Transonic Aileron Buzz Boundary Prediction Using Hopf-Bifurcation Analysis Method

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Abstract

Hopf-bifurcation analysis method is employed to predict the transonic aileron buzz boundary for a delta-wing model with a full-span aileron. The unsteady aerodynamic loads acting on the surface of the aileron are evaluated by solving the three dimensional, unsteady, compressible, full Navier-Stokes equations. The eigenvalues of the Jacobian matrix of the equation of motion of the aileron are evaluated and the critical Mach numbers corresponding to the Hopf-Bifurcation points are obtained. The results of the Hopf-bifurcation analysis are consistent with the results of the time integration calculations and both the calculated results are in good agreement with experimental data.

1 Introduction

In transonic flight, the trailing edge control surfaces of the aircraft may fall into an everlasting "self-excited oscillation", which is known as control surface buzz in aeroelasticity. Early studies of aileron buzz indicate that the occurrence of buzz is closely related with the oscillating shock on the surface of the aileron, and the shock oscillations are not in phase with the motion of the aileron.[2]

Steger and Bailey [1] first calculated the transonic aileron buzz by modern numerical method, and got some good results. But in their simulations, they neglected the spring constant and damping coefficient in the equation of motion of the aileron. And in the aerodynamic calculations, at each time step, they used a simple shearing transformation to form the new grid when the aileron was deflected. Although the computer time was reduced due to these simplifications, the numerical errors were increased. Some experimental investigations on transonic aileron buzz were given by Parker, Spain and Soistmann [2][3]. Other numerical calculations of control surface buzz were presented in ref. [5][6] etc.

In all the above simulations, the method of time integration was used to determine the buzz points. However, in the present paper,we take an alternate approach of determining the buzz points by using the Hofp-bifurcation analysis method. Using this method, not only we can compute the buzz points more efficiently, but also get a clear idea of the influence of the structural parameters on the control surface buzz characteristics.

2 Governing Equations and Numerical Method

In order to calculate the unsteady aerodynamic loads acting on the control surface, the conservative, dimensionless, unsteady, compressible, full Navier-Stokes equations are solved.

In terms of time dependent body-conformed coordinates ξ^1, ξ^2 and ξ^3, these equations may be written as

$$\frac{\partial Q}{\partial \tau}+\frac{\partial E_i}{\partial \xi^i}-\frac{\partial (E_v)_S}{\partial \xi^S}=0;\quad \text{i=1, 2, 3,}\quad \text{S=1, 2, 3} \tag{1}$$

where

$$Q=\frac{1}{J}\begin{bmatrix}\rho\\ \rho u_1\\ \rho u_2\\ \rho u_3\\ \rho E\end{bmatrix},\quad E_i=\frac{1}{J}\begin{bmatrix}\rho U_i\\ \rho u_1 U_i+\frac{\partial \xi^i}{\partial x_1}p\\ \rho u_2 U_i+\frac{\partial \xi^i}{\partial x_2}p\\ \rho u_3 U_i+\frac{\partial \xi^i}{\partial x_3}p\\ (\rho E+P)U_i-\frac{\partial \xi^i}{\partial \tau}p\end{bmatrix}$$

$$(Ev)_S=\frac{1}{J}\frac{\partial \xi^S}{\partial x_j}\begin{bmatrix}0\\ \tau_{1j}\\ \tau_{2j}\\ \tau_{3j}\\ \tau_{kj}u_k+(\kappa+\kappa^t)\frac{\partial T}{\partial x_j}\end{bmatrix}$$

$$U_i=\frac{\partial \xi^i}{\partial x_j}u_j+\frac{\partial \xi^i}{\partial \tau}$$

E_i is the inviscid flux in ξ^i direction and $(Ev)_S$ is the viscous and heat-conduction flux in ξ^S direction. τ, x_i, u_i, ρ, p and E are the time, Cartesian coordinates, Cartesian components of the velocity, density, pressure and the total internal energy per unit mass, respectively. T, κ, κ^t and J are the temperature, heat conductivity, eddy heat diffusivity and the Jacobian, respectively. The elements of the stress tensor τ_{ij} are given by

$$\tau_{ij}=(\mu+\mu_t)\left[\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right]$$

where μ and μ_t are molecular viscosity and eddy viscosity, respectively, δ_{ij} is the Kronecker delta. μ is calculated from the Sutherland law. μ_t is calculated from the Baldwin-Lomax turbulence model[14].

