AN EXPERT DATA MINING SYSTEM FOR NONLINEAR AEROELASTIC RESPONSE PREDICTION

Yau Shu Wong, Ovidiu Voitcu and Cristina Adela Popescu
Department of Mathematical and Statistical Sciences
University of Alberta, Edmonton, Alberta, Canada, T6G 2G1
email: yaushu.wong@ualberta.ca

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Abstract

Mathematical theory and numerical simulations have been successfully used to study nonlinear behavior of aircraft structures. In this paper, a new approach, based on an expert data mining system, is developed to investigate the dynamical response of an aeroelastic system. Our preliminary study indicates that the new method has a lot to offer to the dynamics community, and it is particularly attractive to analyze the nonlinear responses which are noisy, non-stationary and have high dynamics. Applications of the proposed technique to predict limit cycle oscillations and other nonlinear behaviors of a two-degree-of-freedom airfoil oscillating in pitch and plunge are reported.

1 Introduction

Understanding the nonlinear behavior of aircraft structures is a crucial step in the flutter boundary prediction, since structural nonlinearities affect not only the flutter speed, but also the characteristics of the dynamical response. Research on nonlinear aeroelasticity leads to more efficient and safe design of aircraft wings and control surfaces. A detailed review on the nonlinear aeroelastic analysis of airfoils was recently reported by Lee et al. [1]

Traditionally, the dynamic behavior in nonlinear aeroelasticity can be investigated by the mathematical theory and numerical simulations [2]. The describing function technique, the center manifold theory, and more recently the point transformation method have been successfully applied to predict limit cycle oscillations and other nonlinear responses of an aeroelastic system with structural nonlinearities represented by cubic spring, freeplay and hysteresis. Numerical techniques based on the Houbolt's finite-difference scheme and the Runge-Kutta time-integration procedure are also frequently used to study the nonlinear motion affected by structural nonlinearities. In these conventional approaches, a mathematical model is developed and the corresponding system parameters must be known. However, in some applications, such as during the ground vibration test or the actual flight test, only the dynamic response due to a given excitation is available. For such practical problems, the recorded nonlinear behaviors are noisy, non-stationary and have high dimensional dynamics. Consequently, using the traditional approach based on the mathematical analysis and numerical simulations may be difficult to deal with these problems.

Now, we propose to analyze the dynamics from data instead of using mathematical equations and numerical simulations. In this new approach, an expert data mining system (EDMS) is developed, in which a short-term data set resulting from experimental investigations, flight tests or numerical simulations is taken as the input to an EDMS. The output of EDMS then provides a prediction for the long-term dynamic behavior. The idea of studying the nonlinear dynamics using a data mining technique was first proposed in Popescu and Wong [3], in which we applied statistical models to predict the nonlinear aeroelastic behaviors. In the present paper, further development of an EDMS is described, and its effectiveness is demonstrated in applications concerning the limit cycle oscillations of a nonlinear aeroelastic system.

2 Nonlinear Dynamic Prediction

Let t denote the discrete time, and assume a set of consecutive terms of a time series x_1, x_2, …, x_t …, x_{t_1} is known. We are now interested in predicting the future values x_{t_1+1}, x_{t_1+2}, …, x_{t_2}. A popular approach for prediction used in data analysis is to search for a recurrence relation:

$$x_t = \Phi(x_{t-1}, x_{t-2}, \ldots, x_{t-p}) + e_t. \qquad (1)$$

where $y_t = \Phi(x_{t-1}, x_{t-2}, \ldots, x_{t-p})$ denotes the predicted value for x_t, and e_t denotes the prediction error. Since we are dealing with a nonlinear prediction, the nonlinear mapping Φ and the order p of the recurrence are constructed such that the above relation holds for all known data points. Once Φ and p are determined, the sequence of predicted values can be generated by

$$y_t = \Phi(x_{t-1}, x_{t-2}, \ldots, x_{t-p}), \qquad (2)$$

which is generally referred to as a *one-step prediction*, or

$$y_t = \Phi(y_{t-1}, y_{t-2}, \ldots, y_{t-p}), \qquad (3)$$

which is known as a *multi-step prediction*. Clearly, (2) can only be used if at each step t the correct values x_{t-1}, x_{t-2}, …, x_{t-p} are known. For practical applications in the study of nonlinear aeroelastic response, we are interested in the development of a reliable multi-step prediction method.

Choosing the expression for Φ out of an enormous number of possible mappings is an extremely difficult task. In this study, we propose to determine the order p and the nonlinear mapping Φ by using an expert data mining system (EDMS). To the best of our knowledge, little work has been done on developing a reliable method for long-term predictions. The proposed EDMS is particularly designed for such an application.

