RELATIONSHIP BETWEEN INSTABILITY WAVES AND NOISE OF LOW SUPERSONIC JETS

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Keywords: *stability theory, axisymmetric jet, low supersonic flow*

Abstract

The paper is concerned with the transition from dissipative to dispersive cross-stream behaviour of large turbulence structures outside low supersonic axisymmetric cold jets. Such a transition arises in regions of physical space where noise is generated and depends to a large extent on the jet Mach number. It is shown how the issue of determining these regions can be reduced to analysing the pressure near field in terms of a turning-point problem. Physically, at these low supersonic speeds the radiated sound field associated with axisymmetric instability waves is not an insignificant part of the total phenomenon, even if their phase velocities are subsonic. Low frequency instability waves extent from the mixing layer all the way to the far field.

1 Introduction

The far field noise of jets has received considerable attention in the 1960's. With recent renewed interest in high-speed civil transport, there is an urgency to gain a still better understanding of noise generation mechanism.

It is now generally accepted that turbulent mixing noise of circular high-speed jets consists of two distinct components. One component radiates principally in the downstream direction. This is consistent with Mach wave radiation from the large turbulence structures of the jet flow. The characteristics of the other component that is dominant in the upstream direction suggest that it is the noise from the fine scale turbulence of the jet flow (see Tam[12]). Presently a consensus seems to have emerged that the noise generated directly by the large turbulence structures most probably constitutes the principal part of the turbulent mixing noise component. More recently, Tam (1998) extended the same conclusions to non-axisymmetric jets including jets from rectangular, elliptic, plug and suppressor nozzles.

The shape distribution of the large turbulence structures responds to the local profiles of the mean shear flows and may be calculated by using the linear stability theory. In the sequel x, r and t denote the streamwise direction, cross-stream direction and time, respectively. From a stability point of view, the most elementary approach consists in entirely neglecting the spatial development of the medium : if the basic flow is assumed to be uniform in the streamwise direction, the stability properties of normal modes are completely described by a dispersion relation. That is, pressure fluctuations are typically decomposed into elementary instability waves of the form $\phi(r)\exp(i(kx-\omega t))$ of complex wavenumber k and complex frequency ω. The cross-stream distribution $\phi(r)$ is then shown to satisfy an ordinary differential equation. Enforcement of appropriate boundary conditions then leads to an eigenvalue problem whereby eigenfunctions $\phi(r)$ exist only if k and ω are constrained to satisfy a dispersion relation $D(k,\omega;R)=0$, where R denotes a set of control parameters such as the jet Mach number or the jet temperature. If the flow is convectively unstable, transients will gradually move away from the source. It follows that the asymptotic ($t \to \infty$) flow response to a localized harmonic forcing of real frequency ω_f reduces to the steady-state signal arising from the poles of its Fourier transform in space and time (see Huerre and Monkewitz[5]). These poles are readily identified as the zeroes of $D(k,\omega_f;R)$, i.e. the spatial eigenvalues $k(\omega_f;R)$. However, for the above reasoning to apply, the Fourier transform must be analytic in k (except at points where the dispersion relation holds). In the case of a supersonic jet, it is possible to show that compressibility effects give rise to branch cuts in the complex k-plane (Tam and Burton[13]) corresponding to regions where the Fourier transform is non-analytic. This non-analycity stems from the boundary conditions far away from the centre of the jet, where $\phi(r)$ behaves as a multi-valued cylinder function. Accordingly, the cross-stream structure of instability waves may not be always compatible with boundary conditions as ω is displaced along the real axis.

Most supersonic jets are spatially non uniform in the streamwise direction as a result of turbulent (eddy) viscosity. Here, the fine-grained turbulence plays an indirect but crucial role in that it controls the development of the coherent structures. The previous notions can then taken to apply locally in x, as long as the nonuniformities of the medium are small over a typical wavelength of the instability. Thus the non parallelism of the flow is characterized by a slow space scale $X = \varepsilon x$, where ε is a small parameter of the same order of magnitude as the scaled nonuniformities. Decomposition into local normal modes leads to a dispersion relation of the form $D(k,\omega;R,X) = 0$, where X plays the role of a parameter. The stability properties of the flow are then defined at a given station X in the same manner as for the uniform case. In connexion with the above discussion, we have the following issue : for a given real frequency, is it always possible to ensure boundary conditions, or equivalently, at which locations X is the problem formulated as an eigenvalue problem ?