When the initial and boundary conditions are given, equation (1) can be solved by numerical method. In this paper, in time discretion, we use an implicit finite difference scheme, and adopt Yoon & Jameson's LU decomposition method [12]. In space discretion, Harden & Yee's second order TVD scheme [13] is used for the convective terms. The viscous terms are approximated by a second order central difference scheme. A "C-H" type grid of $121\times41\times26$ in the ξ, η and ζ directions respectively is used.

The grid is generated by the conformal mapping method and algebraic method.

The equation of motion of the aileron is

$$\ddot{\delta}+2\varsigma\omega_n\dot{\delta}+\omega_n^2\delta=\frac{1}{I}H(t) \tag{2}$$

where ω_n is the natural frequency of the aileron system, ζ is the damping coefficient, I is the moment of inertia of the aileron, δ is the angle of deflection of the aileron, H is the hinge moment of the aileron, which can be calculated from the pressure distributions obtained by solving the Navier-Stokes equations.

Equation (2) is discretized to second-order accuracy in time as:

$$\frac{\delta^{n+1}-2\delta^{n}+\delta^{n-1}}{\Delta t^{2}}$$

$$+2\varsigma\omega_{n}\frac{\delta^{n+1}-\delta^{n-1}}{2\Delta t}+\omega_{n}^{2}\delta^{n}=\frac{1}{I}H^{n}(t) \quad (3)$$

In numerical calculations, the Navier-Stokes equations and the equation of motion of the aileron are solved simultaneously. On the surface of the wing, we have the following boundary conditions:

$$\frac{\partial T}{\partial n}=0, u=\dot{x}, v=\dot{y}, \frac{\partial P}{\partial n}=-\rho \mathbf{a}\bullet\mathbf{n} \quad (4)$$

(on the aileron surface)

$$\frac{\partial T}{\partial n}=0, u=0, v=0, \frac{\partial P}{\partial n}=0 \quad (5)$$

(on the other part of the wing)

where $\mathbf{a}$ is the acceleration of a point on the aileron surface, $\mathbf{n}$ is the unit normal to the surface.

The initial condition is

$$\delta=\Delta\delta_{0} \quad (6)$$

where $\Delta\delta_0$ is a small deflection angle of the aileron from the equilibrium state.

In order to obtain the time history of the motion of the aileron, two steps of computations are required. In the first step, we calculate the steady flowfield around the wing with a fixed aileron deflection angle $\Delta\delta_0$. This solution represents the initial conditions for the second step. In the second step, set the aileron free to move. And the unsteady Navier-stokes equations and the equation of motion of the aileron are solved simultaneously. At the time step n, the Navier-Stokes equations are solved to get the hinge-moment $H^n(t)$, then the aileron deflection angle δ^{n+1} can be obtained from equation (3), and a new grid is generated, new metrics $\frac{\partial\xi^i}{\partial x_j},\frac{\partial\xi^i}{\partial\tau}$, etc. are computed at each new grid point. An implicit finite difference algorithm described above for Equation (1) is used to advance the flow variables to the next time step. The process is repeated for each subsequent time step.

3 Hopf-bifurcation Analysis

Consider the following nonlinear dynamic system:

$$\dot{X}=f(X,\mu) \quad (7)$$

$$X\in U\subseteq R^{n} \qquad \mu\in V\subseteq R^{m}$$

where X is the vector of the state variables, μ is a parameter of the system.

