CONTROL LAW SYNTHESIS FOR AIRCRAFT AUTOPILOT WITH NONLINEAR ELEMENTS

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Abstract

During control law synthesis for aircraft autopilot there is applied the linear or linearized model of the aircraft. Taylor series method is used for linearization of the aircraft model. The aircraft mathematical model mainly determined for the rigid-body case. In this particular case the high frequency oscillations of the fuselage elastic motion are omitted.

There are some types of nonlinear effects, which can be experienced during modeling of elements of automatic flight control systems. These nonlinearities are dead-zone, saturation, relay characteristics or theirs combination etc, which dynamics should be considered during controller synthesis.

In this paper aircraft dynamics will be considered with its elastic motion of the fuselage. The elastic motion dynamics is added to the rigid aircraft model as the additive type uncertainty. The nonlinear element to be considered during controller synthesis will be dead-zone plus saturation, which represents typical nonlinear transfer characteristics of the two degree-of-freedom electromechanical gyroscopes.

Considering fuselage elastic motion and nonlinearity the control law for the aircraft autopilot will be synthesized. The pitch angle control system will be used for control law synthesis and test of the designed system.

1 Introduction

Most of real physical systems are nonlinear ones. Dynamic processes are described with nonlinear models. Elements of control systems can be considered for nonlinear ones due to its principle of working, structure etc. Most commonly met nonlinear effects are hysteresis, dead-zone, saturation, backlash, relay characteristics, switch, rate limiter, transport delay, geometry of coordinate transformation.

If there is a smooth nonlinearity it can be treated by using a linearized model. The widely applied time domain method for linearization is the Taylor series method based on the small perturbation principle.

The other important method widely applied in control theory is the describing function method, which means harmonic linearization of the unavoidable nonlinearities.

2 Time Domain Linearization of Nonlinear Systems

In case of small perturbations of the nonlinear systems it is possible to linearize the control system. Dynamics of the nonlinear control system can be represented by the following equation [9]:

$$g = g(x_1, x_2, x_3, \dots, x_n) \tag{2.1}$$

In eq (2.1) g represents the output of the system, x_i is the state variable, or input of the control system. Let us consider the following steady-state conditions:

$$G = g(X_1, X_2, X_3, \dots, X_n) \tag{2.2}$$

Using theory about small perturbations nonlinear function (2.1) can be rewritten as follows:

$$\Delta g \approx A_1 \Delta x_1 + A_2 \Delta x_2 + \dots + A_n \Delta x_n = \sum_{i=1}^{n} A_i \Delta x_i \tag{2.3}$$

In eq (2.3) small perturbations can be defined as:

$$\Delta g \approx g - G,\ \Delta x_i = x_i - X_i \qquad (2.4)$$

The linearized equation for Δg can be derived as:

$$\Delta g = g(X_1 + \Delta x_2, ..., X_n + \Delta x_n) - G \qquad (2.5)$$

In other manner we have:

$$\Delta g = \Delta g(\Delta x_1, ..., \Delta x_n) \qquad (2.6)$$

In general, the Taylor series of the nonlinear function can be derived as:

$$g = G + \sum_{i=1}^{n} \left.\frac{\partial g}{\partial x_i}\right|_M (x_i - X_i) + \frac{1}{2}\sum_{i=1}^{n} \left.\frac{\partial^2 g}{\partial x_i^2}\right|_M (x_i - X_i)^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \Big|_M (x_i - X_i)(x_j - X_j) + ... \qquad (2.7)$$

Neglecting higher order terms in eq (2.7) we have:

$$g \approx G + \sum_{i=1}^{n} \left.\frac{\partial g}{\partial x_i}\right|_M (x_i - X_i), \qquad (2.8)$$

or

$$\Delta g \approx \sum_{i=1}^{n} \left.\frac{\partial g}{\partial x_i}\right|_M \Delta x_i \qquad (2.9)$$