This problem was first studied by Tam and Morris[10] and Tam and Burton[13]. They showed that the classical solution based on the locally parallel approximation is but the first term of a multiple-scales expansion of the solution of a more rigorously formulated instability wave theory. The multiple-scales expansion is not uniformly valid and consequently should not be used in the whole physical space outside the jet. Requiring a solution to satisfy an imposed boundary condition in a region where it does not represent the physical entity naturally would lead to unsurmountable difficulties. In addition Tam and Chen[11] showed that for jets with Mach number up to 2.0 and jet temperature to ambient temperature ratio up to 2.5 the most amplified wave (the helical Kelvin-Helmholtz mode) generates intense Mach wave radiation. The near-field in this case consists of both the acoustic and hydrodynamic (non- propagating) fluctuation components and may be evaluated by applying the method of matched asymptotic expansions. However, for jets operating at low supersonic Mach numbers, Millet and Casalis[9] demonstrated, through a combined analytical and numerical approach, that only the axisymmetric mode can radiate sound in the near-field of axisymmetric cold jets. As a consequence, the cross-stream behaviour of axisymmetric fluctuations is no longer exponential in the entire physical domain ; it exists a location X where the cross-stream decay of the pressure amplitude is algebraic in the near-field and a propagating wave is generated. This transition holds in a way which will be given in the present paper.

The main objective of the present article is to obtain a characterization of the cross-stream behaviour of axisymmetric disturbances for low supersonic jets. The paper is organized as follows. The basic equations are given in section 2 where we show that the issue of determining locations X where the problem cannot be expressed as an eigenvalue problem can be reduced to analysing the cross-stream structure of disturbances in the vicinity of the jet. In section 3, we study the cross-stream behaviour of axisymmetric disturbances in the complex k-plane for low supersonic jets when the phase velocity at the location of the peak amplitude is quasi-sonic. In our study we consider the case of cold supersonic jets in the Mach number range of 1.0 to 2.0. In closing, we give numerical results obtained by using a spectral collocation discretization through a multiple domain technique.

2 Basic formulation

Consider small amplitude disturbances superimposed on the mean flow of an inviscid perfeclty expanded supersonic jet. Dimensionless variables with r_0 (radius of the jet at nozzle exit), u_0 (jet exit velocity), r_0/u_0, ρ_0 (jet density at nozzle exit) and $\rho_0 u_0^2$ as the length, velocity, time, density and pressure scales will be used throughout the analysis. The Mach number of the jet and the (dimensionless) ambient gas mean density will be denoted by M and $\overline{\rho}_\infty$, respectively.

Following Tam[12], the large-scale fluctuations are mathematically represented by a linear superposition of instability modes and substituted into the equations of motion. Therefore, the standard form of the WKBJ approximation (see Bender and Orszag[1]) which describes instabilities of weakly non-parallel flows can be used. This analysis is restricted to axisymmetric jets, so that the instability waves can be Fourier-decomposed into azimuthal modes. Thus, with respect to a cylindrical coordinate system (x,r,θ) centered at the nozzle exit, to the lowest order, the solution for an instability wave of angular frequency ω and azimuthal mode number n has the form $\phi_n \exp(in\theta - i\omega t)$, with

$$\phi_n \sim A(X)\hat{\phi}_n(r;X,\omega)\exp\left(\frac{i}{\varepsilon}\int_0^X k(s;\omega)\,ds\right), \tag{1}$$

where $k(X;\omega)$ is a local wavenumber originating in the upper half of the k-plane when the imaginary part of ω goes from a large positive value to zero (for details, see Huerre and Monkewitz[5]), A is an unknown complex amplitude determined at $O(\varepsilon)$ and $\hat{\phi}_n$ is a suitably normalized function, so that

$$\hat{\phi}_n(r;X,\omega) = H_n^{(1)}(i\lambda(k(X;\omega);\omega)r), \tag{2}$$

outside the jet, where $H_n^{(1)}$ is the nth order Hankel function of the first kind and the function λ is given by the irreducible polynomial $\lambda^2 + k_0^2 - k^2 = 0$, with $k_0 = \sqrt{\overline{\rho}_\infty} M\omega$. As discussed in section 1, this mathematical description is valid only inside and in the vicinity of the jet. For large r, the appropriate variables are $X = \varepsilon x$ and $R = \varepsilon r$ as long as λ has a non-zero finite value in the limit $\varepsilon \to 0$. The intermediate solution can easily be obtained by changing the coordinates of the solution (1) to the intermediate variables, i.e. replacing r by $r_\alpha/\varepsilon^\alpha$, where $\alpha < 1$ is a small positive number. Then the intermediate limit of (2) leads to the following expansion for the Hankel function

