NUMERICAL SIMULATION OF FLOWS INTO INDUCTIVE PLASMATRONS

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Abstract

Simulation of sub- and supersonic flows in plasmatrons is considered. A physico-chemical model, numerical method and computation results for equilibrium inductive coupled plasma flows in a plasmatron are given. An effective preconditioning technique along with an implicit TVD scheme is used to solve the Navier-Stokes equations in both subsonic and supersonic regimes. The governing equations include source terms corresponding to the electromagnetic field influence: the Lorentz force components (so called, magnetic pressure) and Joule heat production. The necessary transport coefficients were calculated in advance for equilibrium plasma as the functions of pressure and temperature. Transport properties were calculated by the precise formulae of the Chapman-Enskog method in temperature range $300 \le T \le 15000\,K$ *for five gases: air, nitrogen, oxygen, argon, carbon dioxide. Calculations of equilibrium air, nitrogen, oxygen, argon and carbon dioxide plasma flows for the IPG-4 (Institute for Problems in Mechanics, Moscow) discharge channel geometry with the channel radius* $Rc = 0.04\,m$ *and length* $Zc = 0.46\,m$ *were performed. Creation of both under-expanded and over-expanded jets in the plasmatron channel is considered. A comparison with experimental results is given.*

1 Introduction

A modern application of the inductively coupled plasmas is simulating thermochemical interaction of high-enthalpy gas flows with thermal protection materials (TPM) at the hypersonic flight conditions [1, 2]. The IPG plasmatrons at IPM RAS appeared to be very efficient tools for the TPM aerothermal testing, and the prediction of the TPM catalycity [1]. The efficient capabilities of the 100-kW IPG-4 plasmatron for the simulation of physico-chemical processes accompanying the hypersonic entry of a vehicle aeroshell in the Martian atmosphere have been demonstrated recently [3, 4]. But in fact, the potential capabilities of R&D facilities to simulate reacting flow physics and real surface processes can be revealed, if the measurements are properly combined with CFD modeling. CFD modeling is an indispensable tool for aerothermal testing in order to carry out CFD codes validation, to rebuild flow field in plasmatron and to extract TPM catalycity related to atoms recombination from heat transfer measurements. In general, the problem is rather tricky. Up to now there are few computations of the nonequilibrium air plasma flow coupled with the RF electromagnetic field [5].

A new numerical algorithm presented and computer code have been developed to simulate the problem without dividing all domain into different subregions. The algorithm allows us to simulate both sub- and supersonic regimes in a unique manner.

In the paper we present the capabilities of CFD modeling inductively coupled plasmas and some results of computations carried out by two different codes in the wide range of the IPG-4 plasmatron operating conditions for different equilibrium plasmas. The advanced technology developed for calculations of the plasma transport coefficients based on the rigorous modification of the Chapman-Enskog formalism [6] and appropriate database for thermodynamic and transport properties have been used.

The time-relaxation method along with the implicit scheme and preconditioning, is used in the computer code developed. The code is based on a new approach to use preconditioning techniques with TVD schemes [7, 8]. It allows one to simulate both low Mach number and supersonic flows in a unite manner. The code has been validated by a comparison against computational results obtained in the von Karman Institute for Fluid Mechanics (Brussels) and the Institute for Applied Mechanics of the Russian Academy of Sciences (Moscow) in [8].

Simulation of both under-expanded and over-expanded supersonic jets in the plasmatron has been carried out. A quite reasonable correspondence with an experimental data is observed.

2 Governing equations and method of solution

Plasma flow in an induction plasmatron discharge channel under the LTE conditions is assumed to be governed by the Navier-Stokes equations for compessible gas (taking into account the Lorentz force and Joule heating) and the Maxwell equations for RF electromagnetic field.

The governing equations are closed with constitutive relations which define electrical conductivity, transport properties and equation of state of plasma under LTE conditions. The equation of state, conductivity and transport properties are handled in a table form.

The full system of governing equations and description of the method of solution are given in [8]. The method allows us to simulate both subsonic and supersonic regimes in a uniform manner.