Let parameter μ vary continuously, when μ equals the critical value μ^*, system (7) becomes unstable, i.e. the topology of the system changes suddenly, then we say a bifurcation of the system takes place.

Let $\dot{X}$ be equal to zero, we get.

$$f(X,\mu)=0 \quad (8)$$

Using the Newton’s iteration formula, equation (8) is solved to obtain the equilibrium solutions of system (7). The Newton's iteration formula is given by

$$D_X f^k(X,\mu)(X^{k+1}-X^k)=f^k(X,\mu) \quad (9)$$

where $D_X f^k(X,\mu)$ is the Jacobian matrix, and the symbol k and k+1 represent the values at the kth and (k+1)th iterations respectively. The iteration process will be continued until the errors are less than a given small value.

The Jacobian matrix $D_X f^k(X,\mu)$ has a set of eigenvalues $\lambda_i (1 \le i \le N)$. If the real parts of all the λ_i are less than zero, then the dynamic system is stable at this equilibrium point.

If at the equilibrium point where $\mu = \mu^*$ and $X = X^*$, the Jacobian matrix has a pair of purely imaginary eigenvalues $\lambda_\pm = \pm i\theta$, and all the other eigenvalues have negative real parts, and $\lambda_\pm$ satisfy the following "transversality condition"

$$\frac{d}{d\mu}(\mathrm{Re}\,al[\lambda_\pm(\mu)])_{\mu=\mu^m} \neq 0 \qquad (10)$$

then this equilibrium point is called a Hopf-bifurcation point. When μ varies across through μ^*, the system will change from stable to unstable, and a limit cycle oscillation will occur.

The above principle can be used to analyze the problem of transonic control surface buzz directly. In equation (2), set $x_1 = \delta, x_2 = \dot{\delta}$, we get the equivalent equations of (2) as follows

$$\dot{x}_1 = x_2 \qquad (11a)$$

$$\dot{x}_2 = -2\varsigma\omega_n x_2 - \omega_n^2 x_1 + \frac{1}{I}H(t) \qquad (11b)$$

When the altitude is kept constant, the Mach number M_∞ may be considered as a parameter of the system.

Let $\dot{x}_1 = 0$, $\dot{x}_2 = 0$, we get

$$x_2 = 0 \qquad (12a)$$

$$-2\varsigma\omega_n x_2 - \omega_n^2 x_1 + \frac{1}{I}H(t) = 0 \qquad (12b)$$

Equation (12) is solved using the Newton's iteration method (formula (9)), and the equilibrium point of the system (11) is obtained. Let

$$f(X, M_\infty) = \begin{Bmatrix} x_2 \\ -2\varsigma\omega_n x_2 - \omega_n^2 x_1 + \frac{1}{I}H(t) \end{Bmatrix} \qquad (13)$$

At the equilibrium point, the Jacobian matrix is

$$D_x f(X^*, M_\infty) = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 + \frac{1}{I}\frac{\partial H}{\partial x_1} & -2\varsigma\omega_n + \frac{1}{I}\frac{\partial H}{\partial x_2} \end{bmatrix} \qquad (14)$$

Substituting $\delta, \dot{\delta}$ back for x_1, x_2 in equation (14), and evaluating its eigenvalues, we get

$$\lambda_{1,2} = \alpha(M_\infty) \pm i\beta(M_\infty) \qquad (15)$$

where

$$\alpha(M_\infty) = -\varsigma\omega_n + \frac{1}{2I}\frac{\partial H}{\partial \dot{\delta}} \qquad (16)$$

$$\beta(M_\infty) = \sqrt{\omega_n^2 - \frac{1}{I}\frac{\partial H}{\partial \delta} - \left(\frac{1}{2I}\frac{\partial H}{\partial \dot{\delta}} - \varsigma\omega_n\right)^2} \qquad (17)$$

Numerical simulations indicate that for some aircraft when the Mach number increases from subsonic to supersonic, the real part $\alpha(M_\infty)$ of the eigenvalues $\lambda_{1,2}$ varies from negative to positive, which means that the system changes from stable to unstable. Hopf-bifurcation occurs at the Mach number when $\alpha(M_\infty) = 0$, which is also the buzz point of the system..