$$\lim_{r_\alpha} \hat{\phi}_n(r;X,\omega) \sim \frac{\exp(-\lambda(k(X;\omega);\omega)r)}{\sqrt{i\lambda(k(X;\omega);\omega)r}}, \tag{3}$$

where the limit process is defined by requiring r_α fixed in the limit $\varepsilon \to 0$. Note that for (3) to be valid, the algebraic function $\lambda(k;\omega)$ must satisfy $\lambda = O(1)$ and $\lambda \neq o(1)$. That is, the asymptotic approximation (3) holds only outside neighbourhoods of branch points $\pm k_0(\omega)$.

Spatial branches are obtained from the dispersion relation $D(k,\omega;R\,,X) = 0$ by solving for complex wavenumbers k when ω is given real[1]. For cold supersonic jets in the Mach number range of 1.0 to 2.0 the large turbulence structures are associated with the well-known Kelvin-Helmholtz instability modes (see Tam et al[11]) corresponding to branches of spatial eigenvalues. If $\lambda = o(1)$, the cross-stream behaviour of $\hat{\phi}_n$ is not given by (3) and strongly depends on the azimuthal wavenumber n through the asymptotic behaviour of the Hankel function. It follows that the radiation condition may not be compatible with the assumption $r_\alpha = O(1)$, $\alpha < 1$ and, consequently, the problem may not be formulated as an eigenvalue problem for all X. Regions of zero real part of λ are known to be branch cuts in the complex k-plane. They are defined by $-\pi/2 \leq \arg\lambda \leq \pi/2$. From the radiation condition, the left (right) equality sign is to be removed if ω is negative (positive). Note that for hot supersonic jets, the jet could support another family

[1]Following Tam[12],[11], in this paper, we assume the instabilities to be convective and ignore the transient portion of the response.

of instability waves generated by the presence of a Mach wave system inside the jet (see Tam and Hu[14]). Here, we restrict the discussion to the Kelvin-Helmholtz instability.

Causality implies that there exists a local maximum growth rate for the perturbation of the mean flow, which means that the minimum of the imaginary part k_i of k (for $\omega > 0$) is everywhere defined over all X real, as shown in figure 1. In addition, on following the propagation of instability waves downstream one finds that spatial branches move into the upper half of the k-plane for ω positive. For large X, only a small segment of positive frequencies in the vicinity of the origin leads to amplified waves. Since the mean velocity of the jet gradually decreases in the flow direction, phase velocities of instability waves would eventually be supersonic ($k_r < k_0$, where k_r is the real part of k) before they reach the region of the jet where they become damped ($k_i > 0$). In such cases, the real part of λ is zero for $k_i = 0$ and the cross-stream decay of the amplitude of these neutral instability waves become algebraic in the near-field, as indicated by the asymptotic evaluation (3). The cross-stream behaviour of damped waves is then obtained by analytic continuations, which give rise to solutions which become exponentially large as $r \to \infty$. This is permissible because boundedness of solution at large r is no longer a requirement of the local solution ($r = O(1)$). Thus, at given control parameter settings, the dominant contribution of an instability wave to the far-field is given by a neighbourhood of branch cuts.

Since, in the displacement of the station X along the real axis, the spatial branches move, the cross-stream behaviour of neutral waves may change from exponential (algebraic) to algebraic (exponential) around locations corresponding to regions where their phase velocities change from subsonic (supersonic) to supersonic (subsonic). These exponential-algebraic (algebraic-exponential) transitions take place as a result of the permutation of the regular branches of the algebraic function $\lambda(k;\omega)$. Figure 1 shows a typical spatial branch for two locations X_0 and $X_1 > X_0$. For location X_0, the spatial branch is

