A FAST AND PRECISION INITIAL ALIGNMENT METHOD FOR SINS ON STATIONARY BASE

Xinlong Wang, Gongxun Shen
Beijing University of Aeronautics and Astronautics, Beijing, P.R. China
E-mail: xlwon@sina.com

Keywords: *SINS, initial alignment, Kalman filter, Observability*

Abstract

In this paper, a strapdown inertial navigation system (SINS) error model is introduced, and the observability of the SINS error model is analyzed. Then, on the basis of this SINS error model and the analysis of computer simulation results, a fast and precision initial alignment method of the multiposition is proposed for SINS on stationary base. Consequently, the time of initial alignment is reduced, and the precision of the initial alignment is improved greatly. The computer simulation results illustrate the efficiently.

1 Introduction

Today, the strapdown inertial navigation system (*SINS*) is being used more widely for the positioning and navigation of aeroplanes, ships, missiles, and vehicles, etc. The purpose of initial alignment of *SINS* is to get a coordinate transformation matrix from body frame to computation frame, and drive the misalignment to zero. Unfortunately, the goal can never be achieved in a practical system. From the control theorectical point of view, the basic difficulty associated with the self-alignment technique is that the system is not completely observable. The requirements of *SINS* initial alignment are high accuracy and speed. However, since the observability of *SINS* is weak, and the fast initial alignment contradicts the accuracy of alignment. It can be observed that the *SINS* system matrix may be manipulated by changing sensor position or equivalently by rotating vehicle body, i.e., the multiposition alignment technique [2]. Thus, by choosing different sensor position, the observability of SINS system can be improving. It is demonstrated that the alignment errors in *SINS* can be drastically reduced by employing the multiposition technique, but the speed of the multiposition alignment technique is rather slow. One question arises, that is, how can the best accuracy using the multiposition alignment be obtained in the shortest time ?

We introduce the *SINS* error model for the stationary alignment, and describing a simplified observability rank test of piece-wise constant time varying system and show that optimal two-position alignment not only satisfies complete observability condition but also minimizes alignment errors. Then, on the basis of the analysis of computer simulation results of the multiposition alignment technique [2], a fast estimation method of the azimuth error is integrated with the multiposition alignment method of *SINS* on stationary base. This method greatly accelerates the convergence rate of the Kalman filter that is used for estimation of error angles of the multiposition alignment. Consequently, the time of initial alignment is reduced, and the precision of the initial alignment is improved greatly. Simulation results are given to illustrate the efficiency of the method.

2 SINS Error Model

Here, a local level *NED* (North-East-Down) frame is used as the navigation frame and position and vertical velocity errors are ignored. The *SINS* stationary error model augmented with sensor errors can be written [2][3]

$$\begin{bmatrix} \dot{X}_1(t) \\ \cdots \\ \dot{X}_2(t) \end{bmatrix} = \begin{bmatrix} F & \vdots & T_i \\ \cdots & \vdots & \cdots \\ 0_{5\times5} & \vdots & 0_{5\times5} \end{bmatrix} \begin{bmatrix} X_1(t) \\ \cdots \\ X_2(t) \end{bmatrix} + \begin{bmatrix} W(t) \\ \cdots \\ 0_{5\times1} \end{bmatrix} \qquad (1)$$

The state vectors consist of $X_1 = [\delta V_N, \delta V_E, \Phi_N, \Phi_E, \Phi_D]^T$, $X_2 = [\nabla_N, \nabla_E, \varepsilon_N, \varepsilon_E, \varepsilon_D]^T$, and the process noise vector $W = [W_{\delta V_N}, W_{\delta V_E}, W_{\Phi_N}, W_{\Phi_E}, W_{\Phi_D}]^T$, where V and Φ are velocity and attitude errors, respectively, ∇ is the accelerometer error, ε is the gyro error, and the subscripts N, E and D denote the body axes. $0_{5\times5}$ and $0_{5\times1}$ are zero matrice of indicated dimension, and

