MULTIDISCIPLINARY OPTIMIZATION OF AIRCRAFT PARAMETERS BY THE LOCAL DISTRIBUTED CRITERIA METHOD

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Abstract

The philosophy of the new approach to multidisciplinary optimization of aircraft parameters by a target criterion are formulated in view of requirements of aerodynamics, flight dynamics & control, and strength. The idea of a local decomposition of the initial problem into monodisciplinary subtasks with special criteria – Local Distributed Criteria (LDC), is used. The regular method of LDC formation is proposed on the basis of sensitivity functions. These functions result from transformation of adjoint variables and Lagrangian multipliers extracted from the rigorous solution of the aircraft trajectory optimization problem by the target criterion with use of the indirect method – the Pontryagin maximum principle. The LDC, on the one hand, give monodisciplinary subtasks an independent access to the unified target criterion, and on the other hand, take into account the specific contribution of all aircraft flight phases.

The LDC approach application for the launcher parameter optimization is described. The qualitatively new solutions are demonstrated.

1 Introduction

Computer-aided aerospace design software, which dates back to 60 years, uses, as a rule, the simplified analysis and synthesis procedures on separate disciplines to ensure the convergence of general designing process for reasonable time. It is possible to expect a qualitative growth in the efficiency of aerospace design in the nearest perspective (irrelevant with search for nonconventional types of aircraft) at the expense of integration of modern profound developments in the models and methods accumulated by each of aerospace disciplines, and concentration for the solution of the main (target) problem, for which the aircraft is created.

One of ways to embody such capabilities is offered on the base of:

- the thorough optimization of aircraft control with use of the automated program complex ASTER [1], based on application of the indirect method, the Pontryagin maximum principle [2];
- formation of the local distributed (on flight regimes) criteria (LDC) on the basis of the solution of an adjoint system, ensuring subordination of “monodisciplinary” problems (optimization of the aerodynamic shape and structural scheme) to the unified purpose of reaching the maximum efficiency of the aircraft;
- the maximum use of available software;
- interactions between experts in different areas of aerospace sciences on the basis of equal rights and a mutual interest.

In the basis of the approach is the idea of local decomposition of an initial problem into monodisciplinary subtasks with personal criteria for each discipline. The peculiarity of the approach consists in a way of setting these criteria: they are formed objectively, on the one hand, to give a monodisciplinary problem an independent output to the target application criterion, and, on the other hand, to allow taking into account the specific contribution of all flight regimes.

The basis for practical embodying of the approach is the essential progress of the authors [1] in automation of the optimization problem solution by the maximum principle for branched processes with different types of constraints.

It is supposed, that the developed approach will be most useful for optimization of the aircraft, which characteristics vary essentially on flight segments, as, for example, for aircraft in free flight, supersonic planes, aerospace launchers, etc. The LDC application takes objectively into account the "weight" of each flight regime (not in isolated points only but distributed continuously) through the criterion of target application efficiency.

Advantages of the LCD technique are follows: an integrated approach, natural subordination of monodisciplinary variables and parameters to the single target criterion, an independence in selection a research method within the framework of each discipline, taking into account a non-linear character (including bifurcations) of a functional relation to parameters.

2 Substantiation of the LDC method

Let efficiency of fulfillment of some target task is characterized by criterion

$$\Phi \Rightarrow \max. \tag{1}$$

The efficiency of the target application depends on selection of the control law $\mathbf{u}(\mathbf{x}, t) \in \mathcal{U} \subset \mathbf{R}^m$ and vector parameter $\mathbf{p} \in \mathcal{P} \subset \mathbf{R}^p$:

$$\{\mathbf{u}, \mathbf{p}\}_{\text{opt}} = \arg\max \Phi. \tag{2}$$

Here $\mathbf{x} \in \mathcal{X} \subset \mathbf{R}^n$ is the state vector, $t \in [t_i, t_f]$ is the time. The parameter $\mathbf{p}$ determines the aircraft layout and influences on the right member of the motion equation, on constraints on the admissible control and state vector, including initial conditions. The parameter $\mathbf{p}$ can also be included explicitly into the functional Φ. Thus, the optimization problem (2) breaks up naturally to the control optimization problem:

$$\{\mathbf{u}\}_{\text{opt}} = \arg\max \Phi|_{\mathbf{p}=\text{fix}}, \tag{3}$$

and the non-linear programming problem:

$$\{\mathbf{p}\}_{\text{opt}} = \arg\max \Phi|_{\mathbf{u}=\mathbf{u}_{\text{opt}}}. \tag{4}$$

The problem (3) is the typical flight mechanics problem, while (4) reflects a multidisciplinary problem of the optimal selection of an aerodynamic layout, a structural scheme, a propulsion system, and also a trajectory and control. The flight mechanics problem arises here newly because of a possible explicit dependency of the allowable control and state set on $\mathbf{p}$.