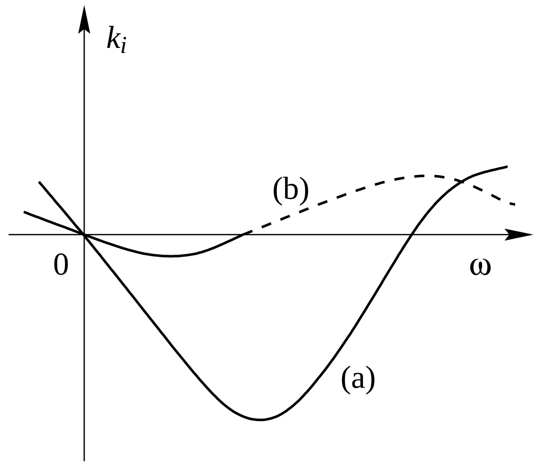

Fig. 1 Sketches of typical spatial branches. Imaginary part k_i of the local wavenumber versus frequency for the axisymmetric ($n = 0$) Kelvin-Helmholtz instability wave at two locations X_0 (a) and $X_1 > X_0$ (b). Dashed line : analytic continuation into $k_i > 0$ (corresponding to solutions which become exponentially large as $r \to \infty$).

contained in the analycity domain of the function $\hat{\phi}_n$ and, consequently, the inner solution is identical with the eigenfunction of the classical locally parallel-flow stability theory. In the other case, the one-term inner solution decays to zero as $r \to \infty$ only in small region close to the origin ($k_i < 0$ for $\omega > 0$) and becomes exponentially large as $r \to \infty$ in the complementary region. We can conclude that the exponential-algebraic transition of the neutral wave is located between X_0 and X_1. In the following section, we show that a similar transition holds in the cross-stream direction when the phase velocity of the neutral axisymmetric ($n = 0$) instability wave is subsonic, in a way which is given by a turning-point problem.

3 A turning-point problem

In this section, we study the cross-stream behaviour of axisymmetric disturbances for low supersonic cold jets. As already indicated, the cross-stream structure of local plane waves, to the lowest order, is governed by the wave equation which admits Hankel functions for solutions. Let us first $z = i\lambda r$ be a new independent variable and reduce the wave equation to its normal form by writing $\psi = \hat{\phi}_0 z^{1/2}$. Then, we have

$$\frac{\partial^2\psi}{\partial z^2} + \left(1 + \frac{1}{(2z)^2}\right)\psi = 0, \qquad (4)$$

which defines a real turning-point problem (see Bender and Orszag[1]) for locations X such that $|k_r| > k_0$ and $k_i = 0$. The turning-point is given by $z_0 = i/2$, that is, $r_0 = 1/(2\lambda)$ where λ is defined by $\lambda^2 = k_r^2 - k_0^2$. Since the cross-stream decay of the axisymmetric instability wave is given by $\hat{\phi}_0$, the turning-point gives the radial position of the dispersive-dissipative transition in physical space for the axisymmetric mode. This result is completely compatible with the asymptotic expansion (3) valid in the intermediate region : for $|k_r| > k_0$ the rapidly varying component of the solution (3) decays exponentially (dissipates) away from the jet. For $|k_r| < k_0$ and $k_i = 0$, the turning point lies in the complex r-plane and the cross-stream behaviour is dispersive for real X. As indicated by (3), the solution is wavelike with very small and slowly changing wavelengths and slowly varying amplitudes as function of r_α.

For $n \neq 0$, it is easy to prove that the turning-point is complex when k belongs to the domain obtained by removing the vertical strip $|k_r| < k_0$ from the complete k-plane. We arrive at the result, that the cross-stream behaviour of neutral instability waves with subsonic phase velocities is dissipative for all $n \neq 0$. Recall that the inequalities $|k_r| > k_0$ and $|k_r| < k_0$ must be considered in the sense of asymptotic analysis, that is, $|\lambda| = O(1)$ and $|\lambda| \neq o(1)$. If $|\lambda| = o(1)$ one might consider trying to apply perturbation methods to find the asymptotic structures. The leading-order of the equation to be solved outside the jet then reduces to the transverse part of the Laplace operator. The form of the solution no longer depends on the phase velocity but is stronger affected by the azimuthal wavenumber n. For axisymmetric instability waves, the general solution corresponds to the asymptotic form of the Hankel function for small argument, that is $\hat{\phi}_0(z) \sim \ln z$. From matching arguments, we deduce that the real turning-point becomes larger than $O(1)$ and has no physical sense. For non-axisymmetric instability waves, the solution which matches to the Hankel function is given by $\hat{\phi}_n(z) \sim 1/z^n$ for $n > 0$. Let us note in connexion with the above discussion, that the branch points $\pm k_0 = \pm\sqrt{\rho_\infty}\omega M$ go to zero as $M \to 0$ for finite values of ω and, consequently, the length of the vertical strip between the two branch points, representing waves for which the convective Mach number of the flow is supersonic, decreases until zero. Hence only axisymmetric pressure fluctuations may exhibit a dispersive behaviour for low-speed jets. In what follows, we consider only supersonic jets and exclude situations where $|\lambda| = o(1)$ by reducing the analysis to the plane of the complex variable k with the exception of small neighbourhoods of branch points.

