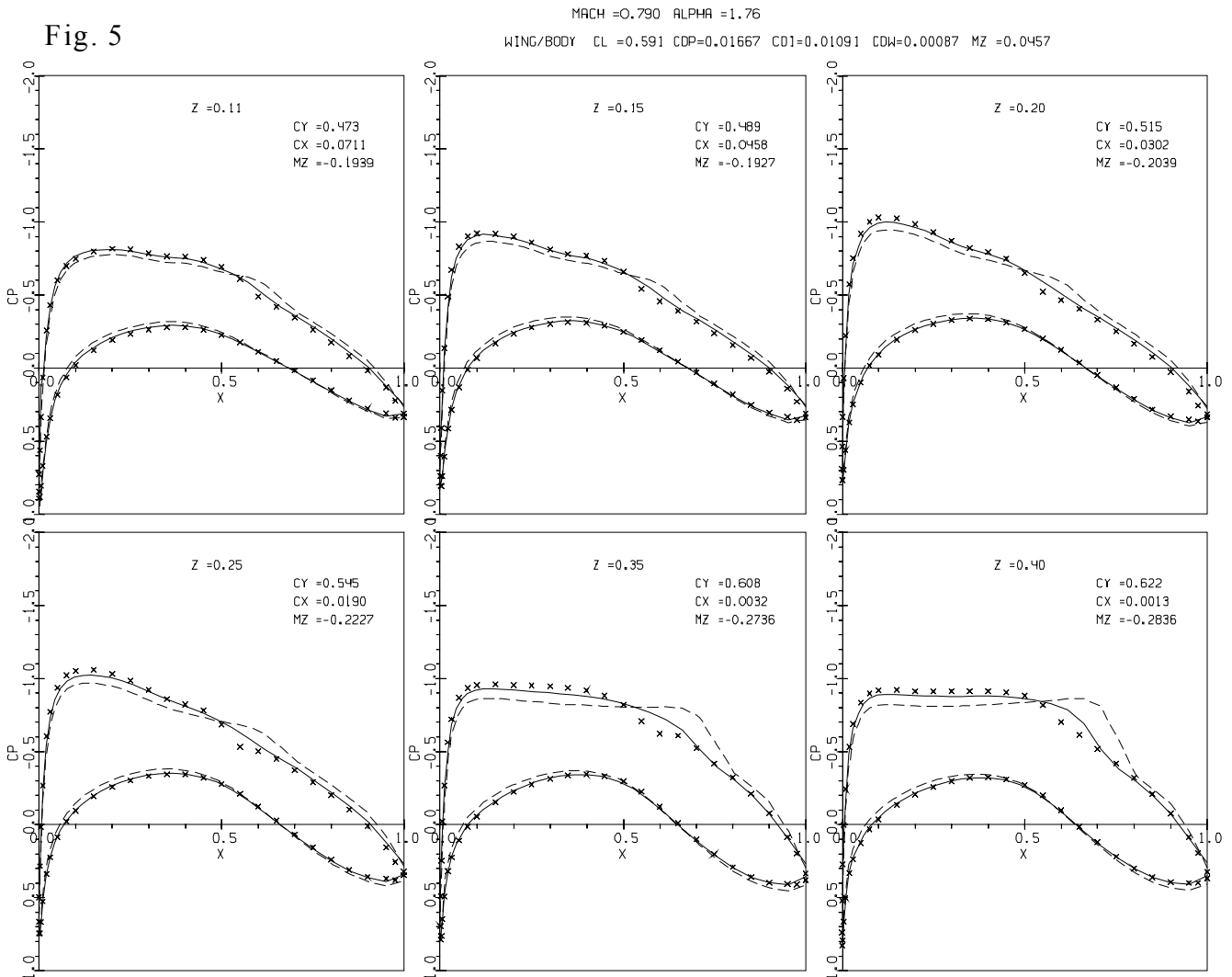
assumption of the developed hybrid method holds. Note, however, that the computations on the third mesh are much more frequently interrupted as a result of the collapse of the iterative process of viscous-inviscid interaction. If the computation on the second mesh is performed successfully whereas the run on the

third mesh is interrupted, the difference  $\Delta$  between the two values of the objective function is limited in magnitude by a specified bound  $\Delta_{max}$  (this value can be estimated at the stage of selecting the initial set of nodes), while the value of the exact function  $F$  is taken to be equal to  $G - \Delta_{max}$ . The introduction of the limiter  $\Delta_{max}$  is also useful to exclude large errors of linear extrapolation, caused by the presence of numerical “noise”. The introduction of a limit on the minimum permitted distance between the neighboring nodes is made with the same purpose.

The problem to be solved was to redesign a wing for the regime  $M_{cruise}=0.80$ . In the course of optimization, 41 variables were used, of which 34 variables related to the geometrical characteristics of airfoil sections at five basic spanwise stations, 6 variables were responsible for the variation of the wing planform. Besides, the angle of attack was a design variable. As an objective function the following complex criterion was selected (see [13]):

$$F = \frac{L/D}{1 + p \frac{\Delta G}{G_0}}$$

where  $G_0$  is the takeoff weight,  $\Delta G$  is the wing weight increment,  $p$  is a free parameter which enables assigning higher priority to aerodynamic characteristics or weight (is taken to be  $p=2.2$ ). By using penalty functions limitations were imposed on the internal volume of the wing, curvature of the wing spars, relative thickness of the outer wing.



