LATERAL FLIGHT CONTROL SYSTEM USING OPTIMAL H^{∞} CONTROLLERS

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Abstract

The present paper describes a design method for the automatic flight control system of the lateral motion of the aircraft; this method is based on an optimal H^{∞} control technique which solution is obtained using the singular perturbations theory. In contrast with the suboptimal design, no ill-conditioned computations appear when a level of attenuation close to its minimal value is used. Moreover the optimal controller derived has a smaller order than the suboptimal one. The case study for the lateral flight control system of the VISTA F-16 aircraft illustrates the theoretical developments.

1 Introduction

One of the most demanding problems in the design of automatic flight control systems is to ensure a satisfactory aircraft dynamic response to the pilot commands. Since the modern aircrafts have highly coupled dynamics, strongly depending on the nominal flight conditions over a wide flight envelope, the conventional design methods mainly oriented towards single–input, single–output systems, are completely inadequate. One of the most powerful design methodologies developed in the last decade is based on the H^{∞} control theory. This theory has been intensively used during the last years (*e.g.* [1], [2], [10]) when automatic flight control systems has been designed both for the longitudinal and lateral motion of the aircraft. In the mentioned papers, it is shown that the design objectives can be accomplished by solving an H^{∞} nonsingular control problem which state–space solution can be found for example in [3], [6], [8]. On the other hand, it is known that in many applications, when using these formulae for a level of attenuation close to its optimum, the numerical computations become ill–conditioned. This means that in most applications suboptimal solutions are in fact determined, rather than optimal ones. In order to avoid the ill-conditioning mentioned above, different methods has been proposed in the literature; among them we mention the ones described in [4], [7].

In the present paper the design problem of a flight control system for the lateral axis of a fighter is addressed. The design approach is mainly inspired from [2] and [10] where it is shown that the design problem is equivalent with an H^{∞} control problem. In this paper an optimal solution of the H^{∞} control problem is used such that no ill-conditioned computations appear when the level of attenuation is closed to its minimal value. Moreover the order of the optimal H^{∞} controller is lower than in the suboptimal case.

The paper is organized as follows: in Section 2 the design objectives for the lateral manual flight control system of the VISTA F-16 aircraft are presented. The transformation of the control problem in an H^{∞} control problem is given in Section 3. Numerical results illustrating the proposed method are presented and discussed in Section 4, where comparative results with the suboptimal H^{∞} design are also given.

2 The Control Problem

According with the MIL-STD-1797 specifications ([9]) the first order roll-mode approximation from the lateral stick input δ_{lat} to the stability axis roll rate $\dot{\mu}$ must have the transfer function:

$$\frac{\dot{\mu}(s)}{\delta_{lat}(s)} = \frac{K_{\mu} e^{-\tau_{\mu} s}}{s + \frac{1}{T_{\mu}}} \tag{1}$$

where $\dot{\mu} = p\cos\alpha + r\sin\alpha$, and α denotes the angle of attack. According with the same specifications, the Dutch roll approximation from the rudder pedal input δ_{ped} to the sideslip β is:

$$\frac{\beta(s)}{\delta_{ped}(s)} = \frac{K_{\beta}\left(s + \frac{1}{T_{\beta}}\right)e^{-\tau_{\beta} s}}{s^2 + 2\xi_D \omega_D s + \omega_D^2} \tag{2}$$

For level 1 of performance, $T_{\mu} \leq 1s$, $\xi_D \geq 0.4$ and $\omega_D \geq 1\, rad/s$ ([9]).
These flying quality requirements are accomplished by the automatic flight control system for the lateral dynamics.