The next section deals briefly with the numerical formulation for performing calculations of instability waves. Using the structure of solutions on a 2-sheeted Riemann surface, to every point of which correspond one value of the function $\lambda(k;\omega)$, the computational domain has been reduced to the vicinity of the jet, where $r_\alpha = O(1)$.

4 Numerical formulation

4.1 Spectral discretization

The approximate solution requires two steps. First the operator associated with the linearized equations is converted to a matrix. In the present case the discretized system is obtained by a spectral collocation discretization through a multiple domain technique (see Khorrami et al[6], Malik[8]). Such a method can be applied with no modification when the viscosity effects are taken into account (the Reynolds number is finite). None of the above boundary conditions arises directly from the effect of viscocity, and are formally the same in an inviscid fluid. Thus, the physical domain is splitted into several domains. Each domain preserves the advantages of spectral collocation and allows the ratio of the mesh spacings between regions to be several orders of magnitude higher than allowable in a single domain. In the second step the resulting nonlinear eigenvalue problem is converted to a generalized eigenvalue problem by using the linear companion matrix method (Bridges and Morris[3]) at axial locations X where homogeneous boundary conditions can be applied. Then, the eigenvalues are obtained by using standard algorithms. In the reverse case, we use a local method : only the eigenvalue which happens to lie in the neighbourhood of a guessed value is computed using iterative techniques such as Newton's method. Thereafter, the wavenumber solution k at the previous location X is used as the initial guess for the wavenumber at the next axial location. Extrapolating the first guessed wavenumber at the next location from previous values often speeds up the convergence. For X close to the nozzle exit, numerical results have shown that the spectrum of the discretized operator is completely compatible with the set of eigenvalues obtained with the vortex sheet model (see Tam and Hu[14]).

To simplify the necessary calculations one can use the inviscid stability theory (the Reynolds number is infinite). In order to continue the inviscid stability calculations into the damped region ($k_i > 0$), it is well-known that a contour deformation must be made into the complex r-plane to avoid large viscous regions. Unfortunately, this stretching transformation leads to a new distribution of nodes along the complex contour and particularly around the critical point. Following anew the multi-domain spectral collocation method, the domain which contains the critical point is divided into two domains in which the standard collocation points are the Gauss-Lobatto points.

4.2 Mean-flow profile

Before numerical calculations can be performed, it is necessary to provide a description of the mean velocity and density profiles in the jet. The mean velocity profiles are taken from experimental measurements of perfectly expanded supersonic jets obtained by Lau *et al.*[7] whose data were fitted by an error function profile and Troutt and McLaughlin[16] who fitted data by a half-Gaussian profile. It is known that a supersonic jet can be divided into three regions, i.e. the core, the transition and the fully developed regions according to the characteristics of the mean velocity profile. Here, we restrict the analysis to the core region where the mean velocity may be approximated by the half-Gaussian profile

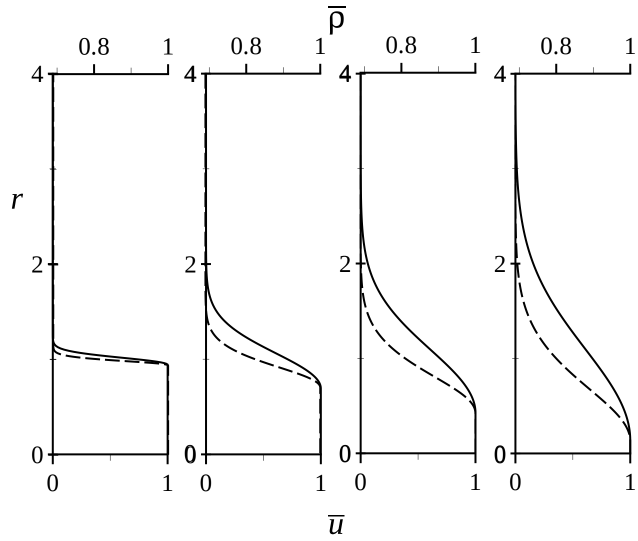

Fig. 2 Mean velocity (solid lines) and mean density (dashed lines) profiles for a cold supersonic jet of Mach number 1.5. From the left to the right : $x = 1$, $x = 5$, $x = 9$ and $x = 13$.