In a similar problem, it has been proven by Takens [4] that for a deterministic nonlinear system, the dynamics can be reconstructed if the embedding dimension d and the time-lag T are known. However, the information on d and T is usually unavailable for realistic problems. Moreover, Takens' theory is applied to a noise-free condition, while for practical applications we are interested in data corrupted by noise. The proposed EDMS is developed so that it is capable to deal with noisy data.

3 Expert Data Mining System

The proposed EDMS consists of three key components, namely the preprocessing, prediction and verification.

3.1 Preprocessing

The aeroelastic response recorded after an experimental testing is usually contaminated with noise. The noise content from a typical ground test is normally very small and the majority is caused by the measurement noise. However, especially due to the effect of turbulence, the amount of noise from a flight flutter testing is often significant.

The wavelet transform [5] which is based on a multi-resolution analysis is used to filter the noise. The technique is efficient and it has proven to be very appropriate for real-time applications. In addition, de-noising with local cosine bases is proposed for signals containing sinusoidal oscillations with moderate duration [6]. Other de-noising procedures, such as using neural networks and the unscented filter, will also be discussed.

Besides the de-noising step, we work with the mean deleted time series and the input data is usually transformed to the interval $[-1, 1]$.

3.2 Prediction

The prediction is the key module of the EDMS, where a long-term prediction is performed. Three different techniques, artificial neural networks, nonlinear time series models and the unscented filter are being applied.

3.2.1 Neural Networks

A neural network is an information processing system composed of interconnected elements known as artificial neurons. Each neuron receives several real numbers as inputs and generates a single real number as output [7]. The output is constructed by computing a weighted sum of the inputs and passing it through a *transfer function*, which can be linear or nonlinear depending on a specific problem being solved. The network is usually constructed via successive layers of neurons, such that the outputs of all neurons in each layer are fed as inputs to each neuron in the next layer. The inputs to all neurons in the first layer form the *network input*, while the outputs of all neurons in the last layer form the *network output*. The network output $\mathbf{y}$ is a function of the network input vector $\mathbf{x}$ and of the strengths of the network connections, known as the network *weights*. The form of this mapping $\mathbf{y} = \varphi(\mathbf{x}, \mathbf{w})$ (where $\mathbf{w}$ is the vector of all network weights) depends on the chosen network architecture, i.e., on the arrangement of neurons.

We now search for Φ in (1) among the nonlinear mappings $\Phi_{\mathbf{w}}(\mathbf{x}) = \varphi(\mathbf{x}, \mathbf{w})$. At each step t, the neural network input vector is $[x_{t-1}, x_{t-2}, \ldots, x_{t-p}]$ (for some p) in the case of a one-step prediction, or $[y_{t-1}, y_{t-2}, \ldots, y_{t-p}]$ for a multi-step prediction. For a given neural network architecture and for a fixed vector $\mathbf{w}$, the network output $y_t = \Phi_{\mathbf{w}}(x_{t-1}, x_{t-2}, \ldots, x_{t-p})$ or $y_t = \Phi_{\mathbf{w}}(y_{t-1}, y_{t-2}, \ldots, y_{t-p})$, provides a prediction for x_t.

According to Cybenko [8], a continuous function Φ on a compact in $\mathbf{R}^p$ can be uniformly approximated with accuracy $\varepsilon > 0$ by a mapping

$$\Phi^{\varepsilon}(\xi_1, \xi_2, \ldots, \xi_p) \stackrel{\text{def}}{=} w_0^{2,\varepsilon} + \sum_{k=1}^{n_\varepsilon} w_k^{2,\varepsilon} \tanh\left(w_{k,0}^{1,\varepsilon} + \sum_{h=1}^{p} w_{k,h}^{1,\varepsilon} \xi_h \right). \quad (4)$$

Hence, the output of a two-layer feed-forward neural network as illustrated in Fig.1 is capable to approximate the nonlinear mapping Φ in (1), where

$$y_{1,t}^2 = \Phi_{\mathbf{w}}(x_{t-1}, x_{t-2}, \ldots, x_{t-p}) = \mathrm{f}^2\left(w_{1,0}^2 + \sum_{k=1}^{n_1} w_{1,k}^2 \mathrm{f}^1\left(w_{k,0}^1 + \sum_{h=1}^{p} w_{k,h}^1 x_{t-h} \right)\right). \quad (5)$$

The vector $\mathbf{w}$ of all network weights is given by

$$\mathbf{w} = \left[w_{1,0}^1, \ldots, w_{n_1,p}^1, w_{1,0}^2, \ldots, w_{1,n_1}^2 \right]. \quad (6)$$

The neural network input vector is given by $[x_{t-1}, x_{t-2}, \ldots, x_{t-p}]$ and the transfer functions are $\mathrm{f}^1(v) = \tanh(v)$, $\mathrm{f}^2(v) = v$, for the first (*hidden layer*) and the second layer, respectively. The *biases* $w_{k,0}^1$, $w_{1,0}^2$ are regarded as particular weights corresponding to constant inputs equal to one.