In the case study presented in this paper, the VISTA F-16 fighter lateral dynamics is considered. It includes the following state variables: β -angle of sideslip, p-body axis roll rate and r-body axis yaw rate. The control variables are: δ_t-differential horizontal tail deflection, δ_f-differential flap deflection and δ_r-rudder deflection. In fact there are only two independent control variables since the asymmetric flaps and the asymmetric horizontal tail are coupled in order to avoid the control saturations. The measured outputs are the roll and yaw rate and the sideslip angle. It is also assumed that the angle of attack α is measured either directly or from inertial data; these data are necessary in the case of maneuvers with high angle of attack. In [2] a dynamic inversion method is used to linearize the lateral dynamics. Then the true control variables δ_t ,δ_f and δ_r (where δ_t and δ_f are assumed coupled) have been transformed in the pseudo-commands sideslip acceleration ($\ddot{\beta}_c$) and stability axis roll acceleration $(\ddot{\mu}_c)$. This transformation has been determined by the dynamic inversion procedure. The relationship between the pseudo-commands and the true control variables $\delta_t, \delta_f, \delta_r$ is given by:

$$\begin{bmatrix} \ddot{\beta}_c \\ \ddot{\mu}_c \end{bmatrix} = T(\alpha)\begin{bmatrix} \delta_t \\ \delta_f \\ \delta_r \end{bmatrix} \tag{3}$$

where $T(\alpha)$ is a 2×3 matrix depending on the angle of attack and on the coupling ratio between δ_t and δ_f (for more details see equations (4.2), (4.6)-(4.8), (4.16) in [2]).Considering the trim flight condition Mach

0.2 and altitude 10,000 ft at high angle of attack ($\alpha = 30\deg$) the following linearized model is obtained (based on equation (A4-14) in [2]):

$$\dot{x} = A_g x + B_g u \tag{4}$$

with :

$$A_g = \begin{bmatrix} -7.2864e-2 & 5.0042e-1 & -8.6388e-1 \\ -8.8073e+0 & -1.5020e-1 & 1.0335e+0 \\ -6.2957e-1 & -1.1955e-1 & -2.5384e-1 \end{bmatrix}$$

$$B_g = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$C_g = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0 & 0.8660 & 0.5 \end{bmatrix}$$

where the state, the control variables and the outputs vector are $x = [\beta \quad p \quad r]^T$, $u = [\ddot{\beta}_c \quad \ddot{\mu}_c]^T$ and $y = [\beta \quad \dot{\mu}]^T$, respectively. A method to accomplish the MIL-specifications (1) and (2), consists in considering two ideal models which dynamics are close to (1) and (2) and to compare their responses with the aircraft responses at the same commands. Then the automatic flight control system is designed to minimize the signal errors between the models and the aircraft responses.

The control configuration based on the model matching procedure described above is given in Figure 1:

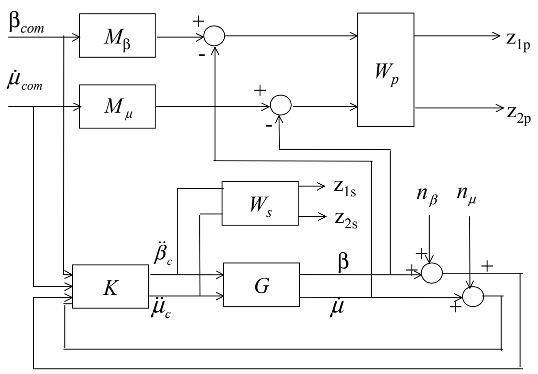

Figure 1: Model matching control configuration

where:

· $\beta_{com}, \dot{\mu}_{com}$ denote the sideslip and stability axis roll rate commands;

· M_β and $M_{\dot{\mu}}$ are ideal models determined according with the MIL specifications and they have the following transfer functions:

$$\frac{\beta(s)}{\beta_{com}(s)} = \frac{\omega_D^2}{s^2 + 2\xi_D \omega_D s + \omega_D^2} \tag{5}$$

and:

$$\frac{\dot{\mu}(s)}{\dot{\mu}_{com}(s)} = \frac{1}{T_R s + 1} \tag{6}$$

respectively, where $\omega_D = 3 rad/s$ denotes the desired Dutch roll frequency, $\xi_D = 0.71$ is the Dutch roll damping and $T_R = 0.33s$ denotes the desired roll mode time constant;

· K is the required automatic flight control system;

· G denotes the lateral linearized dynamics (3) of the aircraft;

· e_β and $e_{\dot{\mu}}$ are error signals between the models and the aircraft responses;

· n_β and $n_{\dot{\mu}}$ are measurement noises;

· W_p and W_s denote dynamic weighting matrices chosen such that $e_\beta, e_{\dot{\mu}}$ are minimized on their specific frequency domain.