$$\bar{u} = \begin{cases} 1 & (r < h) \\ \exp\left(-\ln 2\left(\frac{r-h}{b}\right)^2\right) & (r \geq h) \end{cases} \quad (5)$$

where h is the radius of the potential core and b is the velocity half-width of the annular mixing region. The density is related to the mean axial velocity using a Crocco relationship,

$$\bar{\rho} = \left(\bar{u} + \frac{T_\infty}{T_0}(1-\bar{u}) + \frac{\gamma-1}{2}\bar{u}(1-\bar{u})M^2\right)^{-1} \quad (6)$$

where T_∞ and T_0 denote the ambient temperature and the jet exit temperature, respectively ; γ is

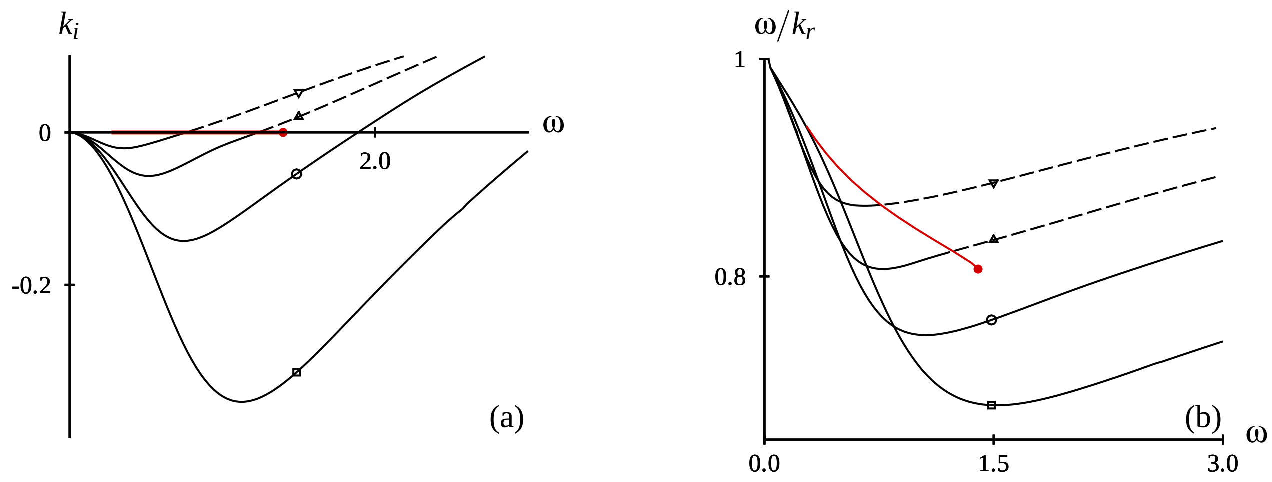

Fig. 3 Evolution of the spatial branch associated with the axisymmetric ($n = 0$) Kelvin-Helmholtz instability mode as the axial location X is increased in the core region, for a cold supersonic jet of Mach number 1.5 : (a) imaginary part k_i of k versus real frequency ω ; (b) phase velocity ω/k_r versus ω. □, $x = 3.0$; O, $x = 5.0$; △, $x = 7.0$; ▽, $x = 9.0$; dashed line, analytic continuation into $k_i > 0$; red line, curve described by the intersection point between the branch cuts and the spatial branch.

the ratio of specific heats. Figure 2 shows mean velocity and mean density profiles for a cold supersonic jet of Mach number 1.5. Note that for cold jets the ratio T_∞/T_0 is a function of the Mach number so that, the only control parameter is the Mach number.

On substituting for $\overline{u}$ and $\overline{\rho}$ equations (5) and (6) into the momentum integral equation, an algebraic relationship can be found between h and b in the annular mixing region. Then, the axial development of the jet is completely defined by the axial variation of the jet half-width $b(X)$. The length of the uniform core is obtained by using the modified formulae given by Tam et all[15]. In the literature db/dx is referred to as the spreading rate σ of the mixing layer. For the Gaussian mean velocity profile, we have the relation $db/dx = \varepsilon = 1.2658/\sigma$. The variation of σ as a function of the jet exit Mach number, M, has been tabulated by Birch and Eggers[2]. As an example, for $M = 1.5$, we have $db/dx = \varepsilon = 0.088$ in the core region. Note also that for supersonic cold jets, experimental measurements have shown that $db/dx \leq 0.10$.