However, using a linear transfer function in the second layer leads to a poor prediction performance due to the error propagation in a multistep prediction process. To overcome this problem, we propose replacing $\mathrm{f}^2(v)$ by

$$\mathrm{f}_c^2(v) \stackrel{\text{def}}{=} (c)^2 \tanh\left\{ \frac{\delta_0}{(c)^2} v \right\}, \quad (7)$$

where c is a scaling parameter and $\delta_0 = 0.1$ is a constant that controls the error propagation [9].

After choosing a specific neural network architecture and a certain value for p, we need to determine appropriate values for the weights such that the neural network can accurately predict the unknown values x_{t_1+1}, x_{t_1+2}, …, x_{t_2} of the time series. A common procedure is to choose the weight vector $\mathbf{w}$ that minimizes the *performance index*

$$E(\mathbf{w}) = \frac{1}{t_1 - t_0} \sum_{t=t_0+1}^{t_1} \left[x_t - y_{1,t}^2(\mathbf{w}) \right]^2 \quad (8)$$

for some fixed $t_0 < t_1$. Recall that at each step $t = t_0 + 1$, $t_0 + 2$, ..., t_1, both the network input $[x_{t-1}, x_{t-2}, \ldots, x_{t-p}]$ and the corresponding correct network output x_t are known, the performance index $E(\mathbf{w})$ is indeed a function of the network weights only. Thus, the neural network is *learning* from a set of examples of the correct network outputs to given inputs, and this is generally referred to as a *network training*. Nonlinear optimization methods such as the conjugate gradient algorithm, variations of Newton's method, etc [7] are commonly applied to minimize the performance index. Note that, during the network training, the network outputs are computed by a one-step prediction, while the unknown values $x_{t_1+1}, x_{t_1+2}, \ldots, x_{t_2}$ are determined by a multistep prediction using the weights resulted after the training. Voitcu and Wong [9] reported the detailed development of the network architecture and the training algorithm for a recurrent neural network. The present neural network illustrated in Fig. 1 is similar to the recurrent network discussed in [9] but more neurons are introduced in the hidden layer and the network has no feedback connection. However, the same training algorithm is essentially applied to both networks.

3.2.2 Nonlinear Time-Series Models

In this section, we now want to predict the subsequent values $[X_{n+1}, X_{n+2}, \ldots]$ of a given time series, $X = [X_1, X_2, \ldots, X_n]$ which contains only a limited number of transient observations.

Linear models have been commonly used for forecasting, but they are particularly suitable for short time predictions. Since the EDMS is designed to deal with long-term predictions for nonstationary data which exhibits a complex nonlinear dynamics, we prefer the nonlinear models rather than the classical linear time-series models. Two nonlinear time series models, namely the amplitude-dependent exponential autoregressive models [10] (EXPAR) and the self-exciting autoregressive models [11] (SETAR), are being implemented in the EDMS.

An EXPAR model of order p can be expressed analytically as

$$\begin{aligned} X_n = (\Phi_1 + \pi_1 e^{-\gamma X_{n-1}^2}) X_{n-1} + \ldots \\ \ldots + (\Phi_p + \pi_p e^{-\gamma X_{n-1}^2}) X_{n-p} + e_n \end{aligned} \tag{9}$$

where $\Phi_i, \pi_i, 1 \le i \le p$ and γ are constants and e_n is a discrete Gaussian white noise process. Such a model does not constrain X_n to be Gaussian and incorporates both the amplitude-dependent frequency and the limit cycle behavior. Notice that, when γ is zero, it becomes the classical linear autoregressive time-series model.

The EXPAR model (9) was further extended [12] to the form

$$\begin{aligned} X_n = (\Phi_1 + f_1(X_{n-1}) e^{-\gamma X_{n-1}^2}) X_{n-1} + \ldots \\ \ldots + (\Phi_p + f_p(X_{n-1}) e^{-\gamma X_{n-1}^2}) X_{n-p} + e_n \end{aligned} \tag{10}$$

where Φ_i, $1 \le i \le p$ and γ are constants, e_n is a discrete Gaussian white noise process and $f_i(X_{n-1}) e^{-X_{n-1}^2}$, $i = 1, \ldots, p$ are the Hermite type polynomials, where

$$f_i(X_{n-1}) = \pi_0^{(i)} + \pi_1^{(i)} X_{n-1} + \ldots + \pi_{r_i}^{(i)} X_{n-1}^{r_i}. \tag{11}$$

In this new form the model admits a more sophisticated nonlinear dynamics. For example, nonsymmetric processes can be generated if the orders r_i of the Hermite polynomials $f_i(X_{n-1})$ are odd. An efficient procedure which does not require a general nonlinear optimization solver to estimate the coefficients in the EXPAR model is reported in [10].