3 Equilibrium Discharge Gas Dynamics and Electrodynamics

We assume a subsonic flow in a cylindrical discharge channel is stationary, laminar and axisymmetric one with a swirl in azimuth direction. High frequency electromagnetic field does not influence gas transport properties, radiative processes are negligible and the flow is under LTE. To simulate the flow we use full Navier-Stokes equations written in the cylindrical coordinate system with account for three velocity components - axial, radial and also tangential one due to the flow spinning, together with the energy equation written for the total gas enthalpy. These equations include source terms corresponding to electromagnetic field influence: the Lorentz force components (so called the magnetic pressure) and Joule heat production. Boundary conditions for the Navier-Stokes equations are following: necessary flow parameters are specified at annular inlet slot at the channel entry section; velocity components equal zero at rigid surfaces; well known "soft" conditions are applied at the channel exit section; symmetry conditions are applied at the axis.

Suppose that the oscillating external current in separate inductor circular coils produces a monochromatic electric field with the complex amplitude $\vec{E}(z,r)$. Following [9], we use the assumption for tangential component of the electric field amplitude $E_\theta(z,r)$: $\partial E_\theta / \partial z << \partial E_\theta / \partial r$, that leads to quasi-1D approximation of Maxwell equations:

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rE_\theta)\right) = -i\omega\mu_0\sigma E_\theta, \quad i\omega\mu_0 H_z = \frac{1}{r}\frac{d}{dr}(rE_\theta)$$

Here z and r are the axial and radial coordinates; $\omega = 2\pi f$ is the circular frequency of monochromatic electric field, σ is the plasma electrical conductivity, μ_0 is the vacuum magnetic permeance, H_z is the complex amplitude of magnetic field axial component. In this approximation the axial component of Lorentz force equals zero. The quasi-1D approximation obtained is an essential simplification for the problem, it leads to a boundary value problem for the ordinary differential equation written above to determine the complex amplitude of vortical electric field $E_\theta(z,r)$; z coordinate is a parameter only in this equation, E_θ depends on z owing to boundary conditions. To determine E_θ we use also symmetry condition at the axis $E_\theta(z,0) = 0$ and the condition at the discharge channel wall [10] as follows:

$$r = R_c : \quad \frac{1}{\mathrm{r}}\frac{\mathrm{d}}{\mathrm{dr}}\left(rE_\theta\right) = i\omega\mu_0 H_{zc0}(z)$$

Here R_c is the channel radius, H_{z0} is the amplitude of magnetic field axial component at the channel wall produced by the inductor RF current outside the plasma flow. The comparison of the full two-dimensional Maxwell equations solutions with the quasi-1D approximation results given in [11] showed a good accuracy of this approximation in a wide range of operating frequencies for the plasma torch geometry under consideration.

4 Calculation of Plasmas Transport Properties

To solve the Navier-Stokes and Maxwell equations, the following transport coefficients are necessary: the viscosity, thermal conductivity and electrical conductivity. In our approach the transport coefficients were calculated in advance for the equilibrium plasma flows as functions of the pressure and temperature. The effects of non-ideal gas and plasma were not accounted for. The transport properties were calculated by the precise formulae of the Chapman-Enskog method [6] in the temperature range $300 \le \mathrm{T} \le 15000\ \mathrm{K}$ for different gases: air, nitrogen, oxygen, argon, carbon dioxide. The first non-zero approximation is rather accurate to calculate transport coefficients for neutral gases, but for ionized gases it can lead to ~50% error [12]. Our calculations for high temperatures were made with $\zeta = 2$ for viscosity, and $\zeta = 4$ for other transport coefficients to provide 5% accuracy, here ζ is the order of approximation by Sonine polynomials, i.e. number of terms in Sonine polynomial expansions of Boltzmann's equation solution that provide transposition in the Chapman-Enskog method. Exploited formulae [6] for the transport properties are more convenient for the calculations than the classic formulae [11] of the Chapman-Enskog method, because the latter formulae are more complicated and use higher order determinants: $\mathrm{N}\zeta \times \mathrm{N}\zeta$ instead of $\mathrm{N}(\zeta\text{-}1) \times \mathrm{N}(\zeta\text{-}1)$.

A comparison has been done against the computation results obtained at the von Karman Institute for Fluid Mechanics and available experimental data in their reliable range for relevant plasmatron temperatures [13].

5 Simulation Results of Inductive Coupled Plasma Flow. Under-expanded and over-expanded supersonic jets

The under-expanded and over-expanded supersonic jets from the IPG-4 plasmatron through a Laval nozzle into the test chamber have been considered in the current research.

The principal scheme of the ICP tourch with a sonic nozzle is given in Figure 1. The outer inductor is represented by parallel current-currying rings which considered as infinitely thin. The nozzle at the end of the channel allows us to reach the sonic velocity for the flow.