In formula (16), $\dfrac{\partial H}{\partial \dot{\delta}}$ can be written as

$$\frac{\partial H}{\partial \dot{\delta}} = qC_e S_e C_{h\dot{\delta}} \qquad (18)$$

where q is the dynamical pressure, $C_{h\dot{\delta}}$ is the aileron damping coefficient.

In order to evaluate the value of $C_{h\dot{\delta}}$, the aileron is specified to oscillate in harmonic motion with the instantaneous deflection angle

$$\delta(t) = \delta_0 + \delta_a \sin \omega t \qquad (19)$$

where δ_0 is the aileron deflection angle in the initial state, δ_a is the amplitude, ω is the frequency.

Computations are carried out with the aileron motion specified by equation (19) to obtain the corresponding instantaneous surface-pressure distributions. After spatial integration of the pressure distributions on the aileron surface, the corresponding instantaneous hinge-moment coefficient $C_h(t)$ is obtained.

This coefficient $C_h(t)$ is expanded into the following series

$$C_h(t) = C_{ho} + C_{h\delta}(\delta - \delta_0) + C_{h\dot{\delta}}\dot{\delta} + C_{h\ddot{\delta}}\ddot{\delta} + \cdots\cdots \qquad (20)$$

where $\delta, \dot{\delta}, \ddot{\delta}$ are obtained from equation (19).

At a given time t, from equation (20) we obtain an algebraic equation with unknowns $C_{h\delta}$, $C_{h\dot{\delta}}$, $C_{h\ddot{\delta}}$, ⋯⋯, at the next time step, we obtain another algebraic equation etc., thus we get a set of algebraic equations. The unknowns $C_{h\delta}$, $C_{h\dot{\delta}}$, $C_{h\ddot{\delta}}$, ⋯⋯, in these equations are estimated by using the parameter identification method (in our calculations, we used least square method). In numerical calculations, δ_a is small. And ω is chosen to be close to the value of frequency of the aileron oscillations generated from the aileron inertia equation (equation (2) in our paper).

If we neglect the structural damping coefficient ς, then from formula (16), we find $\alpha(M_\infty)$ is direct proportional to $C_{h\dot{\delta}}$. Thus Hopf-bifurcation occurs at the critical Mach number M_∞^* where $C_{h\dot{\delta}} = 0$.

4 Results and discussion

A baseline model described in reference [3] is used for aileron buzz investigation. This is a delta-wing model with a full-span aileron. Its planform is sketched in Fig. 1. The natural frequency of the aileron system is 16.5 Hz. Detail descriptions of the model are given in reference [3]. Fig. 2 shows the computational grid at the initial state. The variations of $C_{h\dot{\delta}}$ with M_∞ at dynamic pressure q = 61.8 psf is given in Fig.3. The bifurcation points at which $C_{h\dot{\delta}}$ = 0, $M_\infty = M_\infty^*$, is obtained by interpolation. Similarly we can get the bifurcation points at q = 49.5psf and q = 42.8psf respectively. The critical Mach numbers corresponding to these bifurcation points are all nearly equal to 0.995. The buzz boundary of the aileron system is shown in Fig.4. The experimental results[3] are also provided in this Figure. The time histories of the aileron oscillations are given in Fig. 5. This figure shows that when $M_\infty = 0.92$, the variation of δ with time is convergent, and the aileron system is stable; When $M_\infty = 1.1$, it becomes divergent, resulted in a limit cycle oscillation, the aileron system is unstable. When $M_\infty = 1.0$, the Mach number is close to the critical Mach number M_∞^*. The amplitude of the aileron oscillations begins to increase, which means an aileron buzz oscillation occurs.