The SETAR models are particularly suitable for data arising from the piece-wise linear systems, and they are being considered for aeroelastic system with freeplay or hysteresis nonlinearities. A self-exciting threshold autoregressive model of order $(l; k, \ldots, k)$ or SETAR $(l; k, \ldots, k)$ where k is repeated l times, is a univariate time series $\{X_n\}$ of the form

$$X_n = a_0^{(j)} + \sum_{i=1}^{k} a_i^{(j)} X_{n-i} + \varepsilon_n^{(j)}, \tag{12}$$

conditional on $X_{n-d} \in R_j$, where d is a fixed integer belonging to $\{1, 2, \ldots, k\}$, $R_j = (r_{j-1}, r_j]$,

$j = 1, 2, \ldots l$ with $r_0 = -\infty$ and $r_l = +\infty$ is a partition of the set of real numbers and $r_1, \ldots, r_{l-1}$ are the thresholds. A SETAR $(1, k)$ model is equivalent to a linear autoregressive (AR) model of order k.

Replacing the linear autoregressive models in equation (12) with the nonlinear EXPAR model given by equation (10), we obtain a combination of SETAR and EXPAR models. We propose to use the combined model for studying the time series resulted from an aeroelastic system with the piece-wise linear restoring forces.

3.2.3 Unscented Filter

The extended Kalman filter (EKF) is frequently used for system identification and for performing a short-term prediction. However, the EKF is computationally expensive and can not be applied to non-differentiable nonlinearities such as freeplay or hysteresis models. In the proposed EDMS, we consider the unscented filter (UF) method [13]. Its performances are comparable with the EKF [14], but it does not require the calculation of any Jacobians. Thus, the UF method can be applied for continuous non-differentiable nonlinearities and it is computationally less expensive.

Suppose the nonlinear system is governed by the following differential equations,

$$X^{'}(t) = AX(t) + F(X(t)), \tag{13}$$

where A is the matrix containing the system coefficients, F is a nonlinear function, and X is the state vector.

Now, let consider the corresponding nonlinear discrete system

$$\begin{bmatrix} x(k+1) \\ a(k+1) \end{bmatrix} = \begin{bmatrix} f[a(k), x(k), u(k+1)] \\ a(k) \end{bmatrix} + v(k+1), \tag{14}$$

$$z(k+1) = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \end{bmatrix} \begin{bmatrix} x(k+1) \\ a(k+1) \end{bmatrix} + w(k+1).$$

where $f[\cdot,\cdot,\cdot,\cdot]$ is the process model, $x(k)$ is the state of the system at time step k, a(k) is the vector representing the unknown system parameters, $u(k+1)$ is the input vector, $z(k+1)$ is the observation vector, $v(k+1)$ is a $q-$dimensional noise process and $w(k)$ is the additive measurement noise. We assume that the additive noise vectors, $v(k)$ and $w(k)$, are Gaussian and uncorrelated white sequences.

After we apply the filter using the given noisy observations, the asymptotic behavior can be provided by the predictor. The parameters are fixed to the last values estimated using the filter. For performing the prediction, the corresponding state-space formulation is giving by

$$x(k+1) = f[a_0, x(k), u(k+1)] + v(k+1) \tag{15}$$

$$z(k+1) = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \end{bmatrix} x(k+1) + w(k+1),$$

where a_0 is a constant vector with the estimated values of the parameters [3].

3.3 Verification

The two core components of a typical rule-based expert system are the knowledge base and the reasoning engine. In the EDMS, three methods, the neural networks, the nonlinear time-series models and the unscented filter, are proposed for predictions. Hence, the knowledge base contains the results of the analysis done with the three methods discussed in the previous section. A diagnostic check for each prediction is then performed by comparing the results of these different approaches. The reasoning engine is based on a simple rule, namely the long-term predictions and their classification as limit cycle oscillations, stable or unstable oscillations are given as the system output if the solutions from the three different approaches agree.

The diagnostic check is done in the verification module, which is an important integral part of the EDMS. The input data is divided into the training and the test set. The training set is used for estimating the parameters of the models and the nonlinear mapping. The test set is used for checking the prediction accuracy. We have chosen this cross-validation approach because it emphasizes the predictive aspect of the model selection.