Eq (2.1) can be represented in the following scalar-vector manner:

$$g = g(\mathbf{x}) \qquad (2.10)$$

After linearization of eq (2.10) we have:

$$\Delta g \approx \mathbf{grad}\, g|_M\ \Delta\mathbf{x} = \nabla g|_M\ \Delta\mathbf{x} = \left.\frac{dg}{d\mathbf{x}}\right|_M \Delta\mathbf{x}, \qquad (2.11)$$

in other manner:

$$\Delta g \approx \left[\mathbf{grad}\, g|_M\right]^{\mathbf{T}} \Delta\mathbf{x} = \left[\nabla g|_M\right]^{\mathbf{T}} \Delta\mathbf{x} = \left[\left.\frac{dg}{d\mathbf{x}}\right|_M\right]^{\mathbf{T}} \Delta\mathbf{x} = \left.\frac{dg}{d\mathbf{x}^{\mathbf{T}}}\right|_M \Delta\mathbf{x} \qquad (2.12)$$

In eq (2.12) $\nabla = \left[\partial/\partial x_1, ..., \partial/\partial x_n\right]^{\mathbf{T}}$.

Using this condition we have

$$\mathbf{grad}\, g = \nabla g = \frac{dg}{d\mathbf{x}} = \left[\partial g/\partial x_1, ..., \partial g/\partial x_n\right]^{\mathbf{T}} \qquad (2.13)$$

Eq (2.12) with respect to eq (2.13) can be rewritten as follows [??]:

$$\Delta g \approx [\Delta\mathbf{x}]^{\mathbf{T}}\, \mathbf{grad}\, g|_M = [\Delta\mathbf{x}]^{\mathbf{T}} \nabla g|_M = [\Delta\mathbf{x}]^{\mathbf{T}} \left.\frac{dg}{d\mathbf{x}}\right|_M \qquad (2.14)$$

In case of multi input multi output systems its dynamics is defined by the system of equation given below:

$$\begin{aligned} g_1 &= g_1(x_1, x_2, x_3, \dots, x_n) \\ g_2 &= g_2(x_1, x_2, x_3, \dots, x_n) \\ &. \\ g_m &= g_m(x_1, x_2, x_3, \dots, x_n) \end{aligned} \qquad (2.15)$$

System of equation (2.15) can be written in the following vector-vector form [9]:

$$\mathbf{g} = \mathbf{g}(x) \qquad (2.16)$$

Neglecting higher order terms of the Taylor series of equation (2.16) it can be derived as:

$$\mathbf{g} = \mathbf{g}(\mathbf{x}) \approx \mathbf{g}(\mathbf{X}) + \sum_{i=1}^{n} \left.\frac{\partial \mathbf{g}}{\partial x_i}\right|_M (x_i - X_i) \qquad (2.17)$$

Using eq (2.17) the MIMO nonlinear control system (2.15) can be linearized and we can get a linear model based on small perturbations of the nonlinear functions.

3 Frequency Domain Linearization of Nonlinear Systems – Harmonic Linearization

This method allows analysis of the nonlinear systems in case of large amplitude of sinusoidal input signals. Let us consider the nonlinear control system represented in Fig 1.

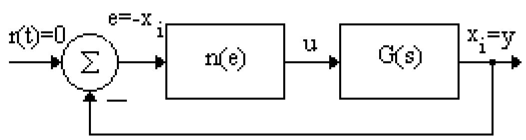

Figure 1 Block Diagram of the Nonlinear Control System

In Figure 1 $G(s)$ represents the linear part of the control system, $n(e)$ is the resulting static nonlinearity of the closed loop system. Respecting zero reference signal the error signal $e(t)$ can be given using following formula [6,9]:

$$e(t) = E \sin\omega\, t \qquad (3.1)$$

In general, output signal of the nonlinearity $u(t)$ is not the pure sinusoid so its Fourier series expansion can be derived as [6,9]:

$$u(t) = A_o + \sum_{n=1}^{\infty}\left(A_n \cos n\omega t + B_n \sin n\omega t\right) \quad (3.2)$$

For symmetric nonlinearity coefficients of the Fourier series are as follows [9]:

$$A_o = 0,\ A_n = \frac{2}{\pi}\int_0^{\pi} u(t)\cos n\omega\, tdt$$
$$B_n = \frac{2}{\pi}\int_0^{\pi} u(t)\sin n\omega\, tdt \quad (3.3)$$