From the mathematical point of view the problem (4) can be solved investigating the relation of the optimal solution of the problem (3) to the parameter $\mathbf{p}$. The continuation method or the neighborhood extremals method [4] can be considered as effective approaches for that.

Let's consider problems (3) and (4) in more detail. Let the vehicle motion is described by the vector equation:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t), \quad t_i \le t \le t_f \quad , \tag{5}$$

where $\mathbf{x}(t)$ and $\mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p},t)$ are piecewise continuous functions.

The state and control vectors are limited to a system of inequalities:

$$\mathbf{x} \in \mathcal{X} = \left\{ \mathbf{x} \in \mathbf{R}^n : \mathbf{X}(\mathbf{x}, \mathbf{p}, t) \le 0, \mathbf{X} \in \mathbf{R}^{n_1} \right\}, \tag{6}$$

$$\mathbf{u} \in \mathcal{U} = \left\{ \mathbf{u} \in \mathbf{R}^m : \mathbf{U}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) \le 0, \mathbf{U} \in \mathbf{R}^{m_1} \right\}. \tag{7}$$

With agree to [4] the state constraint $\mathrm{X}_l(\mathbf{x}, \mathbf{p}, t) \le 0$ of the "k_l" order breaks up to constraints of an equality type on the control (7):

$$\mathrm{U}_j(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) = \frac{d^{k_l} \mathrm{X}_l(\mathbf{x}, \mathbf{p}, t)}{dt^{k_l}} = 0,$$

and on the function of state vector in isolated points of the trajectory:

$$\begin{aligned} D_j(\mathbf{x}, \mathbf{p}, t) &= \mathrm{X}_l(\mathbf{x}, \mathbf{p}, t) = 0, \\ D_{j+1}(\mathbf{x}, \mathbf{p}, t) &= \frac{d\mathrm{X}_l}{dt} = 0, \\ &\dots\dots \\ D_{j+k-1}(\mathbf{x}, \mathbf{p}, t) &= \frac{d^{k_l-1}\mathrm{X}_l}{dt^{k_l-1}} = 0. \end{aligned} \tag{8}$$

Other similar constraints in isolated points of a trajectory can be imposed on the state vector. Generally:

$$\begin{aligned} &\mathbf{D}(\mathbf{x}^-(t_1), \mathbf{x}^+(t_1), \dots, \mathbf{x}^-(t_q), \\ &\quad \mathbf{x}^+(t_q), \mathbf{p}, t_1, \dots, t_q) = 0, \ \mathbf{D} \in R^{n_2}, \end{aligned} \tag{9}$$

where $\mathbf{x}_j^- = \mathbf{x}(t_j - 0)$, $\mathbf{x}_j^+ = \mathbf{x}(t_j + 0)$, $t_1, \dots, t_q$ are instants of jumps of the function $\mathbf{f}$ or state vector

x, output to state constraints, change of the state vector structure etc.

The optimal control problem can be formulated as follows: it is required to find the control $\mathbf{u} = \mathbf{u}(\mathbf{p}, t)$, ensuring fulfillment of requirements (5) - (9) and supplying the maximum to the functional

$$\Phi \equiv \Phi\left(\mathbf{x}^-(t_1),\ldots,\mathbf{x}^+(t_q),\mathbf{p},t_1,\ldots,t_q\right) \Rightarrow \max . \quad (10)$$

To solve the problem we use the indirect optimization method – the Pontryagin maximum principle [2].

Pursuant to a formalism of the maximum principle the Hamiltonian is

$$\mathcal{H} = \mathbf{y}^T \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p},t) + \mathbf{l}^T \mathbf{U}(\mathbf{x},\mathbf{u},\mathbf{p},t), \quad (11)$$

where $\mathbf{y} \in \mathbf{R}^n$ is the adjoint vector, $\mathbf{l} \in \mathbf{R}^{m_1}$ is the vector of Lagrangian multipliers.