$$T_i = \begin{bmatrix} C_{11} & C_{12} & \vdots & 0 & 0 & 0 \\ C_{21} & C_{22} & \vdots & 0 & 0 & 0 \\ \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & C_{11} & C_{12} & C_{13} \\ 0 & 0 & \vdots & C_{21} & C_{22} & C_{23} \\ 0 & 0 & \vdots & C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 2\Omega_D & \vdots & 0 & g & 0 \\ -2\Omega_D & 0 & \vdots & -g & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0 & \Omega_D & 0 \\ 0 & 0 & \vdots & -\Omega_D & 0 & \Omega_N \\ 0 & 0 & \vdots & 0 & -\Omega_N & 0 \end{bmatrix} = \begin{bmatrix} F_{11} & \vdots & F_{12} \\ \cdots & \vdots & \cdots \\ 0_{3\times2} & \vdots & F_{22} \end{bmatrix}$$

The measured signals during the stationary alignment are the horizontal velocity errors. Thus consider the observation model as follows

$$Y(t) = HX(t) + v(t) \qquad (2)$$

And,

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In order to estimate the state vectors of error model by Kalman filter, the observability analysis of error model must be performed.

$$Rank[H \quad HA_i \quad HA_i^2 \quad \cdots \quad HA_i^9] = 7 < 10$$

Due to the system is not completely observable, therefore, only 7 states are observable (the estimation value of state is convergent by Kalman filter); the other 3 states are not observable.

3 Observability Analysis

3.1 Observability Rank Test

Observability analysis of a dynamic system indicates the efficiency of a Kalman filter designed to estimate the states of the system. While the observability analysis of a time invariant system is rather simple, analysis of a time varying system is difficult. If the time varying system is replaced by piece-wise time invariant system, the observability analysis can be performed simply by the stripped observability matrix (SOM) suggested by Meskin and Itzhack [5]. Their work is summarized by the following theorem.

Theorem 1 if $Null(V_i) \subset Null(A_i), 1 \le i \le r$, then [1]:

$$Null(V) \subset Null(Vs) \qquad (3a)$$

$$Rank(V) \subset Rank(Vs) \qquad (3b)$$

For the notations in the theorem, consider the continuous piece-wise time invariant system

$$\dot{x}(t) = A_i x(t), \quad i = 1,2,\ldots,r$$
$$z(t) = Hx(t) \qquad (4)$$

Where, r is the number of the segments. The total observability matrix (TOM) for investigating the observability properties of (4) is constructed as follows.

$$V = \begin{bmatrix} V_1 \\ V_2 e^{A_1\Delta_1} \\ V_3 e^{A_2\Delta_2} e^{A_1\Delta_1} \\ \vdots \\ V_r e^{A_{r-1}\Delta_{r-1}} \cdots e^{A_1\Delta_1} \end{bmatrix} \qquad (5)$$

Where,

$$V_i = [H^T, (HA_i)^T, \ldots, (HA_i^{n-1})^T]^T \qquad (6)$$

Δ_i is the time interval of segment i, And the SOM is defined as follows.

$$Vs = \begin{bmatrix} V_1^T & V_2^T & V_3^T & \cdots & V_r^T \end{bmatrix}^T \qquad (7)$$

If the sufficient condition of Theorem 1 is satisfied, we can examine the observability using the null space of the SOM rather than that of the TOM. However the condition of Theorem 1 is quite restrictive. We now show that the SINS error model defined in (1) and (2) satisfies the sufficient condition.

Proposition 1 The *SINS* error model defined in (1) and (2) satisfies the sufficient condition of theorem 1.