Then the control problem consists in determining K such that the following design objectives are accomplished:

(DO1) The resulting system in Figure 1 is stable;

(DO2) $z_{1p}, z_{2p}, z_{1s}, z_{2s}$ are minimized (in norm);

(DO3) The measurement noises effect is attenuated.

The control problem formulated above can be transformed in the following H^{∞} control problem:

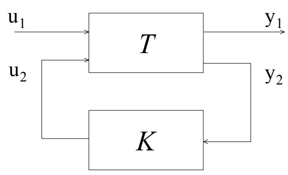

Figure 2: The equivalent H^∞ control problem

where: $u_1 = [\beta_{com} \quad \dot{\mu}_{com} \quad n_\beta \quad n_{\dot{\mu}}]^T$, $u_2 = [\ddot{\beta}_c \quad \ddot{\mu}_c]^T$, $y_1 = [z_{1p} \quad z_{2p} \quad z_{1s} \quad z_{2s}]^T$, $y_2 = [\beta_{com} \quad \dot{\mu}_{com} \quad \beta + n_\beta \quad \mu + n_{\dot{\mu}}]^T$ and T denotes the generalized system which includes the dynamics of $G, M_\beta, M_{\dot{\mu}}, W_p$ and W_s together with the connections between them.
With this transformation, the control problem reduces to the design of an H^∞ optimal controller K for the generalized system T. In contrast with the method used in [1] and [2] where suboptimal H^∞ solutions are considered, in the following developments an optimal H^∞ controller will be used, providing the minimum of $\|T_{y_1u_1}\|_\infty$, where $T_{y_1u_1}$ denotes the transfer matrix of the resulting system in Figure 2. The design procedure of such controller is described in the next section.

3 An Optimal Solution to the H^∞ Control Problem

The interest for optimal solutions of the H^∞ control problem has a practical motivation. Indeed, such solutions could provide the best level of attenuation which implies an improvement of the tracking and disturbances rejection performances.

Unfortunately in most applications, when approaching the optimum of the attenuation level, the usual formulae for the H^∞ control problem become ill conditioned, as it has been remarked in several papers (see [4] and the references therein). In this section a method to determine an optimal solution of the H^∞ control problem, which avoid the ill-conditioned computations mentioned above is briefly described. The complete proof is presented in [8].

Consider the H^∞ control problem associated with the generalized system:

$$
\begin{aligned}
\dot{x} &= Ax + B_1u_1 + B_2u_2 \\
y_1 &= C_1x + D_{12}u_2 \\
y_2 &= C_2x + D_{21}u_1
\end{aligned}
\tag{7}
$$

where the following assumptions are fulfilled:
(***A1***) (A, B_2, C_2) is stabilizable and detectable;
(***A2***) $D_{12}^T D_{12} = I$ and $D_{21} D_{21}^T = I$;
(***A3***) (A, B_1, C_2, D_{21}) and (A, B_2, C_1, D_{12}) have no transmission zeros on the $j\omega$-axis.