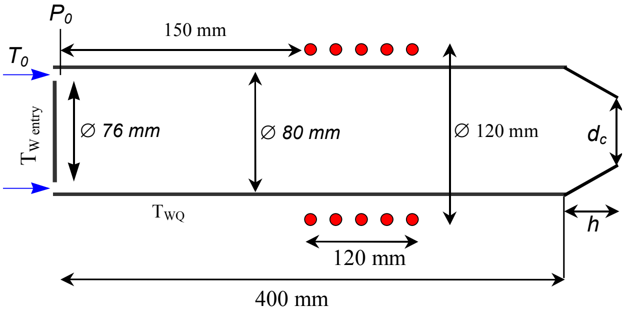

Figure 1. Scetch of discharge channel with sonic nozzle.

After the channel the under-expended jet interacts with a test sample located in the test pressure chamber represented in Figure 2 by a scetch.

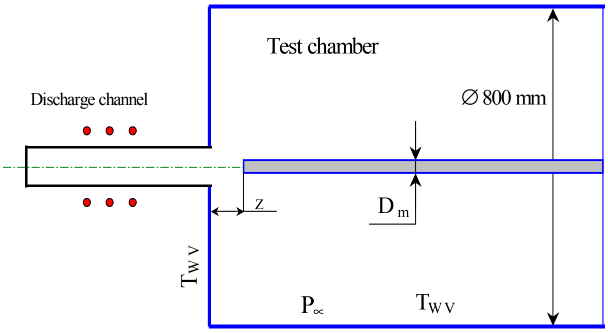

Figure 2. Scetch of test chamber.

The input parameters of the problem are as follows: gas is air, the angle of the flow spinning is 45^0, the operation frequency is *1.76 MHz*, the temperature of the input gas into the channel and walls in the channel and chamber is *300K*.

In the case of under-expanded jets, the gas flow rate $G = 2.4\ g/s$, the power of the plasmatron $N_{pl} = 29\ kW$, its efficiency η is *0.65*, the sonic nozzle diameter d_c is *4sm* and its high is *4.5sm*. The pressure in the test chamber $P_\infty = 6.4\ GPa$ while the pressure $P_0 = 37$ GPa. Strictly speaking, the last value is not necessary for the mathematical statement of the problem and we will turn to this question later.

A comparison of the experimental and computational results is given in Figure 3. The experimental photograph is shown in the upper part. For the comparison, the shadow contour plot picture of the internal energy is in the lower part. The correspondence of the location of "casks", local compression shocks, jet boundary is quite reasonable. We have to remark here that first a comparison with the experiment used was done in [14] using another code and numerical approach. The correspondence with the experimental results obtained has been quite satisfactory as well but the problem has been solved by consideration flows in the channel and test chamber separately. Different codes and numerical methods have been used to simulate these parts. In the second part (test chamber) the value P_0 got from the experiment has been used as the boundary meaning. In our case we solve the problem in a unit manner and P_0 is obtained in the calculations. In the current example we got $P_0 = 41GPa$ that gives us a 10% correspondence with the experimental result.

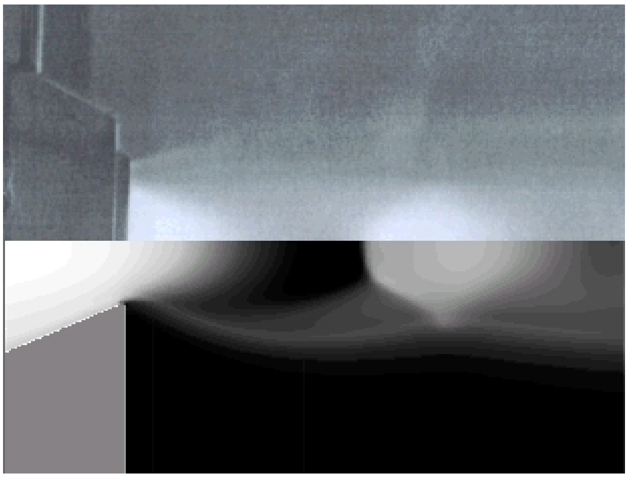

Figure 3: Under-expanded supersonic plasma jet. The upper part is experiment; the lower one is shadow computational picture.