The adjoint vector is determined by the equation [2]:

$$\frac{d\mathbf{y}}{dt} = -\left[\frac{\partial \mathcal{H}}{\partial \mathbf{x}}\right]^{\mathrm{T}}, \quad (12)$$

and boundary (transversality) conditions:

$$\mathbf{y}(t_j+0) = -\left[\frac{\partial \Phi}{\partial \mathbf{x}_j^+}\right]^{\mathrm{T}} - \left[\frac{\partial \mathbf{D}}{\partial \mathbf{x}_j^+}\right]^{\mathrm{T}} \mathbf{n},$$

$$\mathbf{y}(t_j-0) = \left[\frac{\partial \Phi}{\partial \mathbf{x}_j^-}\right]^{\mathrm{T}} + \left[\frac{\partial \mathbf{D}}{\partial \mathbf{x}_j^-}\right]^{\mathrm{T}} \mathbf{n}, \quad (13)$$

$$\mathcal{H}(t_j+0) - \mathcal{H}(t_j-0) = -\frac{\partial \Phi}{\partial t_j} - \mathbf{n}^T \frac{\partial \mathbf{D}}{\partial t_j}.$$

The optimal control is determined by the condition

$$\mathbf{u}_{opt} = \arg\max_{\mathbf{u}} \mathcal{H}. \quad (14)$$

According to Bliss formula [5], adjoint variables and the Lagrangian multipliers in (11) under the transversality conditions (13) have simple physical meaning on the optimal trajectory:

a) The adjoint vector is a function of sensitivity of the functional to variations of the current state vector:

$$\mathbf{y}(t) = \left(\frac{\partial \Phi}{\partial \mathbf{x}(t)}\right)^T; \quad (15)$$

b) The Lagrangian multiplier vector **n** is a factor of sensitivity of the functional to variations of the relevant constraints:

$$\mathbf{n} = \left(\frac{\partial \Phi}{\partial \mathbf{D}}\right)^T. \quad (16)$$

At a small variation $\delta\mathbf{p}$ of the parameter **p** the variation $\delta\Phi$ is determined by the formula:

$$\delta\Phi = \delta_p \Phi + \mathbf{n}^T \delta_p \mathbf{D} + \\ + \int_{t_i}^{t_f} \left(\mathbf{y}^T \delta_p \mathbf{f} + \mathbf{l}^T \delta_p \mathbf{U}\right) dt = \nabla_p \Phi \, \delta\mathbf{p} \Rightarrow \max, \quad (17)$$

where δ_p is the variation of a function caused by $\delta\mathbf{p}$. The variables in (17) correspond to the optimal solution at the nonperturbed parameter **p**. The gradient of the functional on **p**:

$$\nabla_p \Phi = \frac{\partial \Phi(\mathbf{x}^-(t_1),\ldots,\mathbf{x}^+(t_q),\mathbf{p},t_1,\ldots,t_q)}{\partial \mathbf{p}} + \\ + \mathbf{n}^T \frac{\partial \mathbf{D}(\mathbf{x}^-(t_1),\ldots,\mathbf{x}^+(t_q),\mathbf{p},t_1,\ldots,t_q)}{\partial \mathbf{p}} + \\ + \int_{t_i}^{t_f} \left[\mathbf{y}^T \frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p},t)}{\partial \mathbf{p}} + \mathbf{l}^T \frac{\partial \mathbf{U}(\mathbf{x},\mathbf{u},\mathbf{p},t)}{\partial \mathbf{p}}\right] dt, \quad (18)$$

sets the direction of the improving variation in the space of parameters $\mathcal{P}$.

The optimization of aircraft layout parameters **p** by the target criterion (1) is reached as a result of successive steps containing calculations of the characteristics in (17), including gradients in the space of parameters **p,** within the framework of separate disciplines. Improvement of aircraft parameters is possible so long as the gradient of the functional (1) has a positive projection to the cone of permissible variations in $\mathcal{P}$:

$$\nabla_p \Phi \, \delta\mathbf{p} \geq 0, \quad \mathbf{p} + \delta\mathbf{p} \in \mathcal{P}.$$

The variation of the functional (17) has a meaning of the ***local distributed criterion*** for contiguous disciplines. **"Locality"** of the LCD is stipulated by its correspondence with the quickest descent in the space $\mathcal{P}$ at a small variation $\delta\mathbf{p}$ only. For "large" variations the influence functions in the expression of the gradient (17) is generally corrected as a result of the solution of the relevant optimal control problems for changed values of **p** on intermediate steps with small δ**p**. "**Distributiveness**" of the LDC is determined by that the integration function takes into account a change of the influence of the parameter vector **p** on the functional along the optimal trajectory **x***(t)*. Thus, the criterion (17) allows objectively taking into account a distribution of the specific influence of **p** on the target application criterion on all flight regimes. The criterion (17) realizes the integrated approach because while solving the monodisciplinary problems it allows to pass from the space of internal monodisciplinary parameters and variables to the general target application criterion.

The role of such LDC capabilities is clear if, for example, the classical problem of the aerodynamic shape optimization for a supersonic plane is considered. Obviously, the solutions of this problem by any "conventional" criterion (the maximum lift-to-drag ratio, the wave or full aerodynamic drag etc.) will result not only to different, but also, probably, opposite recommendations if subsonic and supersonic flight regimes are considered separately. The LDC make it possible to take objectively into account "the weight" of each flight regime (distributed continuously) to maximize the target efficiency.

