# ON THE TRANSMISSION OF SOUND ACROSS A NON ISOTHERMAL BOUNDARY LAYER

**L. M. B. C. Campos & M. H. Kobayashi**
**Instituto Superior Técnico, Departamento de Engenharia Mecânica/SMA, Av. Rovisco Pais, 1, 1049—001, Lisbon, Portugal**

**Keywords:** *Aeroaccoustics, non-isothermal boundary-layer.*

## Abstract

*The wave equation describing the propagation of sound in an unidirectional shear flow, has had only two exact solutions in the literature, one concerning a linear velocity profile and another concerning an exponential boundary layer, both for an homentropic shear flow and unrestricted Mach number. The same wave equation is shown to apply to the acoustic waves in an isentropic, non-homentropic shear flow, restricted to low Mach number. This is a particular case of the acoustic wave equation in an isentropic, non-homentropic shear flow, valid without restriction on Mach number. The cases dealt with in the literature imply constant sound speed and hence for a perfect gas, isothermal conditions. The present paper concerns an homenergetic mean flow which allows for non-uniform sound speed in a non-isothermal unidirectional shear. The sound field due to a time harmonic line source is considered outside the homenergetic shear flow and compared with the homentropic shear flow for sound incident on a boundary layer over a rigid wall.*

## 1 Introduction

The propagation of sound in shear flows is specified by a wave equation ([22], [34], [45], [43], [39]) which has been studied mostly by numerical and approximate analytical methods, with three motivations in mind: (i) propagation in ducts containing a shear flow, such as jet engine ducts ([51], [40], [37], [23], [48], [49], [14], [15], [31], [50], [36], [28]); (ii) effect of boundary layers, on sound near a wall, such as fuselage or cabin of an aircraft ([1], [17], [18], [42], [21]); (iii) effect of laminar shear layers on sound transmission, e.g. shear layers of a jet exhaust or wake of a control or high-lift device ([38], [20], [2], [3], [33]). The model of a shear layer as a laminar shear flow of finite width ([29]) is intermediate between a vortex sheet ([38], [41]) as a discontinuity of tangential velocity, and a irregular shear layer ([26], [5], [6], [7], [8]), which may entrain turbulence ([35], [46], [25], [24], [9], [10], [11]).

In the present paper sound propagation in laminar shear flow is considered. The simplest velocity profile is the linear shear, which may be matched to uniform streams to represent (i) a boundary layer near a flat wall, (ii) a double boundary layer in a parallel-sided duct or (iii) a shear layer between stream of different velocities. The effect of the uniform flow reduces to a Doppler effect, whereas the linear shear has a critical layer, which has been considered in the literature, sometimes implicitly, by four methods of solution: (a) in terms of parabolic cylinder functions ([19]), (b) in terms of Whittaker functions ([29], [30]), (c) in terms of confluent hypergeometric functions ([47], [32], [33]) and (d) as a linear combination of Frobenius-Fuchs series which are even and odd relative to the critical layer ([13]). These four methods address the acoustic wave equation in linear shear flow, which has two singularities: one regular, at the critical layer and another irregular, at infinity, where the mean flow velocity diverges. In the neighborhood of the regular singularity ([27], [16], [44]), the Frobenius-Fuchs method supplies a pair of linearly independent solutions in power series (in the case of a linear shear flow there is no logarithmic singularity at the critical layer). In the neighborhood of the irregular singularity ([27]), the Frobenius-Fuchs method breaks down, i.e. provides no solution at all or at most one solution; in this case the method of normal integrals or infinite determinants may be used. They are not needed for the linear shear flow, since the wave equation has only two singularities and thus the expansion about the regular singularity (i.e. the critical layer), has infinite radius of convergence (up to the irregular singularity at infinity). Note that the critical layer and other singularities of the wave equation determine the form of its solution.

