MODELING OF FULL-SCALE AIRCRAFT STRUCTURAL TESTS

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Keywords: *aircraft, control, fatigue, full-scale testing, modeling, servo-hydraulic, simulation*

Abstract

Full-scale aircraft structural fatigue tests are extremely complex from a control systems viewpoint. There are usually a large number of actuators with significant interactions between them. Control is made more difficult because the tests are run in load control and the load cells usually move with the actuators. A linear state space model of a single channel structural test was previously developed and experimentally verified as an aid to better understand these tests.

This paper presents an expansion of the model to a multi-channel configuration. A model of a cantilever beam with two actuators is developed as an example and the frequency response of the system presented. The features of the responses are discussed and related to the physical system. The implementation of the state space models within a simulation package with a graphical user interface is also described. This makes the models more useful to test engineers.

1 Introduction

Full-scale aircraft structural tests are extremely complex from a control systems viewpoint. There are usually a large number of actuators and the response of any one actuator depends on the control parameters, the actuator dimensions, the servo-valve characteristics, the other actuators, the structure and the loading attachments and reaction fixtures. The situation is further complicated because these tests are run in load control and it is usually necessary to attach the load cells to the moving actuator rod rather than fixing them to the reaction structure

as in load frame applications. The feedback disturbances caused by the moving load cell significantly reduce the allowable gain that can be used and make test control more difficult [1].

There are also pressures on the test engineer to both set up and complete the test more quickly, in spite of the increasing complexity of the test spectra and loading systems. Thus the need has arisen for tools to help the test engineer better understand the test system and be able to predict the influences of various changes that he can make during the test design phase.

Some earlier work [2] documented the development and experimental verification of a linear state space model of a single channel structural test. It was shown to be sufficiently accurate to allow a greater understanding of these systems and predict the relative stability of different systems. This paper presents the expansion of the model to a multi-channel configuration using modal analysis concepts. A simple example of a cantilever beam is developed and example frequency responses are presented and discussed.

For the modeling to be useful to test engineers, it must be possible to build system models without a detailed knowledge of the state space equations of the subsystems. The incorporation of the state space equations within a simulation package that provides a graphical user interface for building models as block diagrams is discussed.

2 Structural Test System Model

The structural test system model developed in [2] consisted of individual linear state space

models of a servo-valve, an actuator and a simple structure. These were then combined together, as shown in Figure 1, using standard analysis tools to form a single state space system. The structure in this case consisted of a spring, mass and damper.
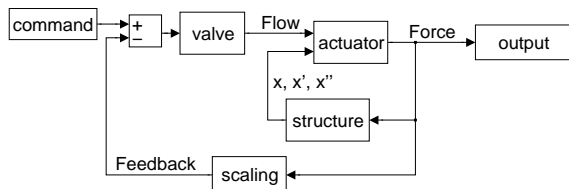

Fig. 1. Model of single actuator test system.

It was noted in [2] that expansion to a multi-channel configuration simply required the state space modeling of the more complex structure.

3 State Space Model of Generalized Structure

Rather than attempting to build the more complex structure in terms of additional springs and masses, it is more appropriate to make use of dynamic finite element models that often exist for the test structure, particularly for aircraft structural tests. In addition, solution techniques are simpler if the model can be formulated in terms of independent differential equations. Thus the generalized structure is modeled using modal analysis concepts.

3.1 Modal Analysis Concepts

The underlying assumption used in modal analysis is that the structural response of any system can be considered as a linear combination of the response to each natural mode. Thus if all the mode shapes of a structure are known, together with the proportion of each mode in response to a forcing function, the combined shape or displacements can be determined. The mode shapes and natural frequencies can be determined either analytically or experimentally. Determination of the proportions of the different modes for a given forcing function can be achieved by use of the modal transformation matrix. This allows

the equations of motion of the system to be separated into a series of independent differential equations, one for every natural mode of the structure. Then the forcing function can be transformed to provide the forcing function for every mode, the solutions obtained for each mode independently and then the total response calculated by combining the contributions from each mode.

3.2 Generalized Structure

A general structure can be described in terms of modal forces, N, and modal coordinates, η, by the equations [3]

$$N_i = \ddot{\eta}_i + 2\zeta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i \qquad (1)$$

where ω_i is the natural frequency and ζ_i is the structural damping for mode i.