3 "Monodisciplinary" components of the LDC technique

The LDC method does not impose any special requirements to software used inside each discipline, except for, may be, the trajectory optimization program. It should provide the LDC calculation, for what an effective mean is the Pontryagin maximum principle. At the same time, an accuracy of obtained results, their reliability, and the calculation time are obviously connected to the efficiency of used monodisciplinary methods.

Described below in this section are the basic features of techniques and software used by the authors for practical implementation of the LDC method of the multidisciplinary optimization.

3.1 The trajectory optimization software ASTER based on the Pontryagin maximum principle

To calculate the LDC enabling to implement the decomposition of the initial integrated problem into monodisciplinary ones, it is necessary, according to (17), (18), to provide a regular numerical procedure for the solution of a multi-point boundary value problem for the state (5) and adjoint (12) differential equations.

For this purpose the technique of rigorous solution of the trajectory optimization problem with regard to practical constraints on the basis of the Pontryagin maximum principle is used in the automated program complex ASTER [1]. The ASTER complex was developed originally for the control and trajectory optimization of spacecraft injected into an Earth orbit, of a sub-orbital flight or interorbital transfers.

Mathematical models of the ASTER complex enable to vary objects of research by quantity of stages, a type of the propulsion system (liquid- or solid-propellant rocket engines, airbreathing engines), types of constraints (on control: angle of attack, thrust, g-load etc.; on a trajectory: Mach number, dynamic pressure, heat flows etc.; on conditions of spent components reentry: impact or landing sites, thermal or g-loads, Mach number etc.). The complex is supplied with the interface software for Microsoft Windows.

The ASTER complex was tested and used for design and feasibility studies of current and advanced space transportation systems from "Proton" to MAKS.

The following information of the ASTER solution is important for the LDC technique:

- the optimal control and trajectory;
- external loads on the nominal (optimal) flight trajectory;
- influence functions of the parameter **p** determining the right member of the motion equation (5) and constraints (6) - (9) on the functional;
- influence functions of disturbances of state coordinates at each point of the optimal trajectory on the target functional.

For example, in the aerodynamic shape optimization problem with strength constraints on the dynamic pressure q: $D_q \triangleq q - q_{adm} \le 0$ and the normal g-load n_z: $U_{nz} \triangleq n_z - n_{z\,adm} \le 0$, the LDC can be written as follows:

$$\nabla_p \Phi = \nu_q \frac{\partial D_q}{\partial \mathbf{p}} + \int_{t_i}^{t_f} \left(F_{c_{D0}} \frac{\partial c_{D0}}{\partial \mathbf{p}} + F_{L/D} \frac{\partial (L/D)_{\max}}{\partial \mathbf{p}} + \right.$$
$$\left. + F_k \frac{\partial k}{\partial \mathbf{p}} + \lambda_{n_z} \frac{\partial U_{n_z}}{\partial \mathbf{p}} \right) dt,$$

where $F_\varphi = \mathbf{y}^T \frac{\partial \mathbf{f}}{\partial \varphi}$, $\varphi = (c_{D0}, (L/D)_{\max}, k)$ includes the zero-lift drag coefficient, the maximum lift-to-drag ratio, and the induced drag coefficient consequently. The aerodynamic coefficients are piecewise continuous functions of the state vector and time. The functions F_φ and λ_{n_z} reflect the distribution through flight regimes of the specific influence of φ and n_z on the functional. The typical behavior of $F_{L/D}$ and λ_{n_z} calculated by the ASTER for the problem of the winged launcher ascent into the low Earth orbit (LEO) is shown in Fig. 1. Figure 2 demonstrates the typical dependence of the sensitivity factor ν_q (see (16)) on q_{adm}.

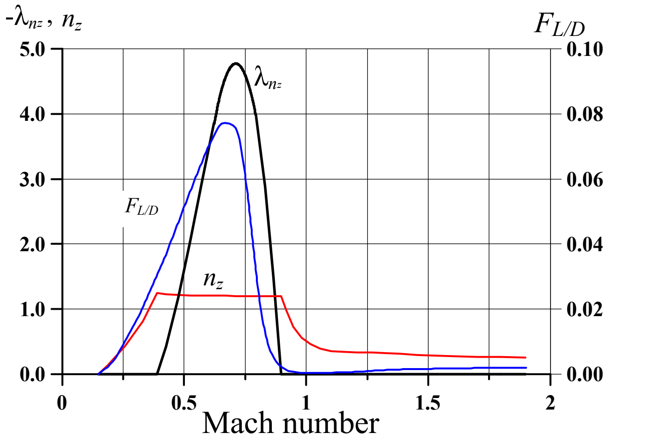

Fig. 1 The Lagrangian multiplier λ_{nz}, the sensitivity function $F_{L/D}$ and normal g-load n_z versus Mach number in the problem of winged launcher ascent into the LEO.