5 General methodology ; numerical results

Different complex pairs (ω, k) were determined by numerical integration of the linearized inviscid, compressible equations of motion, together with boundary conditions given by (2) outside the jet. At each value of the axial location, the phase velocity of the neutral wave was located by searching for the zero of the growth rate $-k_i$, for positive ω. As the location is varied, the spatial branch moves in the complex k-plane. The associated parametric dependence of the real and the imaginary parts k_r and k_i on ω and x is displayed in figure 3, for a cold jet of Mach number $M = 1.5$ and for the axisymmetric Kelvin-Helmholtz mode. The numerical procedure to determine the spatial branches involves the definition of the function $\lambda(k;\omega)$ on its Riemann surface. Subsequently spatial branches are analytically continued into the second Riemann sheet for damped waves ($k_i > 0$), if necessary, as shown in figure 3 for $x \geq 6.3$.

It is interesting to note that the point of zero growth rate describes a curve in the complex k-

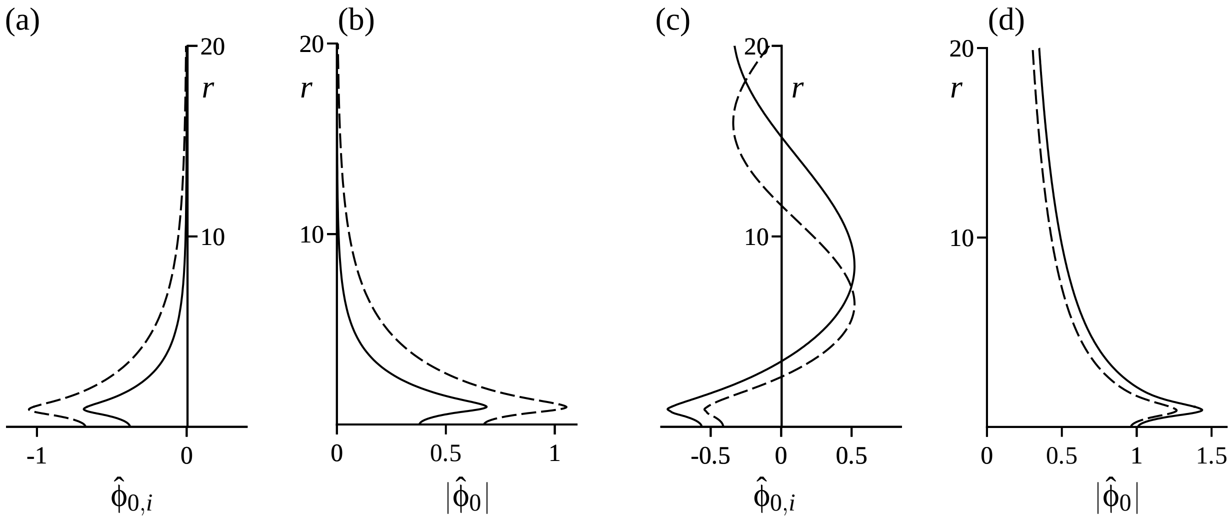

Fig. 4 Cross-stream behaviour of neutral axisymmetric instability waves for a cold supersonic jet of Mach number 1.5, for several axial locations. (a), (b) : $x = 5.5$ (solid line) ; $x = 6.0$ (dashed line). (c), (d) : $x = 6.5$ (solid line) ; $x = 7.0$ (dashed line).