According with the well- known results proved in [6], the H^∞ control problem has a solution if and only if:
(***C1***) The Riccati equations:

$$
\begin{aligned}
&A^T X + XA + \gamma^{-2} XB_1B_1{}^T X - (XB_2 + C_1{}^T D_{12}) \\
&\times (B_2 X + D_{12}{}^T C_1) + C_1{}^T C_1 = 0
\end{aligned}
\tag{8}
$$

and

$$
\begin{aligned}
&AY + YA^T + \gamma^{-2} YC_1^T C_1 Y - (YC_2^T + B_1 D_{21}^T) \\
&\times (C_2 Y + D_{21} B_1^T) + B_1 B_1^T = 0
\end{aligned}
\tag{9}
$$

have stabilizing solutions $X(\gamma) \geq 0$ and $Y(\gamma) \geq 0$, respectively.
(***C2***) $\rho(X(\gamma)Y(\gamma)) < \gamma^2$, where $\rho(.)$ denotes the spectral radius of $(.)$.

Under the conditions above, an H^∞ suboptimal controller can be determined using for example the explicit formulae derived in [6].

Denote by γ_0 the optimum of γ which may be easily computed via a γ- procedure; in fact this procedure consists in decreasing γ until (***C1***) or *(C2)* fails. If (***C1***) fails before (***C2***) the formulae for the H^{∞} controller give reliable computations; by contrast, if (***C2***) fails before (***C1***), the computations become ill-conditioned due to the expression $\left(\gamma^2 I - Y(\gamma)X(\gamma)\right)^{-1}$ included in the controller formulae, which tends to be unbounded near γ_0. Therefore we focus our attention only on the critical situation when (***C2***) fails before (***C1***). In fact, as mentioned in [3], this is the most frequent case appearing in applications.

Thus we complete the assumptions (***A1***) - (***A3***) with the following one:

(***A4***) The optimal value γ_0 of γ is a solution of the transcendental equation:

$$\gamma^2 - \rho\left(X(\gamma)Y(\gamma)\right) = 0 . \tag{10}$$

Let us remark that if the above equation has a solution, this is unique; this remark is based on the fact that the dependencies $\gamma \to X(\gamma)$ and $\gamma \to Y(\gamma)$ are decreasing ([8]) and it allows to determine γ_0 with an assigned level of tolerance, using a bisection procedure.

A key role in the construction of the optimal H^{∞} controller has the *balanced realization* of (7) with respect to the solutions of equations (8) and (9). This balanced realization uses a result of Glover given in [5] stating that for two matrices $X \geq 0$ and $Y \geq 0$ there exists a nonsingular transformation T such that:

$$TXT^T = diag(X_{11}, X_{22})$$

and

$$\left(T^{-1}\right)^T YT^{-1} = diag\left(Y_{11}, Y_{22}\right)$$

with $X_{11} = Y_{11} = diag\left(\sigma_1 I_1, ..., \sigma_r I_r\right)$, where $\sigma_1 > ..... > \sigma_r > 0$, δ_i are $n_i \times n_i$ matrices, $i=1,...,r$, $X_{22} \geq 0, Y_{22} \geq 0$ and $X_{22}Y_{22} = 0$. Then a balanced realization of (7) is given by:

$$\left(T(\gamma)AT^{-1}(\gamma), T(\gamma)\begin{bmatrix} B_1 & B_2 \end{bmatrix}, \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} T^{-1}(\gamma), \begin{bmatrix} 0 & D_{12} \\ D_{21} & 0 \end{bmatrix} \right).$$

Therefore, without loosing the generality of the problem one can consider that (7) is written in balanced form. Based on assumption (***A4***), it follows that $\sigma_1 = \gamma_0$ and therefore the solutions $X(\gamma_0)$ and $Y(\gamma_0)$ of the Riccati equations associated with the balanced realization of (7) have the form:

$$X = diag\left(\gamma_0 I_1, \widetilde{X}_{22}\right) \text{ and } Y = diag\left(\gamma_0 I_1, \widetilde{Y}_{22}\right) \tag{11}$$

respectively where, in order to simplify the notations we omitted to write explicit the dependence of $\widetilde{X}_{22}$ and $\widetilde{Y}_{22}$ upon γ_0. Further, perform the following partitions conformably with the structure of X and Y:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; \begin{bmatrix} B_1 & B_2 \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}; \tag{12}$$

$$C_1 = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix}; \qquad C_2 = \begin{bmatrix} C_{21} & C_{22} \end{bmatrix}$$