To make use of this modal representation, two transformations are required, one to transform the applied forces, F, into modal forces and another to turn the modal displacements back into standard displacements, y. These can be written [3]

$$\begin{aligned} \{N\} &= [U]^T\{F\} \\ \{y\} &= [U]\{\eta\} \end{aligned} \qquad (2)$$

where *[U]* is the modal transformation matrix and can be obtained from the mass normalized modal vectors, typically obtained from a dynamic finite element analysis. The transformation matrix is only required for locations of interest (i.e. actuator locations) and for modes of interest.

3.3 State Space Representation

Assuming that the modal displacements are the only required outputs, equation (1) can be written in state space form as

$$\begin{aligned} \{\dot{H}\} &= [A]\{H\} + [B]\{N\} \\ \{\eta\} &= [C]\{H\} + [D]\{N\} \end{aligned} \qquad (3)$$

where, for 3 modes, for example

$$\{H\} = \begin{Bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{\eta}_3 \end{Bmatrix} \qquad (4)$$

and the state space matrices, A, B, C, and D are defined in [4].

Applying the transformations of equation (2), the system can be described in terms of applied forces and displacements directly as

$$\{\dot{H}\} = [A]\{H\} + [B][U]^T\{F\}$$
$$\{y\} = [U][C]\{H\} + [D][U]^T\{F\} \qquad (5)$$

The structure can thus be treated in the same manner as in [2].

4 Single Actuator Cantilever Beam System

4.1 Cantilever Beam

The natural frequencies and mass normalized displacements of a cantilever beam can be obtained analytically. It is therefore a simple matter to derive the state space matrices for a cantilever beam with actuators at defined locations as shown in [4].

4.2 System Model

The model for a single actuator cantilever beam test system is developed by combining the cantilever beam model with the valve and actuator models to form a simple closed loop system as in the block diagram of Figure 1. This is identical to the single actuator system of [2]. A single state space system can then be obtained and analyzed using standard software tools, e.g. [5,6].

4.3 Model Parameters

The parameters for the valve and actuator models were for the 9.5 l/min valve and 35 cm stroke, 25 kN actuator of Reference 2. The cantilever beam parameters were defined so that the beam stiffness at the free end was the same

as for the spring of Reference 2 and the first natural frequency was the same as for the spring/mass structure. The first four natural frequencies for the beam were about 27, 170, 476 and 930 Hz. The natural frequency of each actuator, considered as a mass on the oil column spring, was 154 Hz.

4.3 Frequency Response

Figure 2 shows the predicted response for the system of Reference 2 without including the effects of the digital controller, together with the equivalent response for the cantilever beam with a single actuator at the tip.
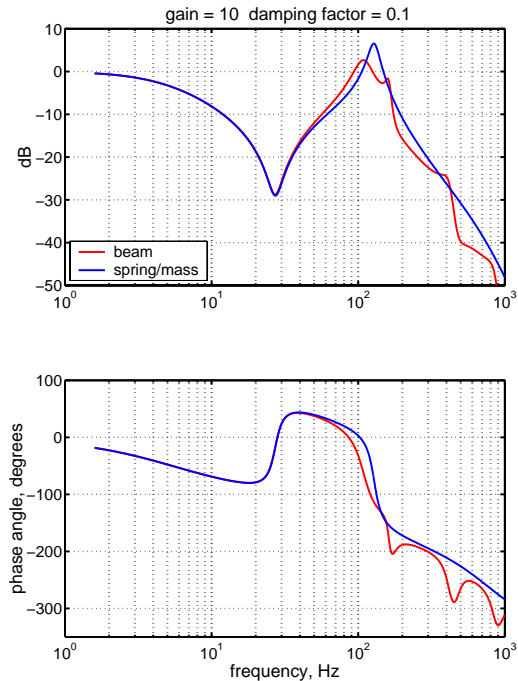

Fig. 2. Comparison of Bode plots for spring/mass system and equivalent cantilever beam system.