The results of some single realizations of the optimization procedure using the conventional and hybrid GAs are presented in fig. 6. It can be seen that the conclusions on the effectiveness of various versions of numerical schemes, made earlier on the basis of demonstration examples, are, as a whole, confirmed by the practical problem of aerodynamic design. In particular, computer time for the case of the hybrid GA with double population is less than that of the conventional GA approximately by a factor of 2.5. The comparison of pressure distributions at cruise regime  $M=0.8$ ,  $C_L=0.575$  for the original ( $F=19.4$ ) and obtained ( $F=21.05$ ) wings is presented in fig.7.

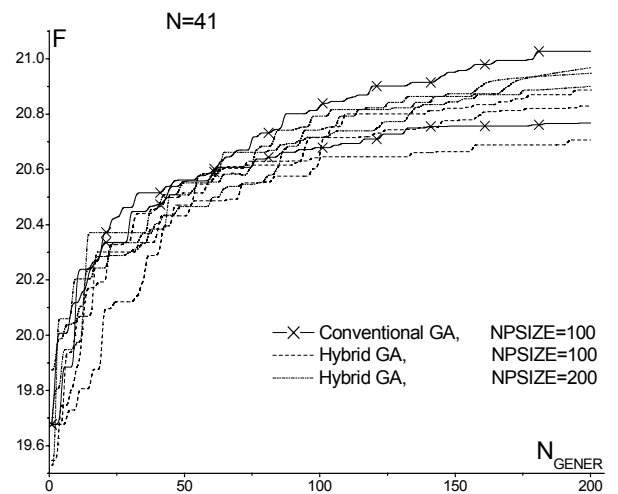
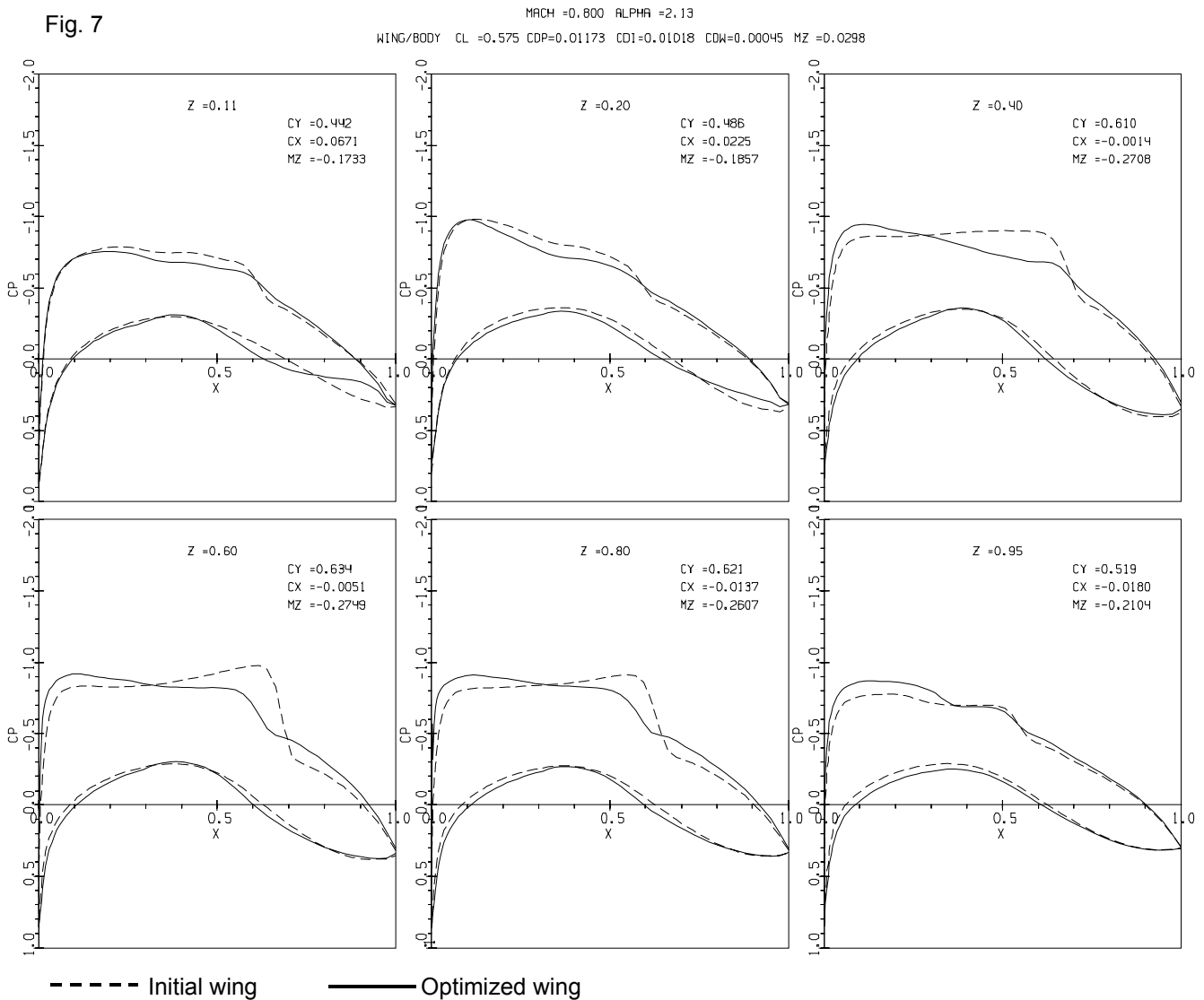