plane, as x is increased from the origin to the end of the core region. This curve may lie in the vertival strip $|k_r| < k_0$ corresponding to supersonic phase velocities as shown by the solid red line in figure 3. According to the definition of branch cuts, this part of the curve gives axial locations $x_i < x < x_s$ and frequencies for which the cross-stream decay of neutral instability waves is algebraic, as shown in figure 4 (c,d), for $x = 6.5$ and $x = 7.0$. For $0 < n < 4$ and $M = 1.5$, the spatial branches remain in the domains of analicity of (pressure) eigenfunctions $\hat{\phi}_n$ so that the amplitudes become exponentially small (dissipate) as $r \to \infty$, as for the axisymmetric instability wave in the region $0 < x < x_i$, $x_i = 6.3$. It follows that the cross-stream decay of pressure disturbances satisfies boundedness conditions for all positive frequency. Figure 4 (a,b) shows the cross-stream decay of the neutral axisymmetric instability wave for $M = 1.5$ and for two locations $x = 5.5$, $x = 6.0$. The above reasoning presents the advantage of uniquely determining which spatial branches are pertinent to acoustic radiation outside the jet. Figure 5 gives the evolution of the lower bound x_i as the Mach number varies in the range $1.0 < M < 2.0$. For low supersonic jets x_i moves outside the core region to go into the fully developed region, where the centreline mean velocity of the jet is a decreasing function of x. One may think of continuing the numerical computations in order to have x_i. But the present approach fails when the order of magnitude of the frequency becomes comparable with that of the slow space variable $X = \varepsilon x$ and, a complete asymptotic analysis must be developed in the case where λ is a small parameter. Note also that for the axisymmetric mode, it was not possible to find an upper bound x_s within the core region whereas it was obtained with no difficulty for other azimuthal wavenumbers. This is illustrated in figures 5-6.

6 Concluding remarks

We have examined the near-field pressure decay generated by instability waves in low supersonic cold jets as a source of noise. As already indicated in section 3, the subsonic-supersonic transition is governed by a turning-point problem, which gives the evolution of the cross-stream transition from a dispersive behaviour to a dissipative behaviour, outside the jet. For cold jets this transition holds only for the axisymmetric instability waves associated with the large turbu-

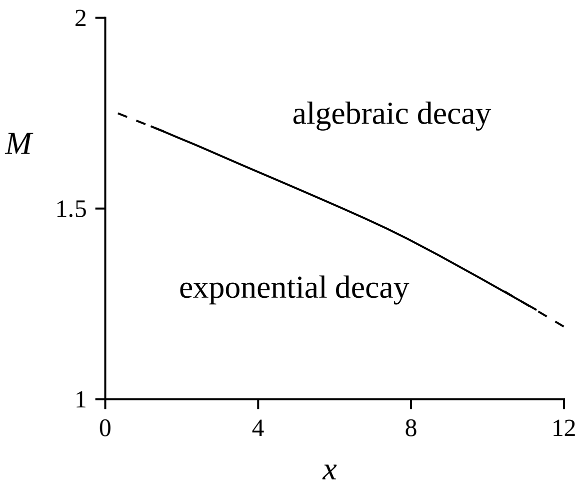

Fig. 5 Evolution of the lower bound x_i for axisymmetric instability waves when the Mach number varies.

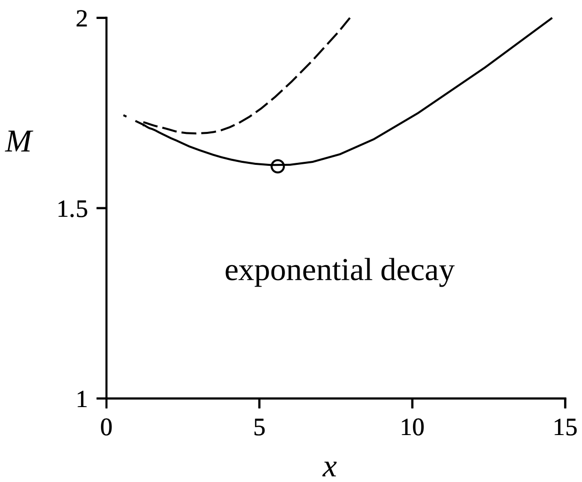

Fig. 6 Evolution of lower and upper bounds x_i and x_s, respectively, for $n = 1$ (solid line) and $n = 2$ (dashed line) when the Mach number varies. The circle gives the point where $x_i = x_s$, corresponding to $\omega = 1.7$, for the helical mode.

lence structures. This axisymmetric structure is then the dominant contributor to acoustic radiation with a cross-stream wavenumber which behaves as the square root of the threshold distance $k^2 - k_0^2$ where k is the wavenumber of the neutral instability wave. This result is in good agreement with the measurements of Yu and Dosanjh [17], obtained for a cold jet with $M = 1.5$.

For supersonic jets at moderate supersonic Mach number (typically $M > 1.6$), the entire history of the spatial evolutions of instability waves need to be taken into account in determining the contribution of each azimuthal wavenumbers and the physical regions concerned by the radiation of sound.

Finally, we note that the generalization of these concepts to subsonic jets is an open question (see Cooper and Crighton[4] for a discussion of acoustic radiation in low-speed axisymmetric jets).

This study has been supported by the French government and by ONERA.

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