All the literature mentioned before, concerning the acoustics of unidirectional shear flow, assumes uniform sound speed; since for an unidirectional shear flow, the mean flow pressure is uniform, it follows from the equation of state, that the mean flow mass density, temperature and entropy are also uniform, i.e. all those results concerns the acoustics or stability of linear, homentropic shear flow. In order to assess the effect of not assuming an homentropic mean flow, in the present paper a linear homenergetic shear flow is considered, for which the stagnation enthalpy, not the entropy, is conserved. In this case the sound speed is related to the mean flow velocity, i.e. is no longer isothermal, i.e. it can support a temperature gradient. This introduces an extra term in the acoustic wave equation; besides, it adds another two singularities, at the critical flow conditions, where the sound speed vanishes. Thus, whereas the acoustic wave equation in a linear shear flow has two singularities in the homentropic case (treated in the literature), in the present homenergetic case it has four singularities: (i) a regular singularity at the critical layer and an irregular singularity at infinity, inherited from the low Mach number (or homentropic) case; (ii) two regular singularities at the critical flow conditions, which occur for high-speed non-homentropic mean flow.

In order to compare the homentropic and the homenergetic models, the sound field due to a time harmonic line source outside the boundary layer for an homogeneuous shear flow is compared with the homenergetic case.

## 2 Homentropic and homenergetic mean shear flows

In [12] the authors derived the wave equation for an unidirecitonal sheared mean flow where the sound speed is allowed to vary in a transverse direction:

$$\frac{d}{dt}\left(\frac{1}{c^2}\frac{d^2p}{dt^2}-\nabla\left(\log\rho_0\right).\nabla p-\nabla^2 p\right) + 2U'\frac{\partial^2}{\partial x\partial y}=0 \tag{1}$$

Now, since the mean flow properties depend only on the transverse coordinate $y$, i.e. the mean flow is steady and longitudinally uniform, it is convenient to use the a Fourier decomposition in time $t$ and longitudinal coordinate $x$:

$$p(x,y,t)=\int_{\mathbb{R}^2}e^{i(\omega t-kx)}\mathrm{d}\omega\mathrm{d}k \tag{2}$$

where $P(y;k,\omega)$ denotes the acoustic pressure perturbation spectrum, for a wave of frequency $\omega$ and longitudinal wave number $k$ at position $y$. The dependence of the acoustic pressure on the latter is generally not sinusoidal, i.e. is specified by substituting (2) in (1), viz.:

$$\begin{aligned}&(\omega-kU)P''+\\&\quad 2[kU'+(\omega-kU)c'/c]P'+\\&\quad(\omega-kU)[(\omega-kU)^2/c^2-k^2]P=0.\end{aligned} \tag{3}$$

All the literature on the acoustics of linear shear flows ([34], [19], [20], [29], [30], [47], [32], [33], [13]) assumes an homentropic flow, i.e. constant entropy; in this case the sound speed is constant, and the wave equation (3) reduces to the well-known form ([22], [45], [39])

$$\begin{aligned}&(\omega-kU)P''+2kU'P'+\\&\quad(\omega-kU)[(\omega-kU)^2/c^2-k^2]P=0,\end{aligned} \tag{4}$$

which is the one considered in all of the references above. The derivation of (3) applies equally well to isentropic, non-hometropic mean flow, in which the wave equation (4) holds only at low Mach number, when the sound speed is constant. In the present paper neither the restriction to homentropic mean flow nor the restriction to low Mach number mean flow is made, so the equation (3) does not reduce to (4), i.e. the mean flow temperature is not assumed to be uniform. Thus the acoustic wave equation (3) describes the propagation of sound in a non-isothermal unidirectional shear flow, if the isentropic condition is retained, but the homentropic condition is not imposed. A temperature profile, which is consistent with isentropic, non-homentropic mean flow, i.e. allows $\rho_0$, $T$, $c$ to vary from one streamline to the next (i.e. as function of $y$), is the condition ([4]) of homenergetic mean flow, i.e. constant stagnation enthalpy; this relates the sound speed $c(y)$ and mean flow velocity $U(y)$ at arbitrary streamline to the stagnation sound speed $c_0$ by

$$[c(y)]^2 = c_0^2 - \varepsilon^2 [U(y)]^2$$

where $epsilon = \sqrt{(\gamma-1)/2}$. Thus the acoustic wave equation in a high-Mach number homenergetic shear flow:

$$\begin{aligned}&(\omega - kU)(c_0^2 - \varepsilon^2 U^2)P'' + \\ &\quad 2U'[k(c_0^2 - \varepsilon^2 U^2) - \varepsilon^2 U(\omega - kU)]P' + \\ &\quad (\omega - kU)[(\omega - kU)^2 - k^2(c_0^2 - \varepsilon^2 U^2)]P = 0\end{aligned} \tag{5}$$

has the following singularities: (i) a critical layer where the Doppler shifted frequency vanishes:

$$0 = \omega_*(y_c) = \omega - kU(y_c) \therefore U(y_c) = \omega/k,$$

i.e. the mean flow velocity equals the acoustic phase speed calculated form the horizontal wavenumber; (ii) two critical flow points, where the sound speed vanishes:

$$0 = c(y_\pm) \therefore U(y_\pm) = \pm c_0/\varepsilon = \pm\sqrt{2/(\gamma-1)};$$