Rewriting the *SINS* error model given in (1) and (2)

$$A_i = \begin{bmatrix} F_{11} & F_{12} & \overline{C}_b^n & 0_{2\times3} \\ 0_{3\times2} & F_{22} & 0_{3\times2} & C_b^n \\ 0_{5\times2} & 0_{5\times3} & 0_{5\times2} & 0_{5\times3} \end{bmatrix} \qquad (8)$$

$$H = [I_2 \quad 0_{2\times3} \quad 0_{2\times2} \quad 0_{2\times3}] \qquad (9)$$

Where, I and 0 are identity and zero matrices of indicated dimensions. Since the observability matrix V_i for (8) and (9) is simplified by elementary row operation as follows.

$$V_i^{'} = \begin{bmatrix} I_{2\times2} & 0_{2\times3} & 0_{2\times2} & 0_{2\times3} \\ 0_{2\times2} & F_{12} & \overline{C}_b^n & 0_{2\times3} \\ 0_{2\times2} & F_{12}F_{22} & 0_{2\times2} & F_{12}C_b^n \\ 0_{2\times2} & F_{12}F_{22}^2 & 0_{2\times2} & F_{12}F_{22}C_b^n \\ 0_{2\times2} & F_{12}F_{22}^3 & 0_{2\times2} & F_{12}F_{22}^2C_b^n \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \qquad (10)$$

Suppose that $x_0 \in Null(V_i^{'})$; that is

$$V_i^{'} x_0 = 0 \qquad (11)$$

When partitioning x_0 into four equal parts, as follows

$$x_0^T = [x_{01}^T \quad x_{02}^T \quad x_{03}^T \quad x_{04}^T]^T \qquad (12)$$

And when using (10) and (12), (11) yields

$$x_{01} = 0 \qquad (13a)$$

$$F_i x_{02} + \widetilde{C}_b^n x_{03} = 0 \qquad (13b)$$

$$F_i W_i x_{02} + F_i C_b^n x_{04} = 0 \qquad (13c)$$

$$F_i W_i^2 x_{02} + F_i W_i C_b^n x_{04} = 0 \qquad (13d)$$

$$\vdots$$

Using (8) and (12), yields

$$A_i x_0 = \begin{bmatrix} \widetilde{W}_i x_{01} + F_i x_{02} + \widetilde{C}_b^n x_{03} \\ W_i x_{02} + C_b^n x_{04} \end{bmatrix}$$

It is evident, from (13), yields

$$\widetilde{W}_i x_{01} + F_i x_{02} + \widetilde{C}_b^n x_{03} = 0$$

$$W_i x_{02} + C_b^n x_{04} = 0$$

Namely,

$$A_i x_0 = 0$$

In other words if $x_0 \in Null(V_i^{'})$ then also $x_0 \in Null(A_i)$ and that is true for any segment i. Consequently

$$Null(V_i) = Null(V_i^{'}) \subset Null(A_i) \quad 1 \le i \le r.$$

The above proposition tells that the complete observability of the 2-position alignment can be tested by the SOM instead of the TOM.

3.2 Degree of Observability

Rank test of observability matrix can determine whether the system is completely observable or not, but it cannot determine the degree of observability. Complete observableness may not supply enough information when numerically implementing a Kalman filter. That is why we have to consider the degree of observability. The error covariance matrix of the Kalman filter can be a good performance index for the degree of observability of a system [2]. For the piece-wise constant time varying system the error covariance matrix P_i is obtained by calculating the discrete Riccati equation,

$$P_i^{-1}(k)=(\Phi_i^T(k,k-1)P_i(k-1)\Phi_i(k,k-1)+Q)^{-1}+H^TR^{-1}H \quad k=1,2,\cdots,n$$
$$P_i(0)=P_{i-1}(n) \tag{14}$$

Where, the subscript i means the number of segment and $\Phi_i(j,k)=e^{(j-k)A_i\Delta_i}$ is the state transition matrix from time k to j on the i-segment.

The optimal position of the multiposition alignment is determined so that the error covariance matrix is minimal. Since the error covariance matrix in the Kalman filter does not have an analytic solution, we have to rely on a numerical solution using (14).