Supersonic flow over a sample cylinder, 2sm in the diameter, is shown in the Fig. 4. The distance between the sonic nozzle, having $d_c = 2.4sm$, and the cylinder equals *2sm*. As in the previous case, the experimental and computational results are presented in the figure. The locations of the shock wave before the cylinder, rarefaction and compression waves quite coincide.

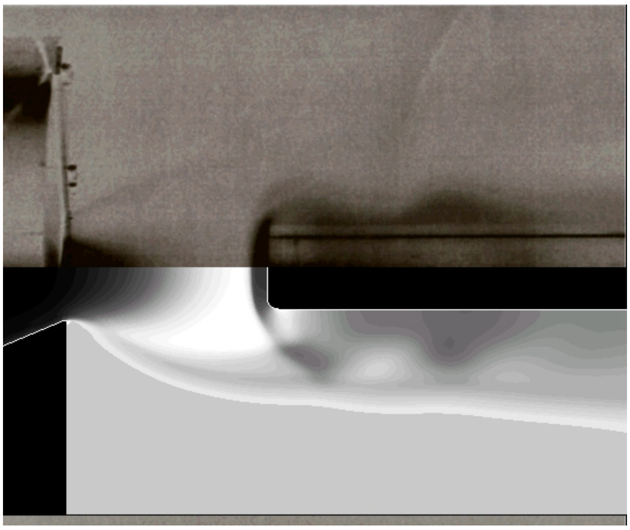

Figure 4: Under-expanded supersonic plasma jet over sample cylinder. The upper part is experiment; the lower one is shadow computational picture.

Further, some examples of modeling over-expanded jets are given. In this case $G = 2.8\ g/s$, $P_0 = 0.1\ atm$, $N_{pl} = 20\ kW$. On the subsonic outlet boundary of the test chamber the pressure is set *1300 Pa*. Thus, rarefaction was created in the test chamber. Mach numbers are changed from $2*10^{-2}$ in the circulated flow up to *2.5* on the nozzle end. In the Fig. 5, the upper part corresponds to streamlines, the lower part – the enthalpy distribution. The pressure was about *700 Pa* on the nozzle end. Thus, the jet is over-expanded in this case and the outflow is accompanied by the typical system of shocks and rarefaction waves. A flow pattern near the nozzle, including the first two “casks”, is presented in the Fig. 6 by the distribution of pressure. The maximal velocity was about *3000 m/s* in the example. All calculations were performed by the unique manner without dividing into different parts.

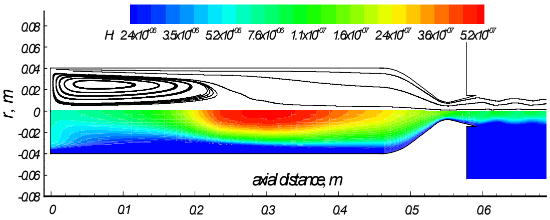

Figure 5: Supersonic outflow from IPG-4 plasmatron. The upper part corresponds to streamlines, the lower one – enthalpy distribution

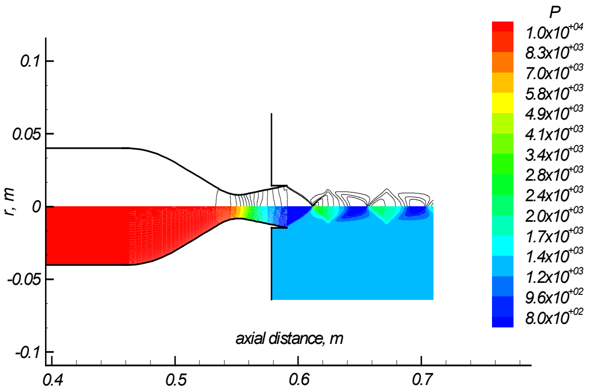

Figure 6: Outflow into test chamber. Pressure (Pa)

6 Conclusion

Numerical modeling equilibrium inductive coupled plasma flows in a plasmatron has been done. An advanced technology developed for the calculations of the plasma transport coefficients based on the rigorous modification of the Chapman-Enskog formalism and appropriate database for thermodynamic and transport properties are has been used.

A new effective preconditioning technique along with an implicit TVD scheme has been used to solve the Navier-Stokes equations in both subsonic and supersonic regimes in a uniform manner. Numerical simulation of under-expanded and over-expanded supersonic jets, including the interaction with a sample in the test chamber, has been performed without dividing the domain investigated into different parts.

The computational results have been validated by a comparison with the experimental ones received in the Institute for Problems in Mechanics of RAS.

Acknowledgment

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