The zero is at the same frequency since the structure appears the same to the actuator system. However, the pole is reduced in both magnitude and frequency because this is primarily a function of the natural frequency of the complete system (structure plus actuator system) and this will be affected by the distributed mass of the beam. There are additional zeros at the other natural frequencies of the beam. These are shown more clearly in

Figure 3, which is for the same system but with lower structural damping. Three additional poles can be observed in this figure, which are related to the additional natural frequencies of the total system and relate to resonance of the total system.
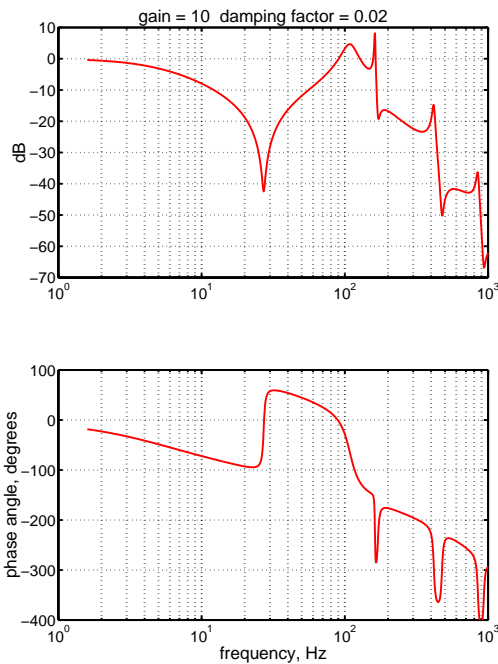

Fig. 3. Bode plot for cantilever beam system with reduced damping.

5 Two-Actuator Cantilever Beam Test System

5.1 Model

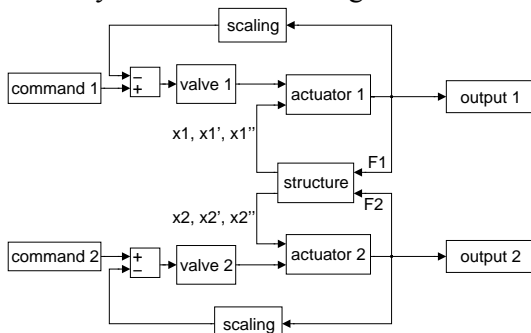The model for a multi-actuator test system is developed in a similar manner to the single actuator system as shown in Figure 4.

Fig. 4. Model of two-actuator beam test system.

The blocks are grouped together and reduced to a single state space system using standard software analysis tools.

5.2 Model Parameters

The cantilever beam parameters were defined as above but with actuators at both the tip and mid span. The damping was maintained at 0.02 so that the various poles and zeros were clearer. The parameters for the two actuators and valves were the same as used for the single actuator.

5.3 Frequency Response

Figure 5 shows the response at the tip and mid span to an input at the tip, while Figure 6 shows the responses due to an input at the mid span.
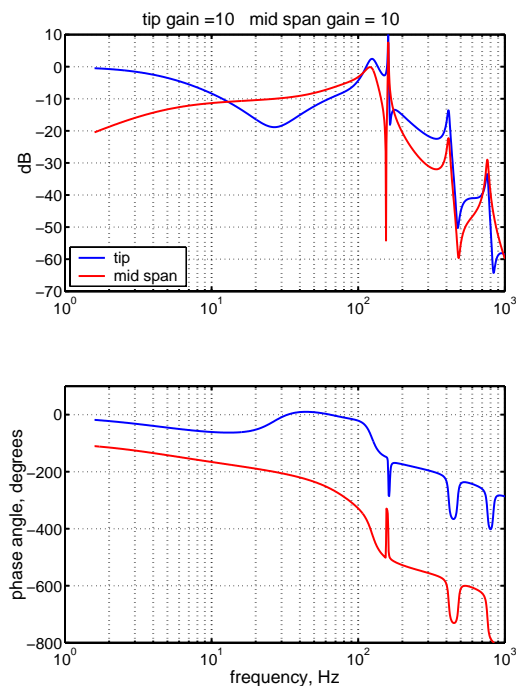

Fig. 5. Bode plot for two-actuator cantilever beam system with an input to the tip.