4 Optimal Multiposition

The rank of observability matrix obtained from SINS model (1) and (2) is 7, i.e., the system is not completely observable system. Additional measurements are needed in order to perform alignment. Since the measurements of attitude or sensor error are difficult to obtain, we intend to get the completely observable system by introducing some changes into the transformation matrix between the body frame and the navigation frame instead of adding more sensors. There are two ways [2] that we can bring the transformation matrix into change at rest. One is to change the attitude of the vehicle, and the other is to rotate the IMU.

Two-position alignment is to perform the alignment from an initial position of SINS using Kalman filter by changing its position once. Using the 2-position alignment, it is possible to estimate all the state variables because the system becomes completely observable. The following theorem illustrate the observability.

Theorem 2 Given the pair (A_i, H) of (1) and (2), where Ω_N is non-zero, let $V(\alpha_1,\alpha_2)$ be the TOM obtained by 2-position. Here α_1 and α_2 denote Euler angles for the first and second position, respectively. Then the rank of the TOM for one axis rotation becomes [2],

1) $Rank(V(\psi_1,\psi_2))=10$ (15a)
2) $Rank(V(\theta_1,\theta_2))\le 8$ (15b)
3) $Rank(V(\phi_1,\phi_2))\le 10$ (15c)

Where, Φ, θ and ψ denote roll, pitch and heading angle, respectively. Theorem 2 shows the rank of the observability matrix of 2-position alignment which is obtained by rotating *SINS* with respect to one axis from the initial position where the body and the navigation frames are coincided. In theorem 2, the change of heading angle always results in a completely observable system. And the change of the pitch angle causes rank deficiency in occasion. However, when we assume that the vehicle is only leveled at rest, the effect of the pitch and the roll axis rotation may be reversed according to the initial heading angle. Therefore, 2-position alignment which changes the heading angle is better than the other. Moreover it is easier to implement.

The optimal position in the 2-position alignment can be calculated by changing heading angle [2]. The optimal position is the one minimizing error covariance matrix defined in Section 3. Since the *SINS* error model used in fixed position alignment is not completely observable for an arbitrary position, the error variance of unobservable state variables remain constant or decrease very slowly. But 2-position alignment causes the system to be completely observable resulting in a fast decrease of error variance.

In order to obtain an optimal 2-position alignment, numerical calculations of the Riccati equation given by (14) are performed by varying heading angle. An initial covariance matrix, spectral density matrix of process noise, and measurement noise covariance matrix R are set to do the numerical calculation. In this work, $P_1(0), Q$, and R for a medium-grade *SINS* are chosen. And they are

$$P_1(0)=diag\{(0.1m/s)^2,(0.1m/s)^2,(1^\circ)^2,(1^\circ)^2,(1^\circ)^2,(100\mu g)^2,(100\mu g)^2,(0.02^\circ/h)^2,(0.02^\circ/h)^2,(0.02^\circ/h)^2\}$$
$$Q=diag\{(50\mu g)^2,(50\mu g)^2,(0.01^\circ/h)^2,(0.01^\circ/h)^2,(0.01^\circ/h)^2,0,0,0,0,0\}$$
$$R=diag\{(0.1m/s)^2,(0.1m/s)^2\}$$