4 Case Studies

The performances of the developed EDMS are demonstrated to predict the asymptotic state of an aeroelastic system. The preprocessing, prediction and verification steps have been implemented using the C++ programming. The input data are generated from a numerical simulation of a two-degree-of-freedom airfoil oscillating in pitch and plunge. For incompressible flow, the following mathematical model [1] is applied:

$$\ddot{\xi} + x_\alpha \ddot{\alpha} + 2\zeta_\xi \frac{\tilde{\omega}}{U^*}\dot{\xi} + \left(\frac{\tilde{\omega}}{U^*}\right)^2 G(\xi) = -\frac{1}{\pi\mu} C_L(\tau),$$
$$\frac{x_\alpha}{r_\alpha^2}\ddot{\xi} + \ddot{\alpha} + 2\frac{\zeta_\alpha}{U^*}\dot{\alpha} + \frac{1}{U^{*2}} M(\alpha) = \frac{2}{\pi\mu r_\alpha^2} C_M(\tau), \tag{16}$$

Here, ξ and α are the plunging deflection and the pitch angle about the elastic axis, $G(\xi)$ and $M(\alpha)$ are the nonlinear plunge and pitch stiffness term and $C_L(\tau)$, $C_M(\tau)$ are integral terms representing the lift and pitching moment coefficients. By introducing four new variables, the integro-differential system can be reformulated [1] as a system of nonlinear differential equations given in (13).

In the present case study, we consider that the structural nonlinearity represented by a freeplay model is imposed in the pitch-degree-of-freedom, namely $M(\alpha)$ is given by

$$M(\alpha) = \begin{cases} M_0 + \alpha - \alpha_f\,, & \text{if } \alpha < \alpha_f \\ M_0 + M_f(\alpha - \alpha_f)\,, & \text{if } \alpha_f \le \alpha \le \alpha_f + \delta \\ M_0 + \alpha - \alpha_f + \delta(M_f - 1)\,, & \text{if } \alpha > \alpha_f + \delta \end{cases} \tag{17}$$

where M_0, δ, α_f, and M_f are the freeplay constants. A linear spring is imposed in the plunge-degree-of-freedom, $G(\xi) = \xi$.

In order to generate the input data set, the nonlinear differential system (13) is numerically solved using a fourth-order Runge-Kutta time integration scheme. The parameters of the system are chosen to correspond to limit cycle oscillations. A typical input data set usually contains 150-430 transient observations, depending on the method used for prediction. The training set is formed with the majority of these data, and the test set contains the remaining observations.

In the following figures, the x- axis displays the non-dimensional time and the y- axis the pitch angle measured in radians, or the non-dimensional plunging deflection.

In Fig. 2, we display a typical time-series for the pitch motion. Ignoring the first 100 data, and taking the remaining 300 data points – as indicated by the two vertical lines – as the training set for the neural network, an excellent prediction is obtained. In Fig. 2, the blue line corresponds to the simulated pitch motion, and the red line denotes the neural network prediction. In the present neural network, the input consists of 75 data points and 25 neurons are selected for the hidden layer. The network performance is not sensitive when the input is varied from 50 to 100 data points, and the number of hidden layer neurons is varied from 10 to 30. The neural network approach can be regarded as a "black-box" tool, once the network architecture and training algorithm are chosen, the user can obtain the long-term prediction by using various numbers of inputs and by varying the number of neurons in the hidden layer. Another attractive feature in applying a neural network is that it is capable to deal with noisy data directly. By introducing a noise with signal to noise ratio 5, the noisy pitch motion is shown in Fig. 3. Using the same numbers of noisy data as network training set, the neural network prediction is illustrated in Fig. 3. The results reported in Fig. 3 also clearly suggest that the neural network is very effective in filtering the noise components from a given noisy data.

The nonlinear time-series EXPAR model based is now applied to the clean pitch motion. The model is selected with polynomials of degree three, $\gamma = 16.3$ and p=16. The training set is taken from data between 40 to 219 as indicated by the two vertical lines. The performance of the EXPAR model is stable to small variation of γ. The EXPAR model requires less training data compared to the neural network approach, and it gives an excellent prediction. However, the method can not deal with data corrupted with noise. For the noisy pitch motion shown in Fig. 2, a wavelet de-noise procedure is first applied to filter the noise. An EXPAR model can then be applied to the de-noised pitch motion for a long-term prediction. However, it is noted that more training data is required when compared to the application to clean data. The results are shown in Fig. 5, where blue represents the wavelet de-noised pitch motion, and red denotes the EXPAR model predictions. The predicted motion is in good agreement with the de-noised signal. A combined SETAR-EXPAR model has also been tested, and the results are similar to those reported in Figs. 4 and 5.

Unlike the neural networks and non-linear time series models, where only the discrete data is needed as input, more information is required when the unscented filter (UF) is used. In particular, the basic mathematical model must be known, but the values of the system coefficients are not required. However, the UF method can be applied directly to the noisy data, and it can also be used to estimate the system parameters. The UF predicted pitch motion (red) is compared with the simulated noisy data (blue) shown in Fig. 6. Clearly, an excellent prediction is achieved. It is also of interest to note that the UF also provides a good prediction for the derivative of the pitch motion.