(iii) the points at infinity $y = \pm\infty$ may also be singularities.

In order to complete the specification of the wave equation (5) a linear shear flow is considered:

$$U(y) = \Omega y, \tag{6}$$

for which the vorticity is constant $\Omega = \mathrm{d}U/\mathrm{d}y =$ const and specifies the position of the critical layer, viz.

$$y_c = \omega/\Omega k,$$

which is generally distinct from the two critical flow points

$$y_\pm = \pm c_0/\varepsilon\Omega.$$

Coincidence would be possible only for $y_c = y_+$ if the phase speed has a precise relation to the stagnation sound speed

$$\frac{\omega}{k} = \frac{c_0}{\varepsilon},$$

for propagation in the positive $x$-direction $k > 0$; alternatively $y_c = y_-$ for propagation in the negative $x$-direction $k < 0$. The change of independent variable

$$\zeta := y/y_c = \Omega k y/\omega,$$

places the critical layer at the point unity $\zeta_c = 1$ and transform the wave equation (5) to:

$$\begin{aligned}&\left(1 - \Lambda^2\zeta^2\right)(1-\zeta)T'' + \\ &2\left(1 - \Lambda^2\zeta\right)T' - \\ &\alpha(1-\zeta)\left[1 - \Lambda^2\zeta^2 - \beta(1-\zeta)^2\right]T = 0,\end{aligned} \tag{7}$$

where $T(\zeta;\alpha,\beta,\Lambda) = P(y;k,\omega,c_0)$. Here the three dimensionless parameters

$$\alpha := (\omega/\Omega)^2, \quad \beta := \omega/kc_0 \quad \Lambda := \varepsilon\omega/kc_0$$

denotes, respectively, (i) the square of the ratio of wave frequency $\omega$ to mean vorticity $\Omega$, which is smaller for larger shear flow effect; (ii) the square of the ratio of horizontal phase speed $u = \omega/k$ to sound speed $c_0$, viz. $\beta = u/c_0$, so that $\beta = 1$ for horizontal propagation, $\beta > 1$ for transversely propagating waves $\omega > kc_0$ and $\beta < 1$ for transversely evanescent waves; (iii) $\Lambda = 0$ for low Mach number flow $c = c_0$ or $\gamma = 1$ or $\varepsilon = 0$, so that $\Lambda \neq 0$ is a measure of high speed effects.

The change of independent variable

$$\xi = \frac{\zeta - 1}{1/\Lambda - 1}$$

shifts the regular singularities at the critical layer and critical flow points to:

$$\zeta_c, \zeta_\pm = 1, \pm\Lambda^{-1} \mapsto \xi_c, \xi_+, \xi_- = 0, 1, \frac{\Lambda+1}{\Lambda-1} := F \tag{8}$$

and leads to the differential equation

$$\xi(\xi-1)(\xi-F)R'' - 2[F - \Lambda\xi/(\Lambda-1)]R' - \alpha(1-1/\Lambda)^2\xi[(\xi-1)(\xi-F) + \beta^2\xi^2/\Lambda^2]R = 0, \tag{9}$$

where $R(\xi;\alpha,\beta,\Lambda) := T(\zeta;\alpha,\beta,\Lambda)$. The point at infinity is an irregular singulary of the wave equation.