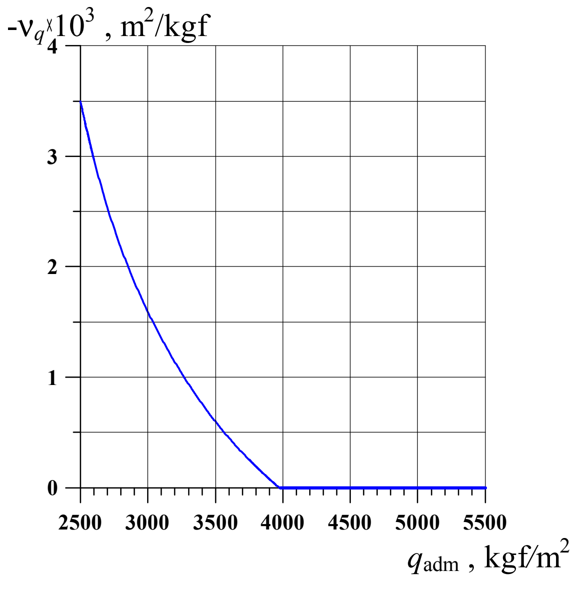

Fig. 2 The dependency of sensitivity factor ν_q on the maximum admissible dynamic pressure q_{adm}.

3.2 Calculation of three-dimensional nonviscous flows about aircraft components based on the universal multi-zone technique

Applied software as mean of automation of computation is widely used for solving numerically a variety of aerogasdynamic problems.

Developed in TsAGI the universal software ARGOLA-2 is based on the Godunov method [9], the principle of stabilization time relaxation technique, and the multi-domain technique of dividing a calculated region into subregions. The Godunov method was modified by replacing the first-order difference scheme by Kolgan gradient scheme of the higher order of accuracy [10, 11]. This approach is known as TVD.

The computation of each subregion and interface between subregions are performed by unified algorithms. The interfaces between subregions are set up automatically regardless of the order of grid cells numbering.

The ARGOLA-2 makes it possible to carry out numerical simulation of steady and unsteady flows of sophisticated geometry and topology over a wide range of defining parameters such as Mach number M_∞, angle of attack α and so on. In particular, the Mach number of the oncoming stream can vary from low subsonic to hypersonic values. At hypersonic flight velocities and high angles of attack, the developed software makes it possible to take into account factors that are not modeled in wind tunnels (flight thermodynamic air properties).

The Godunov method represents the special case of the finite volume method. The whole flow area is divided into a large number of tiny cells for which the laws of mass, momentum, and energy conservation are considered. Two separate problems are solved:

- mass, momentum, and energy flux from one cell to another is determined for each couple of neighboring cells depending on current values of gas parameters in cells;
- change of mass, momentum, and energy in each cell is determined in every short time span Δt depending on fluxes from the neighboring cells.

In the ARGOLA-2, each region is cut into subregions ('curvilinear' cubes), and then subregions are divided into elementary cells, i.e. a 3-D calculation grid is built up. Here, in the case of well-designed grid for the whole calculated region, numerical modeling for different flow envelopes can be made with the same grid that increases the efficiency of the software at all stages of the numerical experiment.

With classical approach for nonviscous flows, the system of Euler's equations with boundary and initial conditions is used to solve specified mathematical task by any existing method. However, in the case of the Godunov approach, this formal stage is redundant since

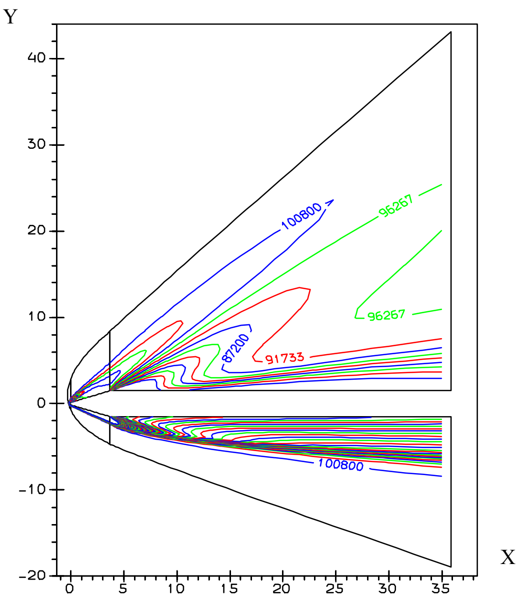

Fig. 3 Pressure isolines in the plane of symmetry Z=0 of blunted cone-cylinder, $M_\infty = 3$, $\alpha = 20°$.