For an input to the tip, the response at the tip is very similar to the single actuator case shown in Figure 3, although the presence of the mid span actuator clearly adds significant damping as the zero is much flatter. Since the mid span actuator is controlling at zero load, the tip actuator effectively sees simply the beam on

its own and so the frequencies of the zeros are simply the natural frequencies of the beam.
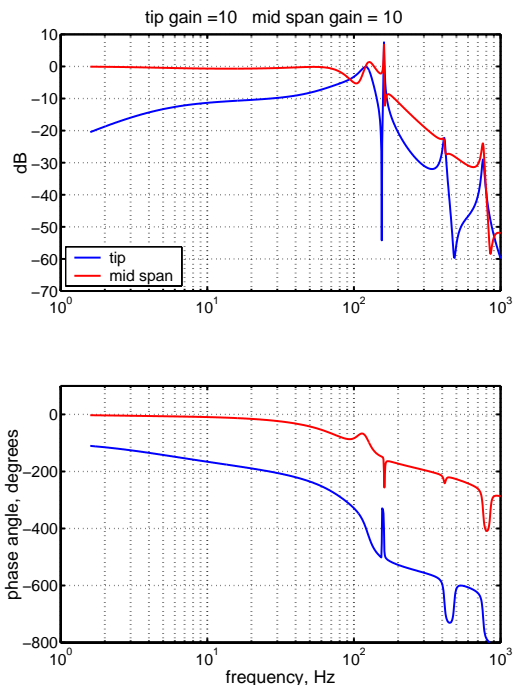

Fig. 6. Bode plot for two-actuator cantilever beam system with an input to the mid span.
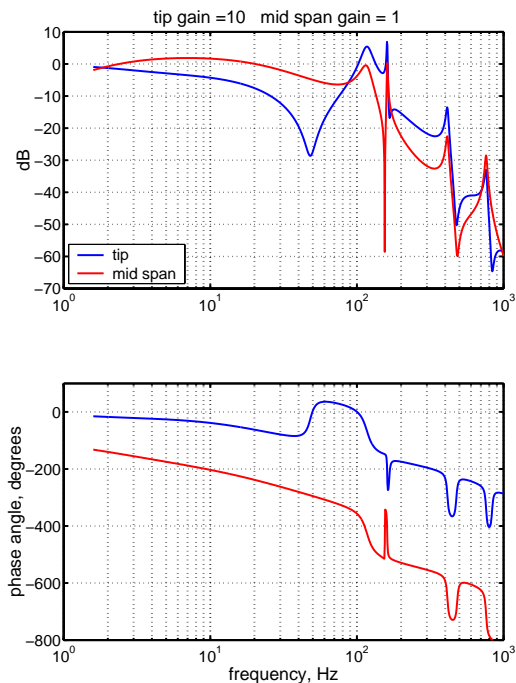

Fig. 7. Bode plot for an input to the tip with decreased mid span gain.

However, if the gain on the mid span actuator channel is reduced, the mid span actuator adds stiffness to the system that the tip actuator sees and the frequency of the zero associated with the first natural frequency is increased as shown in Figure 7. There are no observable differences at the other zeros as these natural frequencies are not altered much by the addition of the mid span actuator.

It is interesting to note that the decrease in gain on the mid span actuator channel increases the magnitude of the first pole of the tip channel. This illustrates one of the difficulties in tuning a multi-channel test of this type since changing the gain on one channel can result in a change in stability on another. The cross coupling effects are also evident in Figure 7 which shows the relatively large response at the mid span for an input to the tip. This may be compared with the frequency response of Figure 8 where the gain on the mid span channel has been increased. In this case the cross coupling effect is significantly reduced as shown by the minimal response at the mid span for a tip input.
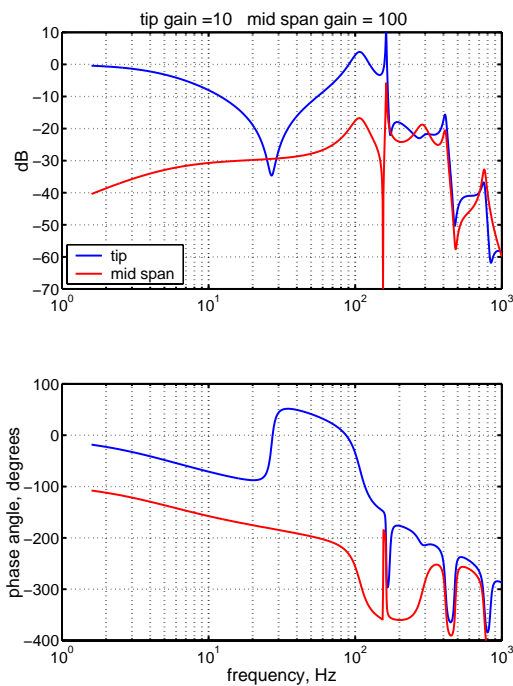

Fig. 8. Bode plot for an input to the tip with increased mid span gain.