Since the critical layer corresponds to the regular singularity $\xi = 0$ of the differential equation (9), the solution in its neighborhood can be determined by the Frobenius-Fuchs method (see [12])

$$R(\xi) = (A + B\log\xi)R_3(\xi) + B\overline{\overline{R}}_0$$

where $A$, $B$ are constants of integration and

$$R_3(\xi) = \sum_{n=0}^{\infty} a_n(3)\xi^{n+3},$$

which vanishes at the critical layer and has recurrence formula for the coefficients, $\sigma \in \mathbb{R}$:

$$\begin{aligned}
&F(n+\sigma+1)(n+\sigma-2)a_{n+1}(\sigma) = \\
&2\left[\Lambda/(\Lambda-1)\right](n+\sigma)(n+\sigma-2)a_n(\sigma) - \\
&\left[\alpha F(1-1/\Lambda)^2 - (n+\sigma-1)(n+\sigma-2)\right]a_{n-1}(\sigma) - \\
&(\alpha/\Lambda)(1-1/\Lambda)^2 \\
&\quad\left[(1+F)a_{n-2}(\sigma) - (\Lambda-1)^2(1+\beta^2/\Lambda^2)a_{n-3}(\sigma)\right]
\end{aligned}$$

$$\overline{\overline{R}}_0 \sum_{n=0}^{\infty} b_n(0)\xi^n,$$

and

$$b_n(0) = a_n(0) + \lim_{\sigma\to 0} \sigma a_n'(\sigma)$$

## 3 Line source outside a boundary layer

The linear shear flow assumed before (6) could be unbounded for the homentropic case and is limited by the critical flow points (8) in the homenergetic case. In either case the linear shear flow can be matched to an uniform stream:

$$U(y) = \begin{cases} \Omega y & \text{if } y \le L \\ \Omega L := U_\infty & \text{if } y \ge L \end{cases}$$

where $L = U_\infty/\Omega$ is the boundary layer thickness and $U_\infty$ the free stream velocity. The critical layer occurs in the boundary layer if $y_c < L$ or $\omega < \Omega kL$. The acoustic field inside the boundary layer has been calculated before and the acoustic pressure $P(y;k,\omega)$ and velocity $\sim P'(Y;k,\omega)$ are to be matched across $y = L$ to the acoustic field in the free stream, thus determining the constants of integration $A$, $B$ in the general solutions. In the free stream the mean flow velocity is constant and the wave equation (3) simplifies to

$$P''_\infty + K^2 P_\infty = S\delta(y - y_0) \tag{10}$$

where $K$ is the vertical wavenumber in the free stream:

$$K := \sqrt{(\omega - kU_\infty)^2/c_\infty^2 - k^2},$$

and a line source of strength $S$ was placed in the free stream at a distance $y_0$ from the wall. The forced solution of (10) is the first term of:

$$P_\infty(y;k,\omega) = -iS/4K\exp[iK|y-y_0|] + C_+\exp(-iKy)$$

and the second term is an upward propagating wave of amplitude $C_+$, reflected from the boundary layer (because the source lies in the free stream). The source strength is chose to be $S = i4K$ and $C_+$ is determined so as to satisfy a rigid wall condition. The dimensionless parameters of the solution in the boundary layer are reconsidered bearing in mind the matching to the uniform stream, viz.:

$$\alpha_\infty := (\omega/\Omega)^2 = (\omega L/U_\infty)^2, \quad \beta_\infty := \omega/Kc_\infty,$$
$$\Lambda_\infty = \varepsilon\omega/Kc_\infty = \varepsilon\beta_\infty.$$

Note the relation between the free stream and stagnation sound:

$$c_0^2 = c_\infty^2 + \varepsilon^2 U_\infty^2 = c_\infty^2(1+\varepsilon^2 M_\infty^2), \quad M_\infty := U_\infty/c_\infty$$

where $M_\infty$ denotes the free stream Mach number. The distance from the wall is made dimensionless dividing by the boundary layer thickness:

$$z := y/L = \Omega y/U_\infty,$$

Finally, using the above non-dimensional parameters, the vertical wave number can be written as

$$K = k\sqrt{(\beta - M_\infty)^2 - 1},$$

so that it is real, i.e. waves propagate in the free stream iff $M_\infty - 1 \leq \beta_\infty \leq 1 + M_\infty$.

## 4 Comparison of homentropic and homenergic cases

The sound field due to a time harmonic line source outside a boundary layer with a linear velocity profile is plotted next as a function of dimensionless distance from the wall in the case of homenergetic and homentropic shear flow.