5 Concluding Remarks

Hopf-bifurcation analysis method is used to predict the transonic aileron buzz boundary for a delta-wing model with a full span aileron. The unsteady, three dimensional Navier-Stokes equations are solved to evaluate the instantaneous aerodynamic loads acting on the surface of the aileron. And the aileron oscillation time histories calculations are also presented for comparison. The results indicate that the present method is in good agreement with experimental data and time histories calculations.

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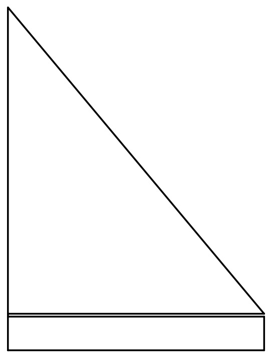
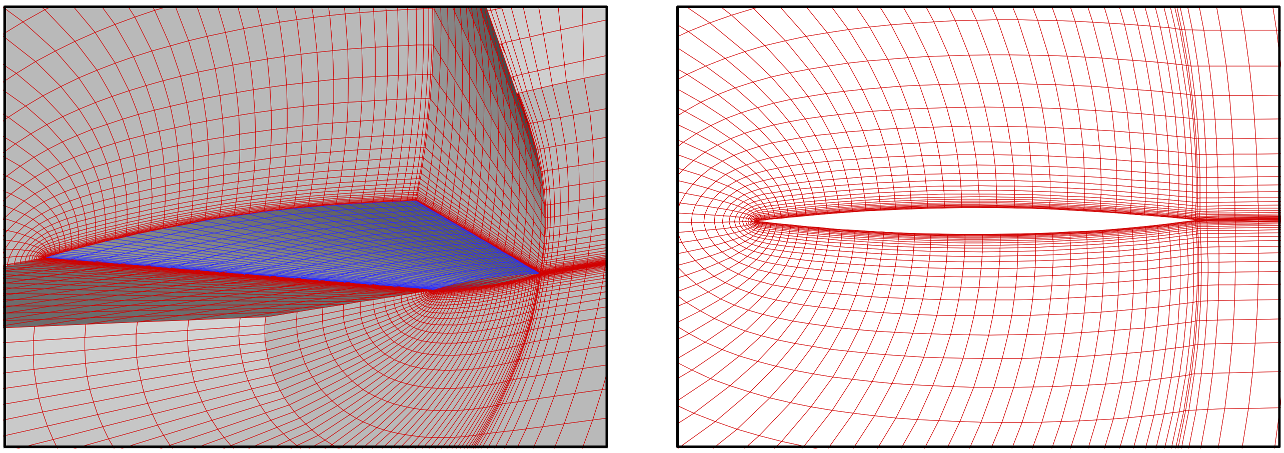