In Figs. 7 to 11, we show the corresponding results when the three different methods are applied to the simulated plunge motion. The predicted motions are in reasonably good agreement with the simulated plunge motion. However, we observe that the long-term predictions indicate some errors due to the phase shift. The errors are more noticeable when the neural network is applied to the noisy data. An improved neural network is currently being studied, which may considerably reduce the phase errors.

The developed EDMS has also been tested on data resulted from nonlinear aeroelastic systems with structural nonlinearity represented by cubic spring and freeplay models. Due to the space limitations, these results will not be reported in this paper. In all case studies, it has been demonstrated by taking a short-term data set as the training set that the EDMS provides accurate prediction for limit cycle oscillations in the pitch and plunge motions. Moreover, it is also capable to predict stable damped oscillations and unstable divergent motions. The neural network can also be designed to extract important features of the nonlinear dynamics. It has been demonstrated by Wong et al. [15] that when a neural network is used in conjunction with a wavelet decomposition software, the frequencies and damping ratios of a damped oscillation can be efficiently and accurately estimated.

5 Concluding Remarks

Accurate and reliable predictions of limit cycle oscillations and other complex dynamic responses of nonlinear aeroelastic systems are important in practical applications. In this paper, we report the development of an expert data mining system (EDMS), in which neural networks, nonlinear time-series models and the unscented filter are proposed as the key component for performing a nonlinear dynamic prediction. The neural network approach is attractive, since it can be considered as a "black-box" tool and it is capable to deal with noisy data directly. However, it is difficult to analyze the accuracy of the resulted predictions. This approach usually requires more training data compared to the other two methods discussed in this paper. The nonlinear time-series models are effective, and need less data for training. However, their performance is significantly affected if the data is contaminated by noise. For the noisy data, a de-noising procedure is first needed to filter the noise. In comparison with the results given by the above two methods, the unscented filter produces more accurate predictions for all problems tested. This technique can deal with noisy data. Moreover, it can be used to estimate the unknown system parameters and to predict the hidden variables such as the derivatives of the pitch and plunge motions. However, the mathematical model for the system under investigation must be available in order to apply the unscented filter. The other approaches using neural networks and nonlinear time series models analyze the nonlinear system dynamics only with the given short-term data as input, and no other information is required.

The EDMS has been implemented into a computer software program, and the preliminary case studies demonstrate that the developed system is capable to provide accurate nonlinear predictions such as limit cycle oscillations, stable and unstable oscillations, when only a short-term data set is taken as input to the EDMS.

The goal of the present study is to introduce a new approach – EDMS – which may have a lot to offer to the aeroelasticity community. Even though the initial assessment of the EDMS performance is encouraging, much more research is needed. Not only we are currently working to improve our predictions using various techniques presented in this paper, but the challenge in our future work is to understand how and what accuracy we can expect from our nonlinear aeroelastic predictions using the proposed EDMS.

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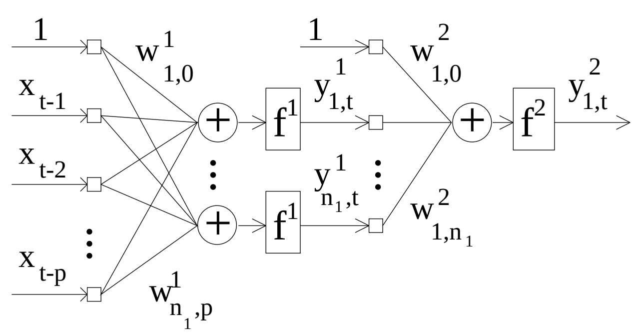

Fig. 1 Two-Layer Neural Network

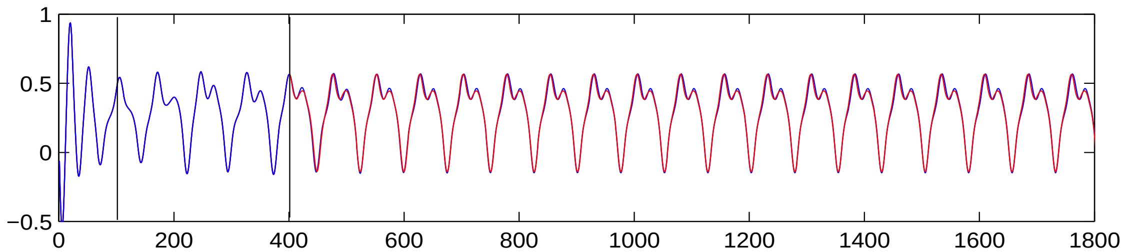

Fig. 2 Pitch motion and ANN prediction (75-25-1 net, train 101-400)