Figures 1–4 concern a comparison of the sound field due to a line source over a rigid wall, at a distance of two boundary layer thickness, for a boundary layer with a linear velocity profile in homentropic (dashed line) or homenergetic (solid line) conditions. The first two plots concern a case of wave frequency equal to the vorticity $\alpha_\infty = 1$ and oblique upstream propagation $\beta_\infty = 4$, for which there is no critical layer in the boundary layer. The amplitude (Figure 1) is almost identical for the homentropic (S) and homenergetic (E) case at low free stream Mach number $M_\infty = 0.1$, but the differences increases with increasing Mach number $M_\infty = 0.3, 0.7, 1$, leading to very different values of the wall pressure in the supersonic case $M_\infty = 3.5$, when the acoustic pressure at the wall is much larger in the homentropic case. The phase (Figure 2) is larger for homentropic case than for the homenergetic case, with a small difference at low Mach number $M : \infty = 0.1$ and a more noticeable difference for increasing Mach number $M_\infty = 0.3, 0.7, 1$. The exception is the supersonic free stream $M_\infty = 3.5$, for which the phase is the same in the homentropic and homenergetic cases; note that for $\beta_\infty = 4$ this is the only case $2.5 < \beta_\infty < 4.5$ of propagation in the free stream.

The final two plots (Figures 3–4) concerns again wave frequency equal to vorticity $\alpha_\infty = 1$, with the condition $\beta_\infty = M_\infty/2$ which places the critical layer at the middle of the boundary layer $z_c = y_c/L = \beta_\infty/M_\infty = 0.5$. The amplitude (Figure 3) of the sound field is always larger in the homentropic case than in the homenergetic case; it is almost uniform in the homentropic case and has a dip in the homenergetic case, close to the border of the boundary layer. With increasing Mach number, the amplitude decreases monotonically in the homentropic case and tends to increase in the homenergetic case. The phase (Figure 4) differs most between the homentropic and homenergetic case for the largest Mach number and is more uniform in the former case.

The near coincidence of the homenergetic and homentropic case at low Mach number results from the sound speed being nearly constant in that case, so that the wave equation (3) simplifies to the usual form (4); as the Mach number increases, the extra term in (3) compared with (4) plays a larger role, which in turn entails both a quantitative as well as a qualitative difference in the sound field generated by both methods.

## References


[1] Almgren G. Acoustic boundary layer influence in scale model simulation of sound propagation. *Journal of Sound and Vibration*, Vol. 105, pp 321—327, 1976.

[2] Balsa T. The far-field of high-frequency convected, singularities in shear flows with application to jet noise prediction. *Journal of Fluid Mechanics*, Vol. 74, pp 193—208, 1976.

[3] Balsa T. Reffraction and shielding of sound from a source in a jet. *Journal of Fluid Mechanics*, Vol. 76, pp 443—456, 1976.

[4] Batchelor G. K. *An Introduction to Fluid Dynamics*. Cambridge University Press, Cam-

bridge, Great Britain, 1967.

[5] Campos L. M. B. C. On the spectral broadening of sound by turbulent shear layers. part I: Scattering by moving interfaces and reffraction in turbulence. *Journal of Fluid Mechanics*, Vol. 89, pp 723—749, 1978.

[6] Campos L. M. B. C. On the spectral broadening of sound by turbulent shear layers. part II: Spectral broadening of experimetal and aircraft noise. *Journal of Fluid Mechanics*, Vol. 89, pp 751—779, 1978.

[7] Campos L. M. B. C. Sur la propagation du sons dans les ecoulements non-uniformes et non-stationaieres. *Reveu d'Acoustique*, Vol. 67, pp 217—233, 1983.

[8] Campos L. M. B. C. On waves in gases. part i: Acoustics of jets, turbulence and ducts. *Review of Modern Physics*, Vol. 58, pp 117—182, 1986.

[9] Campos L. M. B. C. Effects on acoustic fatigue loads of multiple reflection between a plate and a turbulent wake. *Acustica*, Vol. 76, pp 109—117, 1992.

[10] Campos L. M. B. C. On the correlation of acoustic pressures induced by a turbulent wake on a nearby wall. *Acta Acustica*, Vol. 82, pp 9—17, 1996.

[11] Campos L. M. B. C. On the spectra of aerodynamic noise and aeroacoustic fatigue. *Progress in Aerospace Sciences*, Vol. 33, pp 353—389, 1997.