Then, according with[8], the optimal solution of the H^{∞} control problem is given by $K_0 = \left(A_0, B_0, C_0, D_0\right)$ with :

$$\begin{aligned}
A_0 &= A_{22} + B_0\widetilde{C}_{22} + B_{21}F_{12} + B_{22}F_{22} \\
B_0 &= A_{21} + B_{21}F_{11} + B_{22}F_{21} \\
&\quad - \left(Z_{22}\widetilde{C}_{22}^T + B_{21}D_{21}^T\right)\left(\widetilde{C}_{21}^T\widetilde{C}_{21}\right)^{-1}\widetilde{C}_{21}^T \\
&\quad + Z_{22}\widetilde{C}_{22}^T + B_{21}D_{21}^T \\
C_0 &= -D_0\widetilde{C}_{22} + F_{22} \\
D_0 &= F_{21}\left(\widetilde{C}_{21}^T\widetilde{C}_{21}\right)^{-1}\widetilde{C}_{21}^T
\end{aligned} \tag{13}$$

where:

$$\begin{bmatrix} F_{11} & F_{12} \end{bmatrix} = \gamma_0^{-2} B_1 X$$

$$\begin{bmatrix} F_{21} & F_{22} \end{bmatrix} = -\left(B_2^T X + \widetilde{Y}_{22}\widetilde{X}_{22}\right)^{-1}\widetilde{Y}_{22}$$

The proof of the fact that (13) is an optimal solution of the H^{∞} problem is based on the singular perturbations method ([8]).

To summarize, the computation of the optimal solution to the H^{∞} control problem consists in the following steps:

Step 1. Compute the unique solution γ_0 of (10);

Step 2. Determine the balanced realization of (7) with respect to the solutions of (8) and (9) and denote by

$$\left(A, \begin{bmatrix} B_1 & B_2 \end{bmatrix}, \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \begin{bmatrix} 0 & D_{12} \\ D_{21} & 0 \end{bmatrix}\right)$$

this realization;

Step 3. Perform the partitions (11) and (12);

Step 4. Determine the optimal H^∞ controller K_0 with the realization (13).

4 The Aircraft Lateral Control System

Before starting the design of the control system we analyzed the lateral dynamics properties of the aircraft when no control system is used. To this end we determined the sideslip and the stability axis roll rate responses at step inputs β_{com} and $\dot{\mu}_{com}$, respectively; these are illustrated in Figures 3 and 4, respectively.

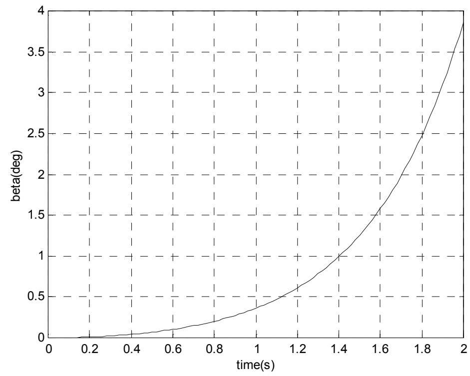

Figure 3: Sideslip response to step input (without controller)

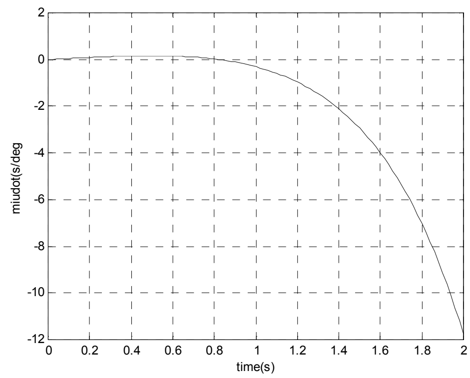

Figure 4: Stability axis roll rate to step input (without controller).