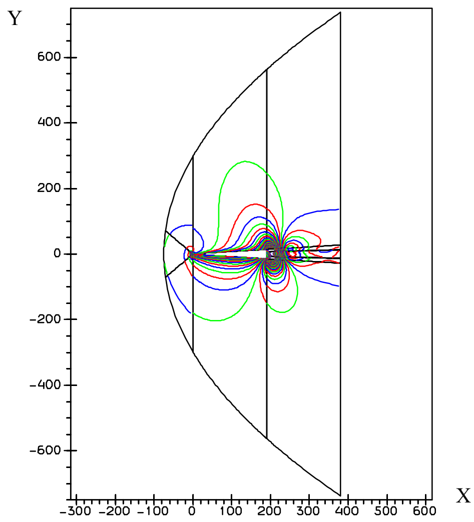

Fig. 4 Pressure isolines in the plane of symmetry Z=0 of the elliptic cone, M_∞=0.8, α=10°.

Euler's equations do not appear explicitly in the Godunov method. Instead, a direct discrete modeling of gasdynamic flow process based on integral conservation laws is carried out.

The ARGOLA-2 is used in applied research of a variety of aircraft. In particular, within the framework of developing the LCD technique, parametric calculations of the flow about sample body like "blunted cone-cylinder" with the cone half-angle 20°, bluntness radius 0.15 , and the radius of the cylindrical part 1.5 were carried out. As an example, pressure isolines in the plane of symmetry Z=0 are shown in Fig. 3 with $M_\infty = 3$ and the angle of attack $\alpha = 20°$. Looking at the behavior of isobars in the shock layer, one can see the bow shock and the inner suction wave near the leeward surface.

Numerical computations were also accomplished for the elliptic cone with a triaxial ellipsoid bow of the radius 0.25 in the plane of symmetry. Numerical modeling was performed for nonviscous perfect gas ($\gamma = 1.4$) with the following parameters of oncoming flow: $0.6 \le M_\infty \le 20$, $0 \le \alpha \le 30°$, pressure $p = 10^5$ Pa, density $\rho = 1$ kg/m^3. Total number of cells was taken up to $4 \cdot 10^5$.

The calculated pressure isolines p=const in the plane of symmetry Z=0 for the flow of elliptic cone are shown in Fig. 4 with $M_\infty = 0.8$, angle of attack $\alpha = 10°$, and the ratio of semiaxis in the cross section 0.5. The shown picture gives an idea of the flow peculiarities and complex topology of the perturbed flow field, including behind the bottom cross section.

3.3 The technique and software for airframe design

Optimizing structural parameters is one of the most challenging issues in the multidisciplinary approach to airframe design. Used as the "single-discipline", the internal optimality criterion for forming and choosing a configuration to maximize the vehicle target efficiency is most commonly the structural mass - or another functional including masses. The design process is bound to meet constraints for: geometry, stiffness, displacements, stresses, manufacturing process parameters, etc.

Within the design parameters space these constraints establish an allowable area within which the optimum values are sought. The general methodology of the search process is depicted in Fig. 5.

Described below is the operation of the complex for designing configurations and structural concepts of launch vehicles (boosters) which is based on the methodology and automated fine (high-dimension) finite-element models. It is seen from Fig. 5 that the procedure is iterative, including three major optimization blocks:
- for determining design loads,
- for establishing configuration parameters, and
- for defining structural concept.

Results of each block are continually monitored by the control module incorporating the database which intended for:
- analyzing input data,
- forming the variation domain for basic parameters,
- compiling approximation functions for geometric description,
- establishing constraints and restrictions and analyzing optimality criteria for the entire set of structural variables, and so on.

The database contains also the statistical information obtained as the result of certification analysis. Each of the three blocks corresponds to interaction with adjacent disciplines.

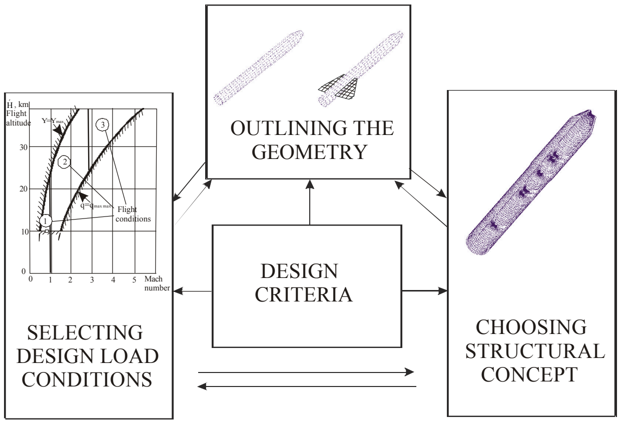

Fig. 5 The methodology of designing an optimal airframe configuration.