In general case takes place the following equations

$$A_n = (E, j\omega),\ B_n = (E, j\omega), \quad (3.4)$$

i. e. they depend on input signal amplitude and the frequency. Applying the describing function method it is supposed that all high frequency overtones are damped on the linear part of the control system, which has low-pass nature, i. e. we can write that

$$u_1(t) = B_1 \sin\omega t + A_1 \cos\omega t \quad (3.5)$$

Let us introduce following equations:

$$C_1 = \sqrt{A_1^2 + B_1^2}, \quad (3.6)$$

$$\sin\varphi_1 = A_1 C_1^{-1},\ \cos\varphi_1 = B_1 C_1^{-1} \quad (3.7)$$

Using eqs (3.6) and (3.7) eq (3.5) may be rewritten as follows [6]:

$$u_1(t) = C_1 \sin(\omega t + \varphi_1) \quad (3.8)$$

The input signal and the out put signal of the nonlinearity may be represented using following equations from electrotechnics [6,9]:

$$e(t) = Im\, E(j\omega)e^{j\omega t} = Im\, Be^{j\omega t} \quad (3.9)$$

$$u_1(t) = Im\, U_1(j\omega) = = Im\, C_1 e^{j(\omega t + \varphi_1)} = Im\, C_1 e^{j\omega t} e^{j\varphi_1} \quad (3.10)$$

From eqs (3.9) and (3.10) we have:

$$E(j\omega) = B,\ U_1(j\omega) = C_1 e^{j\varphi_1} \quad (3.11)$$

Generally, describing function can be defined as follows [9]:

$$N = (E, j\omega) = \frac{U_1(j\omega)}{E(j\omega)} = \frac{C_1(E, j\omega)}{B} e^{j\varphi_1(E, j\omega)} \quad (3.12)$$

From eq (3.12) it is easily can be seen that it depends on amplitude E and the frequency ω. It must be noted that equations of describing functions of nonlinearities can be simplified if they are considered to be single-valued odd symmetric ones. Describing functions are widely applied for stability analysis of nonlinear systems.

4 Nonlinearities of Control Systems

Conventional automatic flight control system is for automatic stabilizing of the aircraft spatial motion and includes many sensors, amplifiers, motors and actuators. Block diagram of the pitch attitude control system can be seen in Figure 2.

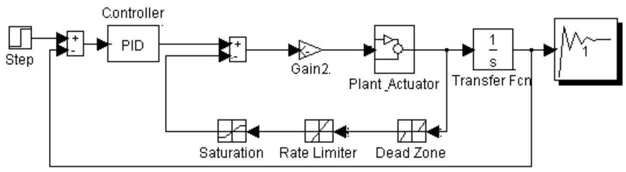

Figure 2 Block Diagram of the Pitch Attitude Control System

The pitch angle autopilot is based upon the pitch rate stability augmentation system. In case of the so-called full state feedback this loop is realizes the feedback by pitch rate. For measuring of the pitch rate in conventional systems pitch rate gyro is widely used.

Most common nonlinearities of elements of autopilots are dead zone, saturation, rate limiter, transport delay, relay characteristics, backlash and their linear or nonlinear combination. In this article three types of nonlinearity, namely dead zone, saturation and rate limiter will be taken into account.

Dead zone of the potentiometer of the rate gyro is installed especially to minimize noises from vibrations. In this particular case dead zone of $\pm 0{,}4$ deg/s will be considered. Saturation and rate limiter is defined by maneuverability of the aircraft, which are ± 18 deg/s and $\pm 1{,}8$, respectively. During automatic control of the pitch attitude the limited angular deflection of the elevator is considered. It is supposed to be limited to the value of ± 1 deg. Block diagram of the nonlinear transfer characteristics of the conventional pitch rate gyro can be seen in Figure 2.

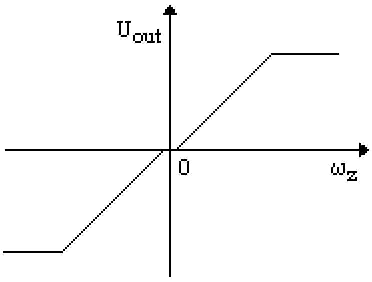

Figure 2 Nonlinearity of the Pitch Rate Gyro

Real aircraft behaves elastically. Any elevator deflection or external disturbance e. g. turbulent air can lead to elastic motion. There are two main methods for modeling of the elastic aircraft. First one is the classical transfer function method [1,2,4]. Second one is the state space representation method [3,5,7]. This paper deals with classical representation of the elastic motion of the aircraft.