One can see that these responses have an unbounded variation indicating an unstable behavior of the lateral dynamics of the aircraft at the considered nominal flight conditions. In fact this instability is also emphasized by the fact that the state matrix A_g given by (4) are: 2.1377; -2.2453; -0.3693.

As shown in Section 2, according to MIL-STD-1797 specifications, $\beta(t)$ and $\dot{\mu}(t)$ must have a variation given by (5) and (6), respectively; their responses at step input are plotted in Figure 5 and 6, respectively.

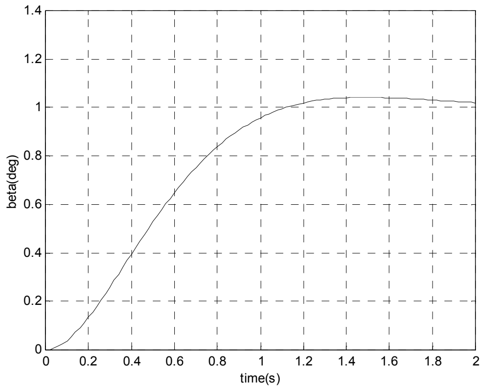

Figure 5: Ideal model response of the sideslip at step input.

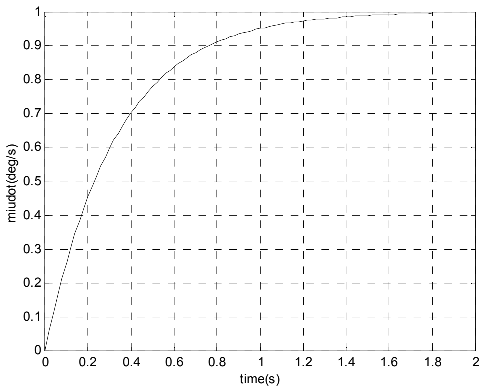

Figure 6: Ideal model response of the stability axis roll rate at step input.

As shown in Section 2, the design objectives for the manual lateral control system can be accomplished by solving the H^{∞} control problem illustrated in Figure 2.

In the following a realization of the generalized system corresponding to the H^{∞} control problem using the control structure in Figure 1 is determined. Denote by

$$M_1 = (A_{m1}, B_{m1}, C_{m1}, D_{m1})$$
$$M_2 = (A_{m2}, B_{m2}, C_{m2}, D_{m2})$$

the realizations of the models (5) and (6), respectively, and let

$$W_p = (A_p, B_p, C_p, D_p)$$
$$W_s = (A_s, B_s, C_s, D_s)$$

be the realizations of the weighting matrices W_p and W_s , respectively. Then, according with the control configuration illustrated in Figure 1, direct computations give the following realization of the generalized system (7):

$$A = \begin{bmatrix} A_g & 0 & 0 & 0 & 0 \\ 0 & A_{m1} & 0 & 0 & 0 \\ 0 & 0 & A_{m2} & 0 & 0 \\ -B_P C_g & B_p \begin{bmatrix} C_{m1} \\ 0 \end{bmatrix} & B_p \begin{bmatrix} 0 \\ C_{m2} \end{bmatrix} & A_p & 0 \\ 0 & 0 & 0 & 0 & A_s \end{bmatrix};$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ B_{m1} & 0 & 0 & 0 \\ 0 & B_{m2} & 0 & 0 \\ B_p \begin{bmatrix} D_{m1} \\ 0 \end{bmatrix} & B_p \begin{bmatrix} 0 \\ D_{m2} \end{bmatrix} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$B_2 = \begin{bmatrix} B_g \\ 0 \\ 0 \\ 0 \\ B_s \end{bmatrix}; \qquad (14)$$

$$C_1 = \begin{bmatrix} -D_p C_g & D_p \begin{bmatrix} C_{m1} \\ 0 \end{bmatrix} & D_p \begin{bmatrix} 0 \\ C_{m2} \end{bmatrix} & C_p & 0 \\ 0 & 0 & 0 & 0 & C_s \end{bmatrix};$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ C_g & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$D_{12} = \begin{bmatrix} 0 \\ D_s \end{bmatrix}; \qquad D_{21} = I,$$

where the zero and the identity matrices have appropriate dimensions.