Optimum parameters are sought within an iterative process. Design load determination is associated with flight path optimization. Establishing the configuration parameters is associated with external geometry optimization. Lastly, the structural concept selection stage includes optimizing principal loads and defining the rational structural concept (with its respective optimal thickness and cross-sectional areas). The iterations are carried out until all optimality conditions are met.

Calculation of strength, stiffness and mass variables is based on finite-element design models capable of fine discretization.

The necessary dimension of the finite-element model is determined automatically - by proceeding from the requirement of suitable modeling of stress fields and structural and manufacturing features of major components. In the case of a large number of variables the structural optimization is an extremely cumbersome and time-consuming process which almost impractical. A solution for particular configurations can be performed with use of specialized automated design module that rely on representing the structural parameters vector $\{ a_i \}$ as a function $a_i = F_i(\{A_j\})$ of a small number of basic parameters $\{A_j\}$. The types of approximating functions $\{F_i\}$ depend on the allowable domains of the basic parameters, $\{\Delta A_j\}$.

The algorithm is exemplified in Fig. 6 showing results of studies for determining how the structural mass depends on ellipticity of the launcher cross section. Two structural concepts

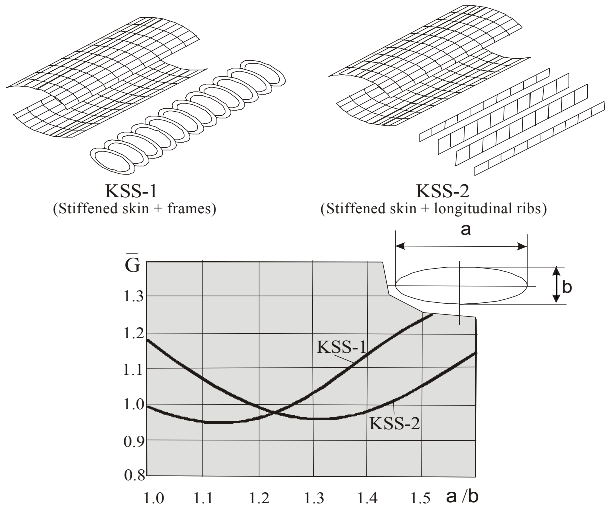

Fig. 6 Dependence of the structural weight on structural concept type.

were considered, differing in their composition of primary elements:

- KSS-1 = stiffened skin + frames,
- KSS-2 = stiffened skin + longitudinal ribs.

The vehicle was assumed to be subjected to pressurization, longitudinal compression, and bending.

The other example is the preliminary study of strength, stiffness and mass of a cylindrical booster as functions of transverse loads caused by the lifting force. The aerodynamic loads add bending to the usual pressurization and longitudinal compression. Therefore, the structure must be strengthened by adding a load-carrying material. The major challenge here is to minimize the extra mass, provided that all restrictions are obeyed. Figure 7 demonstrates loading the cylindrical booster subjected to aerodynamic forces and the thrust.

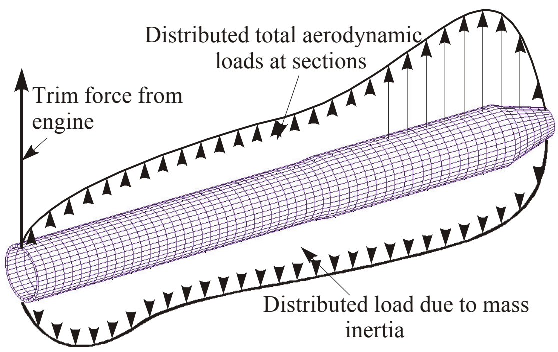

Fig. 7 Scheme of additional loading of the launcher.

4 The LDC approach applications: the launcher parameter optimization

Intensive search for ways to drop the costs of the ascent into an Earth orbit is pursuing at present. Two tendencies stand out here: to modify rocket launchers of a conventional type and to advance aerospace planes. From the point of view of flight mechanics the key distinction between them consists in the attitude to a role of the atmosphere.

The representatives of the first tendency traditionally treat the surrounding medium as a source of a resistance, which should be minimized. This reason (in the absence of additional reentry requirements) results in the selection of the corresponding aerodynamic layout of the "rocket" type and the "traditional" control schedule, which is characterized by a vertical start, zero angles of attack in flight in dense atmospheric layers, and the quasilinear time program of pitch angle in rarefied atmosphere [7,8]. In the classification of the optimal control laws [3] this traditional type of the control laws is referred to as type *B* ("ballistic").