The response at the mid span for an input at the tip in Figure 5 exhibits two zeros and four poles. The poles occur at the same frequency as for the tip response since the poles for the tip response are caused by resonance of the total system. The first zero is at the natural frequency of the actuator. At the natural frequency of the actuator, it would be difficult to generate a load in that actuator since no load is required for displacement at the natural frequency. The second zero is at the third natural frequency of the beam because at this frequency, there is a node at the mid span and therefore no load will be transmitted to the actuator.
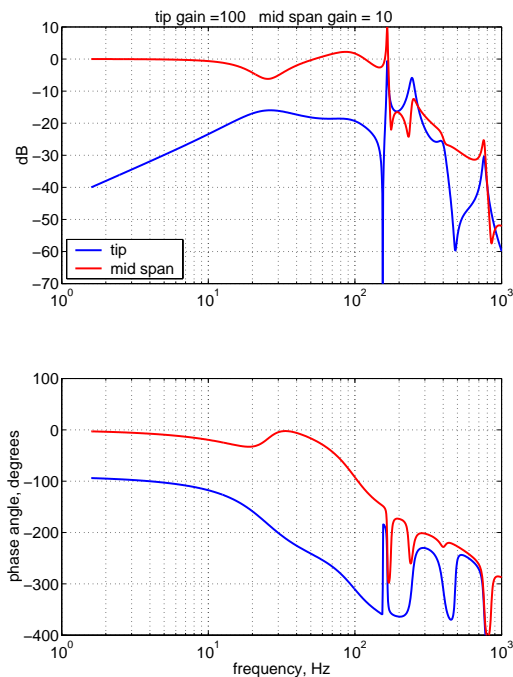

Fig. 9. Bode plot for an input to the mid span with increased tip gain.

The frequency responses to an input at the mid span, shown in Figure 7, have a similar form to those for the tip input. The mid span response shows four poles and four zeros. The first zero is at a frequency considerably higher than the first natural frequency of the beam as the tip actuator has a larger stiffening effect than the mid span actuator. This is confirmed in Figure 9, which shows the effect of increasing

the gain on the tip channel. The first zero now lies closer to the beam first natural frequency. The poles are again at the natural frequencies of the total system.

The response at the tip shows two zeros and four poles and is very similar to the response of the mid span to an input at the tip. The poles, as expected, are at the frequencies of the total system. The first zero is at the natural frequency of the actuator and the second is at the third natural frequency of the beam. It is not possible to excite the beam at this frequency at the mid span location because it is a node. Thus, there can be no response at the tip.

6 Use of the Model for Test Simulation

The structural test system models used above consisted of individual linear state space models of servo-valves, actuators and structures that were combined, using standard Matlab analysis tools, to form a single state space system. While this system is extremely efficient in terms of computing time, it requires a detailed knowledge of Matlab and the Control System Toolbox and the building of new system models using the basic component models is both time consuming and tedious. For the modeling to be useful to test engineers, it must be possible to build and use system models in a much simpler manner without the necessity of a detailed knowledge of Matlab.

6.1 Implementation within Simulink

Simulink [7] provides a graphical user interface for building models as block diagrams, using click-and-drag mouse operations. Thus, if the model elements previously developed in Matlab are converted into Simulink blocks, the user can build system models by simply connecting the relevant blocks using the mouse.