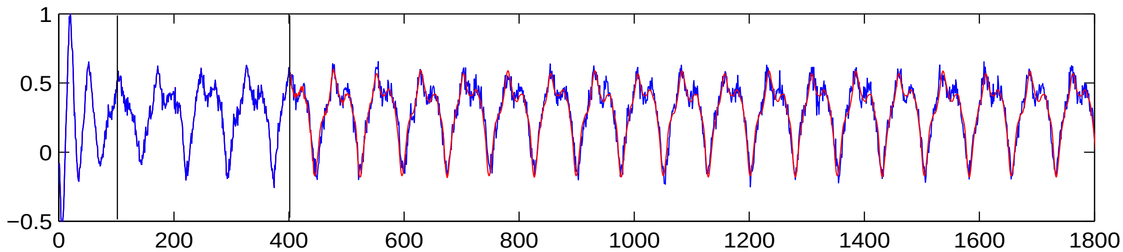

Fig. 3 Noisy pitch motion and ANN prediction (75-25-1 net, train 101-400)

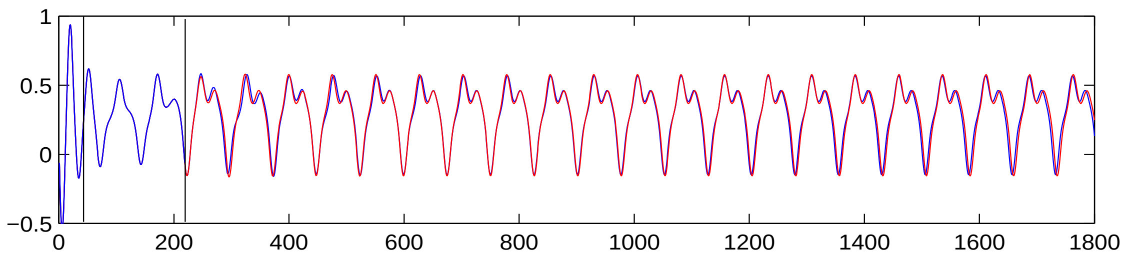

Fig. 4 Pitch motion and EXPAR model prediction (train 40-219)

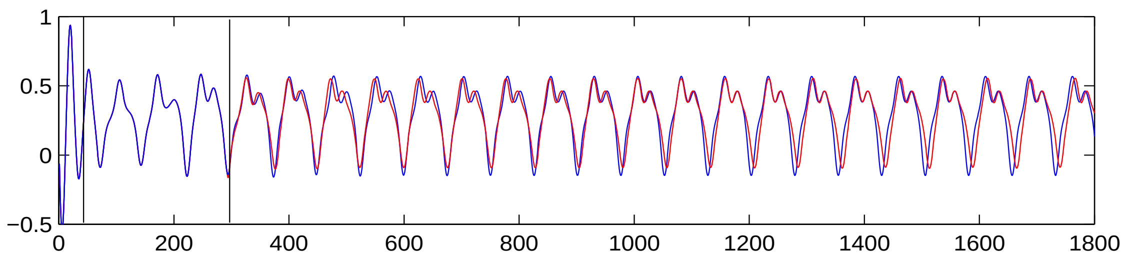

Fig. 5 De-noised pitch motion and EXPAR model prediction (train 40-295)

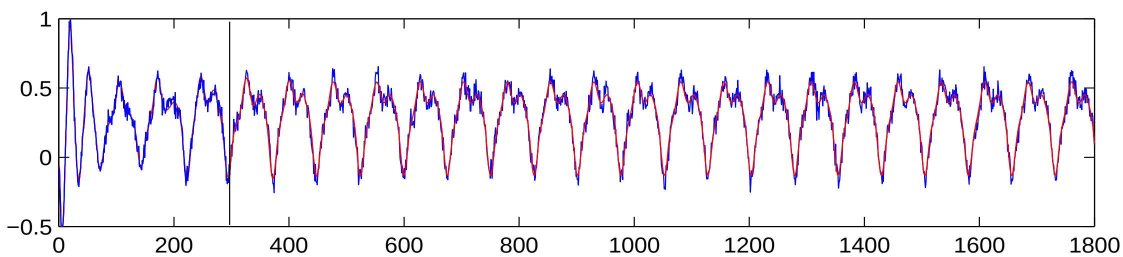

Fig. 6 Noisy pitch motion and UF prediction (train 1-295)

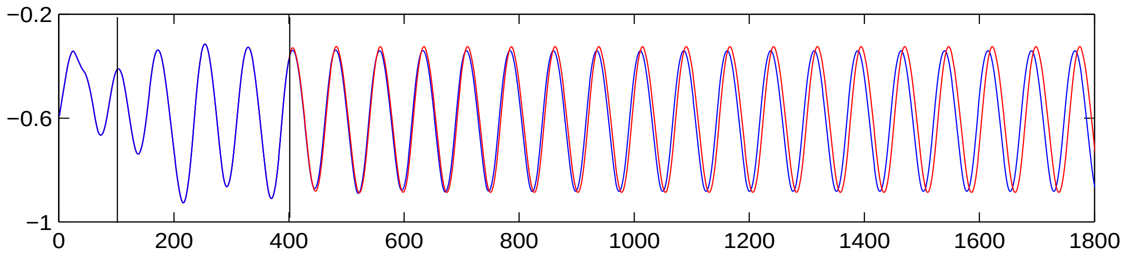

Fig. 7 Plunge motion and ANN prediction (75-25-1 net, train 101-400)