5 Mathematical Model of the Elastic Aircraft

It is easily can be seen that output signal of the pitch rate sensor can be determined as follows:

$$\omega_{z_E}(s)=\frac{sK_1}{s^2+2\xi_1\omega_1 s+\omega_1^2}\delta_E(s) \qquad (5.1)$$

In eq (3.1) K_1 is gain of the ith elastic degree of freedom, ω_1 and ξ_1 are natural frequency and damping ratio of the ith elastic degree of freedom, respectively.

In [1,8] parameters of the first overtone of the fighter fuselage bending motion are given as follows:

$$K_1=10\,s^{-2},\omega_1=10\,s^{-1},\xi_1=0{,}05 \qquad (5.2)$$

It is supposed that the longitudinal motion control system is affecting only the short period motion. The simplified mathematical model of the longitudinal motion of the aircraft for the flight conditions H=1000 m and M=0.4 is given as [1,8]:

$$\omega_{z_R}(s)=-\frac{A(1+sT_\theta)\omega_\alpha^2}{s^2+2s\xi_\alpha\omega_\alpha+\omega_\alpha^2}\delta_E(s) \qquad (5.3)$$

In eq (5.3) let us consider the following parameters of the aircraft:

$$A=1{,}5\,s^{-1};T_\theta=2\,s;\omega_\alpha=5\,s^{-1};\xi_\alpha=0{,}5 \qquad (5.4)$$

The resulting output signal of the pitch rate gyro can be determined as a sum of the rigid and elastic aircraft output signals defined by eqs (5.1) and (5.3), respectively. We have:

$$\omega_z(s)=\omega_{z_E}(s)+\omega_{z_R}(s) \qquad (5.5)$$

Transient response of the open loop pitch rate of the elastic and the rigid aircraft can be seen in Figure 3.

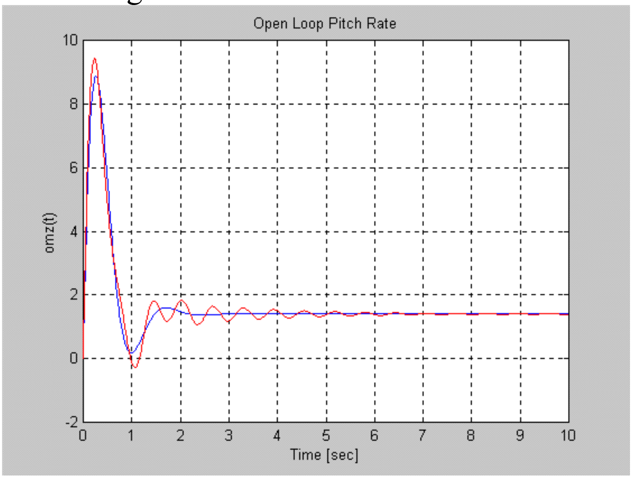

Figure 3 Step Response of the Aircraft
Rigid Aircraft Elastic Aircraft

From Figure 3 it is easily can be seen that the elastic aircraft behaves oscillatory to reference input. The pitch rate goes to its final value through its large peaks and the response time is too large.

Frequency domain behavior of the rigid and elastic aircraft can be seen in Figure 4.

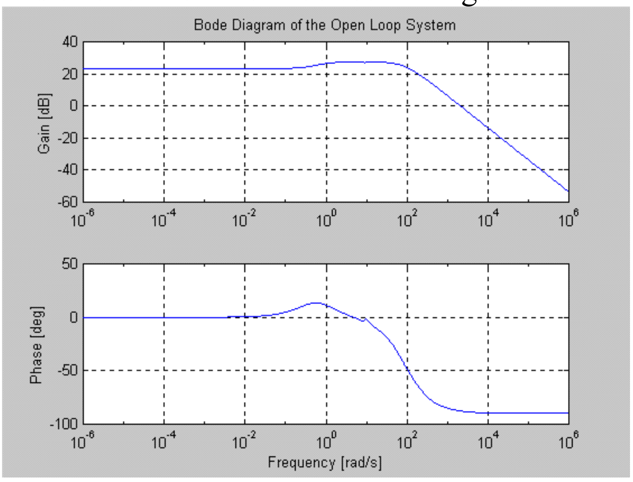

Figure 4 Bode Diagram of the Elastic Aircraft

From eq (5.5) it is easily can be seen that pitch rate sensor measures pitch rate of the rigid and the elastic aircraft. Some of conventional aircraft are supplemented with autopilot without filters in the feedback path of the pitch rate. First and second order filters are often involved to eliminate effects from elastic motion of the aircraft.