There are several ways of implementing model elements within Simulink. However, the most efficient system is one where the state space matrices for each component are calculated only at the start of simulation from a list of basic parameters. This has been achieved using model callback routines that run a script
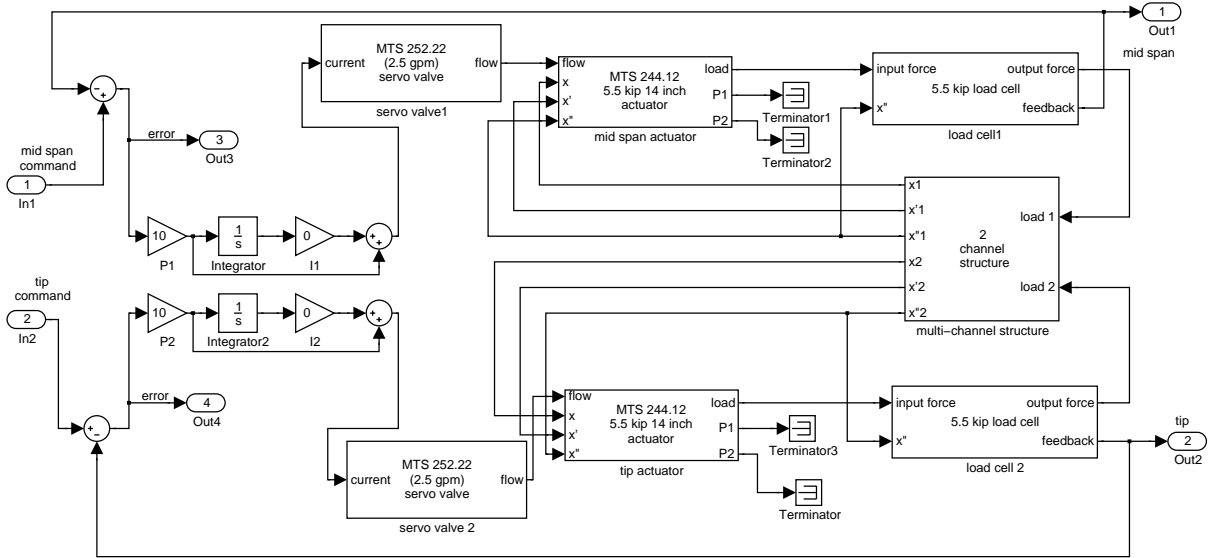
Fig. 10. Simulink model of two-actuator test system.

when the block is opened or at the start of simulation. The scripts were written to calculate the state space parameters and then load these parameters into the Simulink state space block. Each block has a specific icon and dialog box that allows the user to change a limited number of parameters.

Figure 10 shows a Simulink model of the two-actuator cantilever beam example used earlier, with basic inputs and outputs. By using a signal generator as an input and an oscilloscope as an output, the user could simulate the response to a test load spectrum. However, this is not very efficient because, although all the individual blocks are linear, Simulink does not take full advantage of the linearity.

6.2 Improving Simulation Speed

The simulation time can be reduced by a factor of about 50 by extracting the linear model. This is achieved using a simple Matlab command that places the state space matrices of the complete system in the workspace. A new model with a single state space block as shown in Figure 11 is then used, with the matrices within the block defined to use those in the workspace. These matrices are the same ones as would be derived using the earlier method.
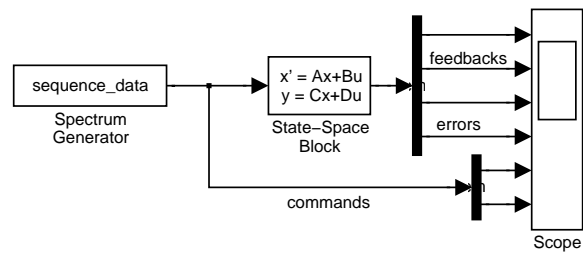

Fig. 11. Single state space block model of two actuator system.

6.3 Example Test Simulation

One of the most difficult tasks in setting up and running a full scale structural test is defining the segment times, i.e. the time between end points. Segment times are often set by assuming they are limited by the flow into the slowest actuator, where the flow is calculated as the nominal valve flow at full opening multiplied by the error and the proportional gain [8]. To examine how well this works, the model was used to determine the simulated response of the two-actuator cantilever beam system to a variable amplitude loading sequence.