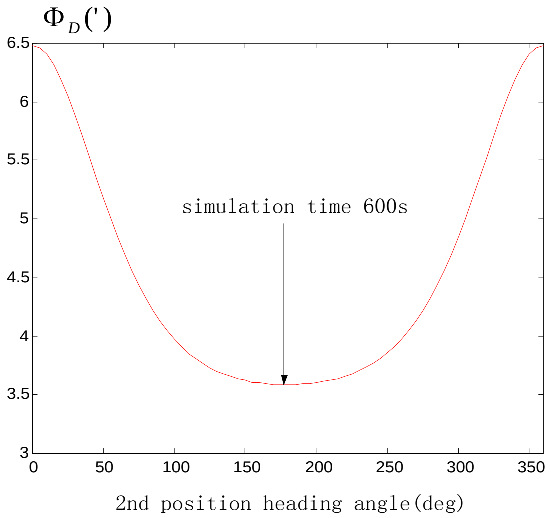

Fig.1 Tradition Multiposition Alignment Method

The number of iteration performed for calculating P_i is 600, which is equivalent to 600s in time-scale [2]. Heading axis rotation is introduced at 300s. Since the heading error converges slowly to a large value, it is one of the most crucial state variables in *SINS*. Therefore 2-position alignment is focused on the convergence rate and value of the heading error**.** Fig.1 illustrates how to seek the optimal position for heading angle rotation. The figure shows the 1-σ value of the heading error at 600s for the various heading angle rotation ranging from 0 deg to 360 deg at an interval of 15 deg. As seen from the figure, the second position with the least error and the best sensitivity to position change is obtained when the heading angle is rotated by 180 deg. After 600s,the heading error has decreased down to 3.7$'$ .It is due to unobservable state ε_E, but the estimation accuracy and speed of Φ_N and Φ_E is very high. In order to accelerate the multiposition alignment, the convergence rate of Φ_D has to be increased.

5 Effective Fast Azimuth Alignment Method

From the first four formula of the *SINS* stationary error model (1) yields:

$$\delta\dot{V}_N = 2\Omega_D \delta V_E + g(\Phi_E + \frac{\nabla_N}{g}) \qquad (16)$$

$$\delta\dot{V}_E = -2\Omega_D \delta V_N - g(\Phi_N - \frac{\nabla_E}{g}) \qquad (17)$$

$$\dot{\Phi}_N = \Omega_D \Phi_E + \varepsilon_E \qquad (18)$$

$$\dot{\Phi}_E = -\Omega_D \Phi_N + \Omega_N \Phi_D + \varepsilon_E \qquad (19)$$

Base on formula (16) and (17), estimates of Φ_N and Φ_E can be written,

$$\hat{\Phi}_N = \Phi_N - \frac{\nabla_E}{g} = -\frac{1}{g}(\delta\dot{V}_E + 2\Omega_D \delta V_N) \qquad (20)$$

$$\hat{\Phi}_E = \Phi_E + \frac{\nabla_N}{g} = \frac{1}{g}(\delta\dot{V}_N - 2\Omega_D \delta V_E) \qquad (21)$$

(18) adds to (19), yields

$$\Phi_D + \frac{\varepsilon_E}{\Omega_N} - \frac{\nabla_E}{g}(\frac{\Omega_D}{\Omega_N}) = \frac{1}{\Omega_N}[\dot{\Phi}_E + (\Phi_N - \frac{\nabla_E}{g})\Omega_D] \qquad (22)$$

Notices, $\dot{\hat{\Phi}}_N = \dot{\Phi}_N$, $\dot{\hat{\Phi}}_E = \dot{\Phi}_E$,

$\frac{\Omega_D}{\Omega_N} = -\tan L$, substituting (20) and (21) into (22), the best estimation of azimuth becomes to

$$\hat{\Phi}_D = \Phi_D + \frac{\varepsilon_E}{\Omega_N} + \frac{\nabla_E}{g}\tan L = \frac{1}{\Omega_N}[(\dot{\hat{\Phi}}_E + \Omega_D \hat{\Phi}_N)] \qquad (23)$$

Where, $\hat{\Phi}_N$ and $\dot{\hat{\Phi}}_E$ are the estimates of the leveling error about north axis and the leveling error rate about east axis. Equation (23) shows the azimuth error can be computed directly from the estimates of Φ_N and Φ_E. Therefore, the convergence rate of $\hat{\Phi}_D$ can be greatly increased, which shows that the azimuth error can be computed from the estimates of the leveling error about north axis and the leveling error rate about east axis. Note that the estimation of azimuth error does not explicitly depend upon gyro output signals. This phenomenon can be used in an alternate filter design for leveling and azimuth multiposition alignment simultaneously.