[12] Campos L. M. B. C and Kobayashi M. H. On the propagation of sound in a non-isothermal shear flow. *To appear*.

[13] Campos L. M. B. C, Oliveira J. M. G. S, and Kobayashi M. H. On sound propagation in a linear shear flow. *Journal of Sound and Vibration*, Vol. 219, No 5, pp 739—770, 1999.

[14] Eversman W. Effect of boundary layer on the transmission and attenuation of sound in an acoustically treated circular duct. *Journal of the Acoustical Society of America*, Vol. 39, pp 1372—1380, 1971.

[15] Eversman W and Beckenmeyer R. J. Transmission of sound in ducts with thin shear layers: Convergence to the uniform flow case. *Journal of the Acoustical Society of America*, Vol. 52, pp 216—225, 1972.

[16] Forsyth A. R. *Treatise of Differential Equations*. Sixth edition, MacMillan, 1926.

[17] Goldstein M. E. Scattering and distortion of the unsteady motion on transversely sheared mean flows. *Journal of Sound and Vibration*, Vol. 91, pp 601—602, 1979.

[18] Goldstein M. E. High frequency sound emission from moving point miltipole sources embedded in arbitrary transversely sheared mean flows. *Journal of Sound and Vibration*, Vol. 80, No 4, pp 499—522, 1982.

[19] Goldstein M. E and Rice E. Effect of shear on duct wall impedence. *Journal of Sound and Vibration*, Vol. 30, pp 79—84, 1973.

[20] Graham E. W and Graham B. B. Effect of a sheared layer on plane waves in a fluid. *Journal of the Acoustical Society of America*, Vol. 46, pp 169—175, 1969.

[21] Hanson D. Shielding of propfan cabin noise by the fuselage boundary layer. *Journal of Sound and Vibration*, Vol. 92, pp 591—598, 1984.

[22] Haurwitz W. Zur theorie der wellenbewegungen in luft und wasser. *Veröffentlichen des Geophysik Institut von der Universität Leipzig*, Vol. 6, pp 334—364, 1932.

[23] Hersh A. S and Catton I. Effects of shear flow on sound propagation in rectangular ducts. *Journal of the Acoustical Society of America*, Vol. 50, pp 992—1003, 1971.

[24] Ho C. M and Kovasznay L. S. G. Acoustical shadowgraph. *Physics of Fluids*, Vol. 19, pp 1118—1123, 1976.

[25] Ho C. M and Kovasznay L. S. G. Propagation of a coherent acoustic wave through a turbulent shear flow. *Journal of the Acoustical Society of America*, Vol. 60, pp 40—45, 1976.

[26] Howe M. S. The attenuation of sound by a randomly irregular impedance layer. *Proceedings of the Royal Society London A*, Vol. 347, pp 513—535, 1976.

[27] Ince E. L. *Ordinary Differential Equations*. Dover, New York, 1956.

[28] Ishii S and Kakutani T. Acoustic waves in parallel shear flows in a duct. *Journal of Sound and Vibration*, Vol. 113, pp 127—139, 1987.

[29] Jones D. The scattering of sound by a simple shear layer. *Philosophical Transactions of the Royal Society London A*, Vol. 284, pp 287—328, 1977.

[30] Jones D. Acoustics of a splitter plate. *Journal of the Institute of Mathematics and its Applications*, Vol. 21, pp 197—209, 1978.

[31] Ko S. H. Sound attenuation in acoustically lined circular ducts in the presence of uniform flow and shear flow. *Journal of Sound and Vibration*, Vol. 22, pp 193—210, 1972.

[32] Koutsoyannis S. P. Characterization of acoustic disturbance in linearly flows. *Journal of Sound and Vibration*, Vol. 68, pp 187—202, 1980.

[33] Koutsoyannis S. P, Karamcheti K, and Galant D. Acoustic resonances and sound scattering by a shear layer. *AIAA Journal*, Vol. 18, pp 1446—1450, 1980.

[34] Kücheman D. Störungebewegungen in einer gasströmung mit grenzschicht. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 30, pp 79—84, 1938.

[35] Lighthill M. J. On the energy scattered from interaction of turbulence with sound and shock waves. *Proceedings of the Cambridge Philosophical Society*, Vol. 44, pp 531—548, 1953.