The loading sequence consisted of a series of equal loads applied on the two actuators, followed by a series of opposing loads on the

two actuators. The time between endpoints was based on simple flow requirements for the critical actuator, the tip actuator in this case. As a result, the maximum errors for the tip actuator, as shown in Figure 12, are constant at about 3% and occur at the mid point of the segment.
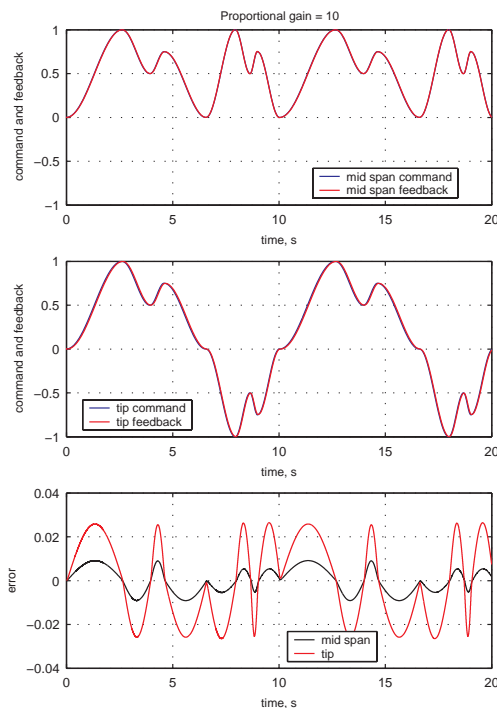

Fig. 12. Simulation for variable amplitude sequence with a proportional gain of 10.

Since this system becomes unstable if the proportional gain is increased significantly, it is common practice to use integral gain to reduce the errors. Figure 13 shows the effect of adding an integral gain of 25.

The maximum error has been reduced to about 1%, but it is no longer constant for each segment. Thus the times between end points for this sequence are no longer optimum, since for a 1% error, many of the segments could be run faster.

However, being able to run a model in advance of the test allows a test engineer to check how well a test may run and see the effect of modifying segment times on the overall test times.

With further experimental validation of additional valves and actuators and the multi-channel structure, it may also be possible for the test engineer to experiment with hardware components, such as different valve sizes, in addition to control parameters, prior to the test. It may then be possible to optimize the test set up in the design phase.
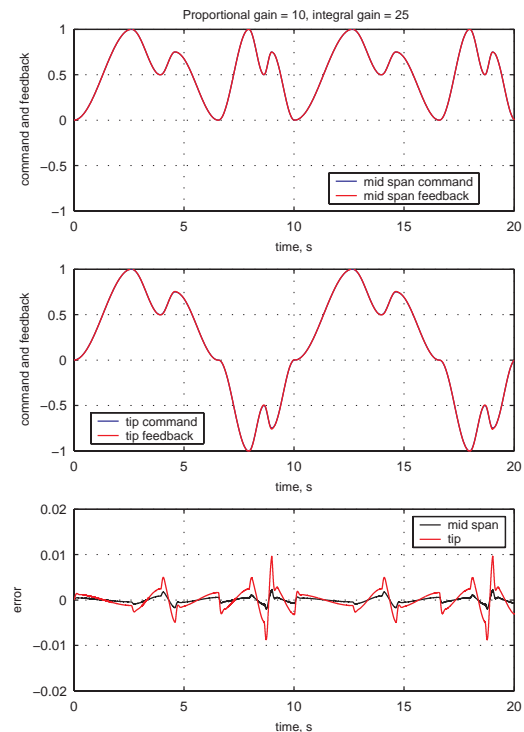

Fig. 13. Simulation for variable amplitude sequence with a proportional gain of 10 and integral gain of 25.

7 Conclusions

The extension of a state space model of a single actuator, moving load cell structural test to a multi-channel test has been described using modal analysis concepts. An example using an analytically determined cantilever beam model has been presented and the frequency responses obtained for the example system.

For a response at the same location as the input, there are poles at the natural frequencies of the total system (structure plus all actuator systems) and zeros at the natural frequencies of the system that is being driven (structure plus

actuator systems other than driving actuator). If the gain on the other actuator systems is sufficiently large, they are effectively de-coupled in terms of the driving actuator, and the zeros are at the natural frequencies of the structure.

For a response at an actuator location other than the input location, there are again poles at the system natural frequencies. There is also a zero at the natural frequency of the actuator at the response location and an additional zero if the actuator is attached at a node of the structure.

The implementation of the state space models within Simulink has been described briefly. This makes the models simpler to build and use once the basic component blocks have been developed and makes them more accessible to test engineers.

The multi-actuator model provides a useful tool for understanding the behaviour of multi-actuator test systems and the interaction between actuators. With further experimental validation, it may also be useful for predicting system behaviour during the design phase of a structural test.

Acknowledgements

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