The numerical values for the equalized system G and for the ideal models M_β and M_μ are given in Section 2. The weightings W_p and W_s used are:

$$W_p(s) = \frac{0.1s+1}{s+0.03} I_2, \; W_s(s) = \frac{s+100}{s+300000} I_2 .$$

Using a bisection procedure the optimal level of attenuation γ_0 verifying (10) has been determined; thus one obtained $\gamma_0 = 0.5823$. Then, using the procedure described in the previous section, we determined the optimal H^{∞} controller, which provides this level of attenuation. This controller represents in fact the required lateral control system.

Let us first notice that the resulting system with the configuration given in Figure 1 is stable. Indeed, the eigenvalues of the resulting system in Figure 1 obtained with the optimal controller (excepting the weights W_p and W_s)

are:

$-2.13 \pm 2.1126i; -132.16 \pm 111.87i; -100.04;$
$-11.543 \pm 13.35i; -10.006; -9.6054; -2.2456;$
$-2.13 \pm 2.1126i; -3.0303; -3.0303; -0.3639.$

Notice that no ill-conditioned computations appeared when determining the optimal solution of the H^∞ control problem, in contrast with the suboptimal design near the optimum. Moreover, as shown above, the order of this optimal controller is smaller than in the suboptimal case. Then the performances of the lateral control system have been evaluated. To this end the tracking errors between the true responses of the aircraft and the ideal models corresponding to the step commands β_{com} and $\dot{\mu}_{com}$ have been determined. These responses are plotted in Figures 7 and 8 respectively, indicating that the aircraft responses are conformably with the MIL-STD-1797 specifications.

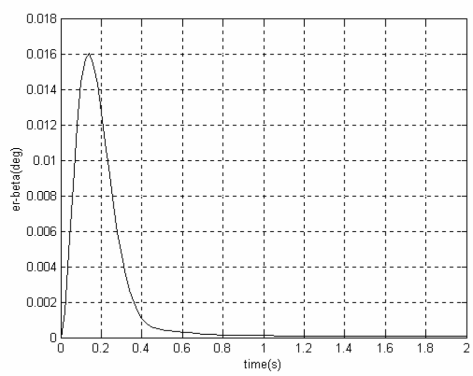

Figure 7: Sideslip tracking error

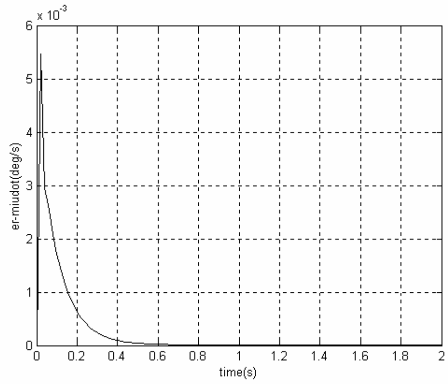

Figure 8: Stability axis roll rate tracking error

Remark. From the low-triangular structure of the state matrix A of the generalized system (14) it results that if A_g is stable then the stabilizing solution of (9) is $Y = 0$ and in this case condition (***C1***) fails before (***C2***) (see Section 3). Then the optimal solution of the H^∞ control problem can be solved using the known formulae corresponding to the sub-optimal case.

5 Conclusions

In this paper it is shown that the problem of maneuverability qualities improvement of an aircraft can be addressed via a H^∞ technique. The optimal solution of this problem is obtained using a new result described in the paper, based on the singular perturbation method. It is shown that in contrast with the results provided by the suboptimal H^∞ controller, the approach proposed in this paper leads to well-conditioned computations and it gives a controller which order is smaller than the order of the suboptimal solution.

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