Fig.1 Planform of the baseline model



(a) Portion of the 3-D grid (b) Cross section at the wing root

Fig.2 Computational grid at initial state

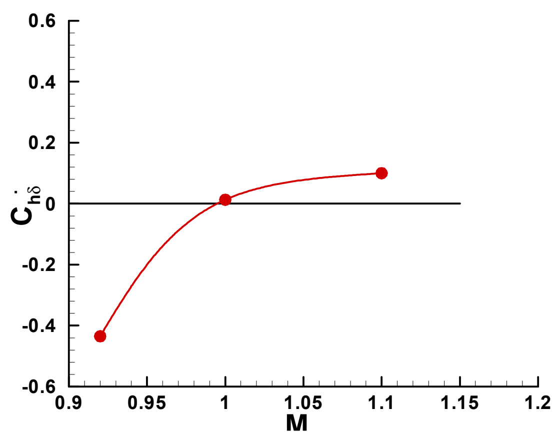


Fig.3 variation of $C_{h\dot{\delta}}$ with M∞

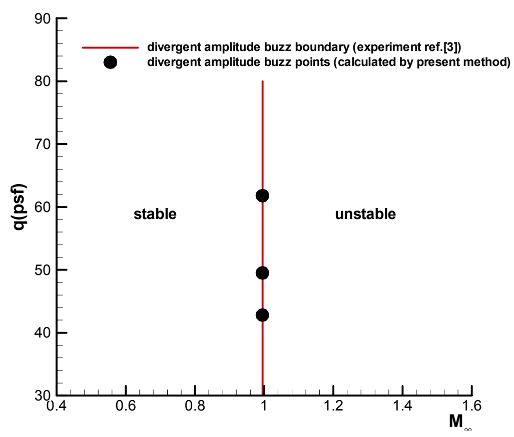
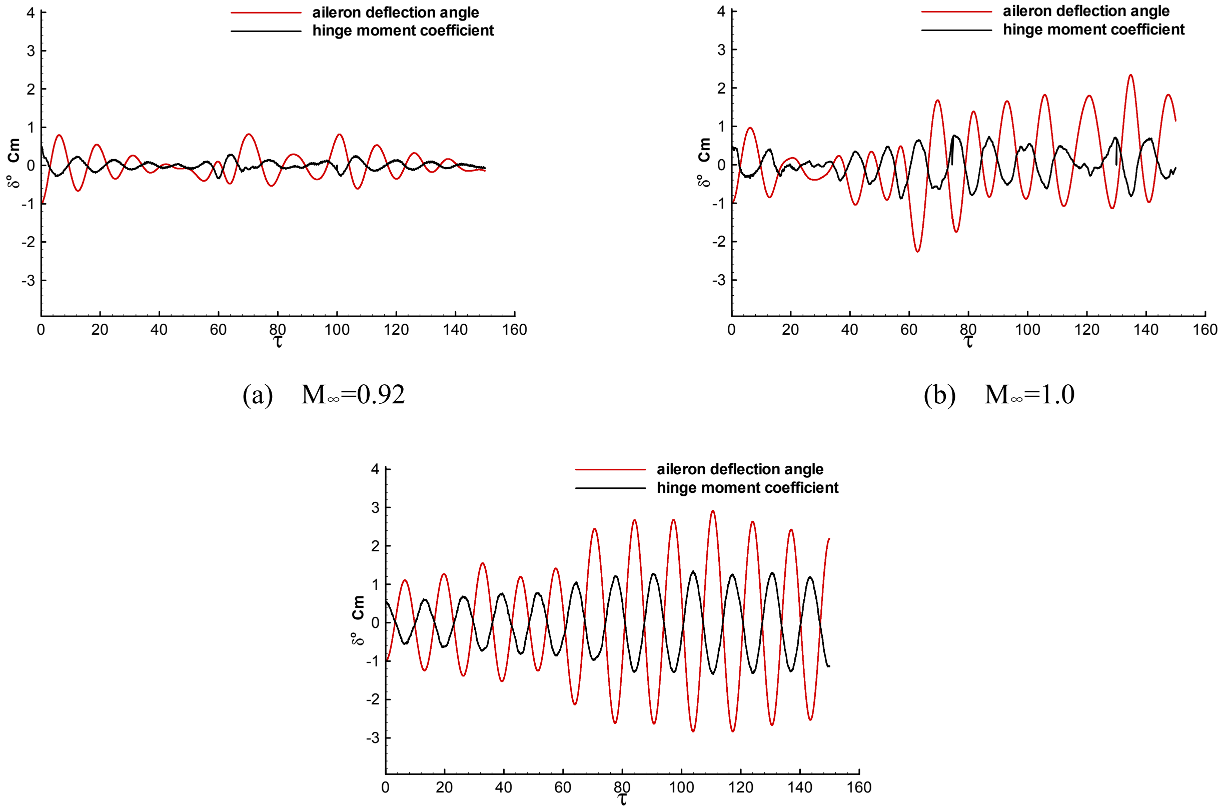


Fig.4 Aileron Buzz boundary for baseline model



(a) M∞=0.92

(b) M∞=1.0

(c) M∞=1.1

Fig.5 Time integration of baseline buzz model at different Mach number