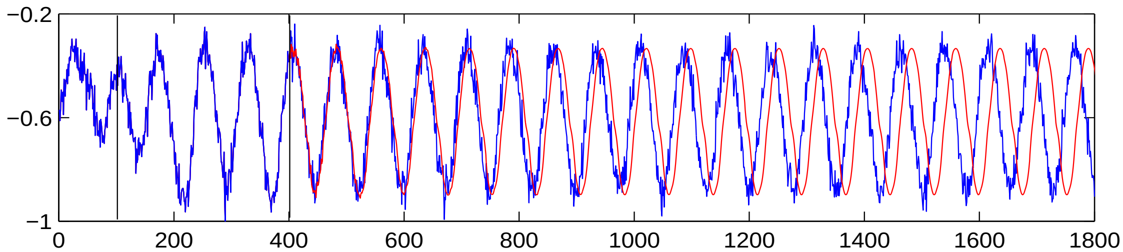

Fig. 8 Noisy plunge motion and ANN prediction (75-25-1 net, train 101-400)

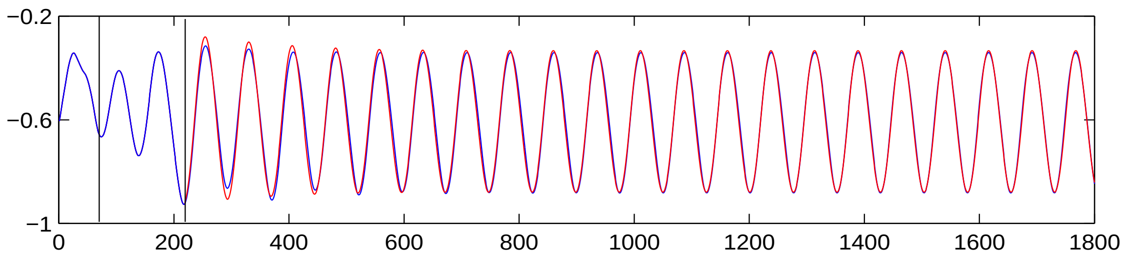

Fig. 9 Plunge motion and EXPAR model prediction (train 70-219)

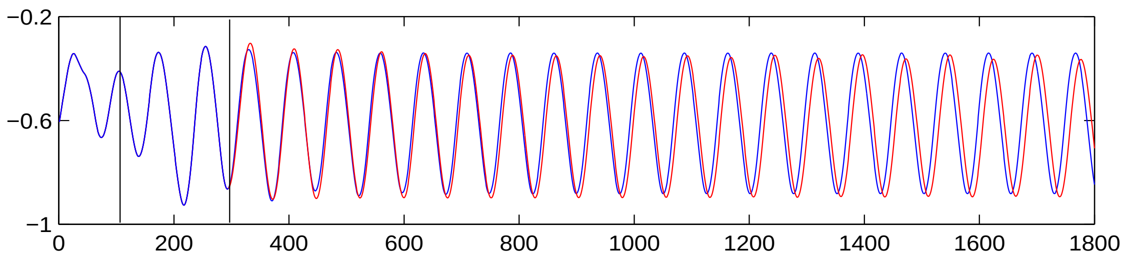

Fig. 10 De-noised plunge motion and EXPAR model prediction (train 105-295)

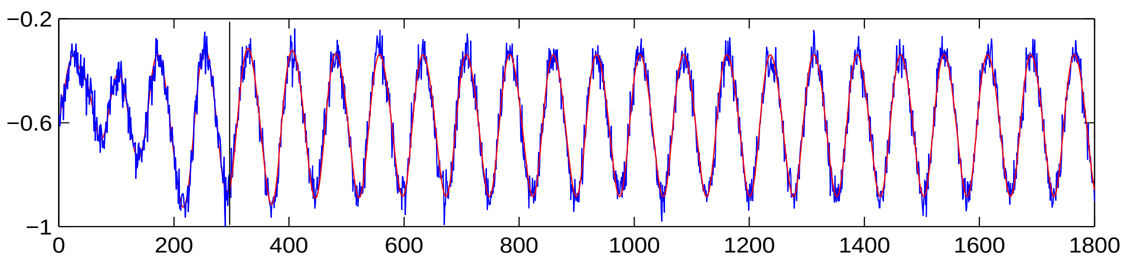

Fig. 11 Noisy plunge motion and UF prediction (train 1-295)