Fig. 6 Evolution histories for practical design problem

Fig. 7



The gain in the objective function for the optimized wing is mainly caused by an increase in the lift-to-drag ratio due to the reduction in the calculated value of wave drag from  $C_{DW}=0.00115$  to  $C_{DW}=0.00045$ . The wing sweep increased slightly from  $25^{\circ}$  to  $25.8^{\circ}$ , the relative wing thickness decreased from  $t/c=0.124$  to  $t/c=0.121$ , but the wing area decreased by 1.8% and, as a result, the wing's weight remained practically unchanged.

#### 4 Conclusions

The presented examples are indicative of the applicability of the developed hybrid method to solving practical optimization problems when numerical methods of various

levels of complexity are available, and particularly to aerodynamic design problems. The optimization program is structurally completely separated from the direct computation programs and can be used for other applications. Further increase in the effectiveness of the hybrid GA is in sight due to selection of other low-level models, for example, more fast inviscid computations on the second mesh (see fig.5 and [14]), or even such a universal approximator as the neural network [15].

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## References

- [1] P.E. Gill, W. Murray, M.H. Wright. Practical optimization. *Academic Press*, London, 1981.
- [2] N.P. Buzoverya. Airfoil optimization by the method of quadratic approximation (In Russian). *TsAGI Transactions*, N 2497, 1993.
- [3] D.L. Knill, A.A. Giunta, C.A. Baker, B. Grossman, W.H. Mason, R.T. Haftka, L.T. Watson. Response surface models combining linear and Euler aerodynamics for supersonic transport design. *J. Aircraft* Vol. 36, No 1, January-February 1999.
- [4] H.S. Chung, J.J. Alonso. Comparison of approximation models with merit functions for design optimization. *AIAA-2000-4754 in 8<sup>th</sup> AIAA/USAF/ NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*. Long Beach, California, 6-8 September, 2000.
- [5] Haftka R.T. Combining global and local approximations. *AIAA Journal*, vol.29, №9, 1991 pp.1523-1525.
- [6] Rasmussen J. Accumulated approximation – a new method for structural optimization by iterative improvements. *Proceeding of the Third Air Force/NASA Symposium on Recent Advances in Multidisciplinary Analysis & Optimization*. San Francisco CA Sept. 24-26. 1990 pp. 253-258.
- [7] I. Egorov. Approximation methods usage to reduce time expenditures for multidisciplinary optimization problems solution. *AIAA-2000-4896 in 8<sup>th</sup> AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*. Long Beach, California, 6-8 September, 2000.
- [8] Michalewicz Z., Genetic algorithms + data structures = evolution programs, Springer-Verlag, Berlin, 1994.
- [9] P. Hajela Nongradient methods in multidisciplinary design optimization – status and potential. *J. Aircraft* Vol. 36, No 1, January-February 1999.
- [10] A.Vicini, D.Quagliarella. Inverse and direct airfoil design using a multiobjective genetic algorithm. *AIAA Journal*, vol.35, №9, 1997.
- [11] A.L. Bolsunovsky, N.P. Buzoverya, K.S. Nickolaeva Optimization procedure for design of high-lift airfoils with specified stall behavior. *Journal of Advanced Engineering Design*, Vol. 40, №1, 2000.
- [12] Karas O.V., Kovalev V.E. Computation of transonic flows around a wing-plus-fuselage configuration taking viscous effects and a thin separated region into account. *La Recherche Aeronautique*, 1994, №1, pp. 23-38.
- [13] A.L. Bolsunovsky, N.P. Buzoverya, Karas O.V., Kovalev V.E. Advances in aerodynamic design methods for subsonic aircraft cruise configuration. (In Russian). *TsAGI Transactions* (to be published in 2002).
- [14] N.M. Alexandrov, E.J. Nielsen, R.M. Lewis, W.K. Anderson. First-order model management with variable-fidelity physics applied to multi-element airfoil optimization. *AIAA-2000-4886 in 8<sup>th</sup> AIAA/USAF/ NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*. Long Beach, California, 6-8 September, 2000.
- [15] Sviridenko Yu.N., Doropheev E.A. Using of neural network technologies in the field of aerodynamic design (In Russian). *TsAGI Transactions* (to be published in 2002).