[36] Mani R. Sound propagation in parallel sheared flows in ducts: the mode estimation problem. *Proceedings of the Royal Society London A*, Vol. 371, pp 393—412, 1980.

[37] Mariano S. Effect of wall shear layers on the sound attenuation in acoustically lined rectangular ducts. *Journal of Sound and Vibrations*, Vol. 19, pp 261—275, 1971.

[38] Miles J. M. On the disturbed motion of a vortex sheet. *Journal of Fluid Mechanics*, Vol. 4, pp 538—554, 1958.

[39] Möring W, Müller E, and Obermeier F. Problems in flow acoustics. *Reviews of Modern Physics*, Vol. 55, pp 707—724, 1983.

[40] Mungur P and Plumblee H. E. Propagation and attenuation of sound in a soft-walled annular duct containing a sheared flow. SP 207, NASA, 1969.

[41] Munt R. M. Interaction of sound with a subsonic jet inssuing from a semi-infinite jet-pipe. *Journal of Fluid Mechanics*, Vol. 83, pp 609—620, 1977.

[42] Myers M. K and Chuang S. L. Uniform asymptotic approximations for duct acoustic modes in a thin boundary-layer flow. *AIAA Journal*, Vol. 22, pp 1234—1241, 1983.

[43] Nayfeh A, Kaiser J, and Telionis D. Acoustics of aircraft engine-duct systems. *AIAA Journal*, Vol. 13, pp 130—153, 1975.

[44] Poole E. G. C. *Linear Differential Equations*. Dover, 1935.

[45] Pridmore-Brown D. Sound propagation in a fluid flowing through an attenuating duct. *Journal of Fluid Mechanics*, Vol. 4, pp 393—406, 1958.

[46] Schmidt D. W and Tilmann P. M. Experimental study of sound phase fluctuations caused by turbulent wakes. *Journal of the Acoustical Society of America*, Vol. 47, pp 1310—1324, 1970.

[47] Scott J. N. Propagation of sound waves through a linear shear layer. *AIAA Journal*, Vol. 17, pp 237—245, 1979.

[48] Shankar P. N. Acoustic reffraction by duct shear layers. *Journal of Fluid Mechanics*, Vol. 47, pp 81—91, 1971.

[49] Shankar P. N. Acoustic reffraction and attenuation in cylindrical and annular ducts. *Journal of Sound and Vibration*, Vol. 22, pp 233—296, 1972.

[50] Swinbanks M. A. The sound field generated by a source distribution in a long duct carrying sheared flow. *Journal of Sound and Vibration*, Vol. 40, pp 51—76, 1975.

[51] Tack D. H and Lambert R. F. Influence of shear flow on sound attenuation in lined ducts. *Journal of the Acoustical Society of America*, Vol. 38, pp 655—666, 1965.

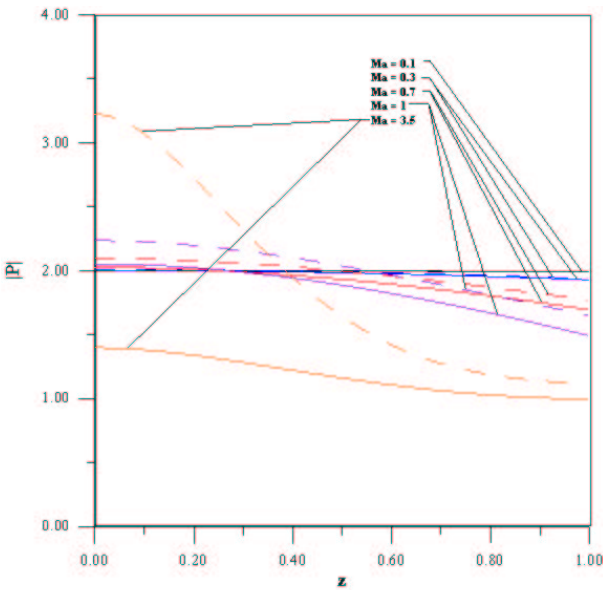


**Fig. 1** Comparison of sound field in a linear boundary layer generated by homenergetic (solid lines) and homentropic (dashed lines) models: magnitude. $\alpha_\infty = 1$, $\beta_\infty = 4$.