6 Numerical Example

Let us consider aircraft pitch attitude control system given in Figure 2. The rigid plant model is given by eq (5.3) and the elastic plant model is given by eq (5.1). Eqs (5.1) and (5.3) are to be considered with their parameters given by eqs (5.2) and (5.4). The rigid and elastic models are grouped in one block in the Simulink model. The NCD Toolbox is attached to output of the control system. The constraints upon the output signal can be seen in Figure 5.

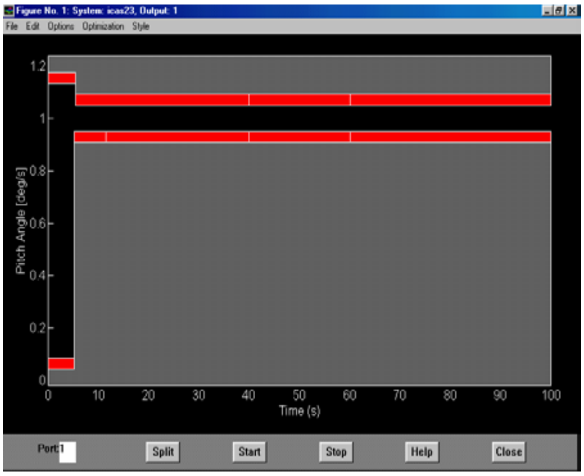

Figure 5 Constraints on Pitch Angle Behavior

It is well - known that parameters of the elastic motion overtones depend on flight parameters e. g. height of the flight, airspeed etc. During PID-controller synthesis for the pitch attitude control system it was supposed that nominal value of the gain K_1 varies in the range of its ± 5 %. Control problem to be solved can be formulated as follows: find the PID-controller for the outer loop of the pitch attitude control system with block diagram represented in Figure 2.

Dynamic performances of the closed loop control system may be determined using flying and handling qualities of piloted airplanes MIL-F-8785C.

7 Conclusions

The paper deals with basic equations of the nonlinear systems. The time domain Taylor series method and the frequency domain harmonic linearization method were outlined for SISO and MIMO control systems. Future work will be made in the field of design application.

8 Acknowledgements

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9 References

[1] Aslanian, A. E.: *Automatic Flight Control Systems*, Air Force Institute of Kiev, University Press, 1984 (in Russian).
[2] Blakelock, J. H.: *Automatic Control of Aircraft and Missiles*, John Wiley and Sons, New York-London-Sydney, 1965.
[3] McLean, D.: *Automatic Flight Control Systems*, Prentice Hall International Ltd., 1990.
[4] Bishplinghoff, R. L. - Ashley, H.: *Principles of Aeroelasticity*, John Wiley and Sons Inc., New York - London, 1962.
[5] Szabolcsi, R. - Gáspár, P.: Flight Control System Synthesis for the Aeroelastic Jet Fighter Aircraft, *Preprints of the Robust Control Design, ROCOND '97*, pp (453-458), 1997.
[6] Brogan, W. L.: *Modern Control Theory*, Prentice-Hall International, Inc, Englewood Cliffs, 1991.
[7] Szabolcsi, R. Design of the Pitch Attitude Control System for the Aeroelastic Fighter Aircraft, *Bulletins for Applied Mathematics, BAMM-1240/96, LXXX.*, pp. (29-40), Göd, Hungary, October 10-13, 1996.
[8] Szabolcsi, R. – Szegedi, P. Robustness Analysis of the Flight Stability Augmentation Systems, *Proceedings of the Pannonian Applied Mathematical Meetings PAMM, PC-136 Conference*, January 24-27, 2002 (in Print).
[9] Csáki, F.: *Modern Control Theory*, Academic Press, Budapest, 1970 (in Hungarian).