It is evident, from (20)-(21) and (23), that the errors in the estimation are

$$\delta\Phi_N = -\frac{\nabla_E}{g} \tag{24}$$

$$\delta\Phi_E = \frac{\nabla_N}{g} \tag{25}$$

$$\delta\Phi_D = \frac{\varepsilon_E}{\Omega_N} + \frac{\nabla_E}{g}\tan L. \tag{26}$$

This result is identical with the accuracy that is often shown in the self-alignment schemes [4]. It should be pointed out that only when $\hat{\Phi}_E$ is nearly stable, can $\hat{\Phi}_E$ be changed to the input of the digital low-pass filter, and $\hat{\Phi}_D$ is estimated by (23).

6 Computer Simulation Results

In term of the initial conditions of the above Kalman filter and multiposition alignment method , $\hat{\Phi}_E$ is alternated to the low-pass filter, and the estimation of $\hat{\Phi}_D$ is performed by (23). The computer simulation results are showed in Fig.2. It is obviously observed that the convergence rate and precision of $\hat{\Phi}_D$ is greatly accelerated, $\hat{\Phi}_D$ nearly converges simultaneously with $\hat{\Phi}_N$ and $\hat{\Phi}_E$, and the error of azimuth is reduced to about 1.9 $'$. Consequently, the time of multiposition alignment of *SINS* is reduced, and the precision of the initial alignment is improved greatly.

Since in this paper, a new multiposition alignment method has been proposed to improve the performance of the stationary alignment of *SINS*. The above results show that the fundamental limitation of the tradition multiposition alignment can be successfully accomplished by a new azimuth estimation method using to it. In other words, *SINS* at rest can be fast and precision converted to the navigation mode since the initial alignment rate and precision are considerably improved by the new multiposition alignment.

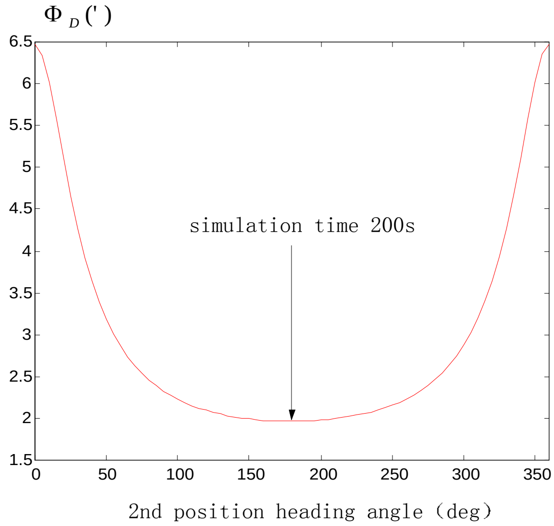

Fig.2 A New Multiposition Alignment Method

References

[1] Jiang,Y.E, Lin,Y.P. Error estimation of INS ground alignment through observability analysis. *IEEE Transaction on Aerospace and Electronic Systems*, Vol.28, No.1, pp 92-97, 1992

[2] Jang Gyu Lee, Chan Gook Park,Heung Won Park Multiposition alignment of strapdown inertial navigation system. *IEEE Transaction on Aerospace and Electronic Systems*, Vol.29, No.4, pp 1323~1328, 1993

[3] Fang J C, Wang D J. A fast initial alignment method for strapdown inertial navigation system on stationary base. *IEEE Transactions on Aerospace and ElectronicSystems*,Vol.32, No.4, pp 1501~1505, 1996.

[4] Bar_Itzhack I Y, Berman N. Control theoretic approach to inertial navigation systems. *Journal of Guidance and control*, Vol.11, No.3, pp 237-245, 1988.

[5] Meskin D G, Bar_Itzhack I Y. Observability analysis of piece-wise constant system-part I: theory. *IEEE Transactions on Aerospace and Electronic Systems*, Vol.28, No.4, pp 1056-1067, 1992.