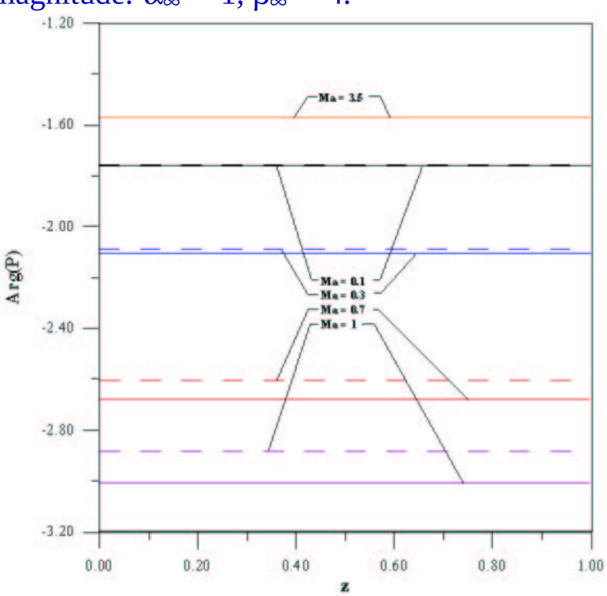


**Fig. 2** Comparison of sound field in a linear boundary layer generated by homenergetic (solid lines) and homentropic (dashed lines) models: phase. $\alpha_\infty = 1$, $\beta_\infty = 4$.

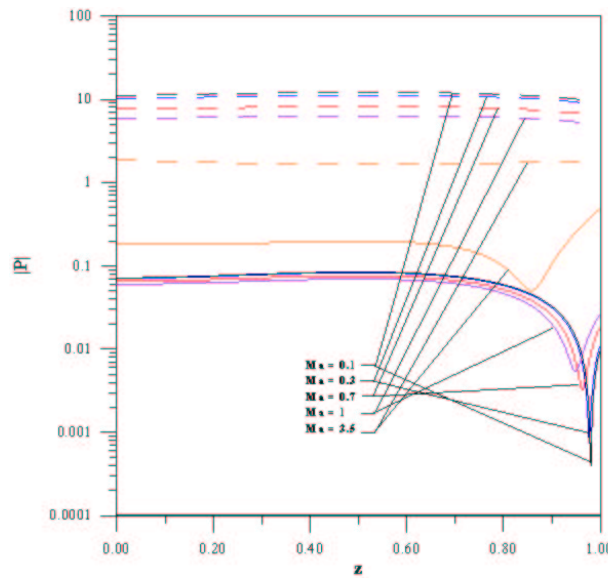


**Fig. 3** Comparison of sound field in a linear boundary layer generated by homenergetic (solid lines) and homentropic (dashed lines) models: magnitude. $\alpha_\infty = 1$, $\beta_\infty = M_\infty/2$.

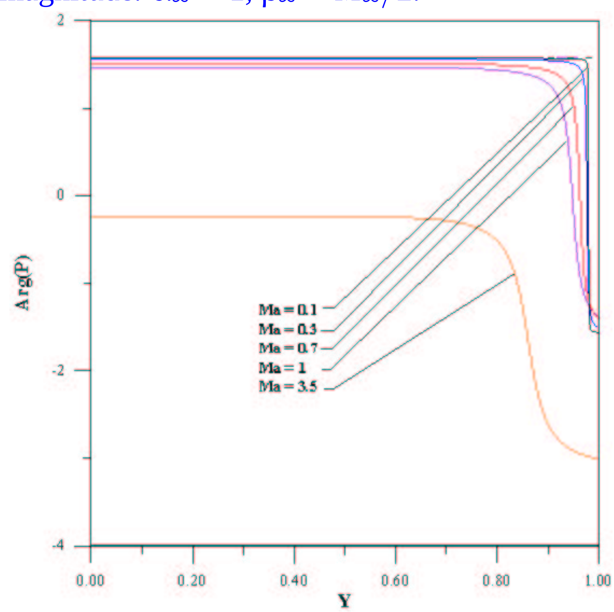


**Fig. 4** Comparison of sound field in a linear boundary layer generated by homenergetic (solid lines) and homentropic (dashed lines) models: phase. $\alpha_\infty = 1$, $\beta_\infty = M_\infty/2$.