The representatives of the second tendency, on the contrary, lean on the atmosphere (both in a literal and a figurative sense). For this purpose the aerospace planes are given the perfect aerodynamic shape with high lift capabilities. For the planes the optimal control laws (type *A*, "aerodynamic", in the classification [3]) are qualitatively differ from traditional ones: the time program of pitch angle has the oscillatory structure in dense atmospheric layers, the quasi-horizontal start is optimal, higher dynamic pressure is realized on the optimal trajectories [3].

The multidisciplinary optimization technique allows to perform an objective investigation and to determine clearly, in terms of geometrical parameters of a layout, the boundary dividing areas of the optimality of two indicated classes of aircraft.

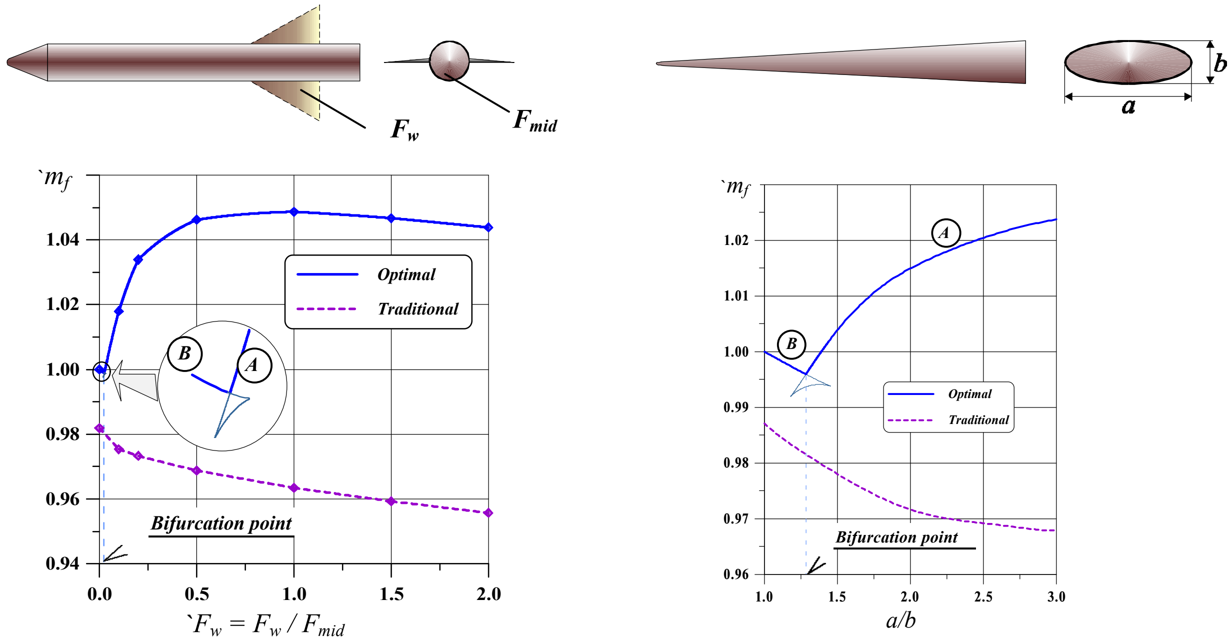

Fig. 8 The relative injected mass `$m_f$ versus the relative outer wing area `F_w for the winged cylindrical vehicle and the ellipticity parameter *a/b* for conical one. The bold lines are relevant to the global optimum of *A* and *B* types.

Presented in Fig. 8 are the typical relations of injected mass to geometrical parameters of the launcher layout such as the cross-section ellipticity (the width-to-height ratio) *a/b* and the relative outer wing area `F_w [6]. Markers A and B sign the curve segments corresponding to the relevant qualitatively different types the optimal control laws.

It is seen that the new *A*-type control laws can be optimal alongside with traditional *B*-type ones for aerodynamic shapes which look at first glance like conventional rockets. It is important that small changes of shape parameters of the launchers can result in a qualitative (bifurcation) change of the optimal trajectory and control law structure. In turn, it leads to essential non-linear relations (with gradient jumps) of the target criterion (`m_f) to all vehicle parameters (the thrust-to-weight ratio, the aerodynamic shape, etc.) [6].

Shown in Fig. 9 are the relations of the maximum dynamic pressure q and product ($q\alpha$), indicating a local load level, to the relative outer wing area `F_w . Although the optimal control laws of *A*-type are accompanied by higher aerodynamic loads on the launcher construction, the loads do not exceed the reasonable (in practice) level for current launchers of the “rocket” type.

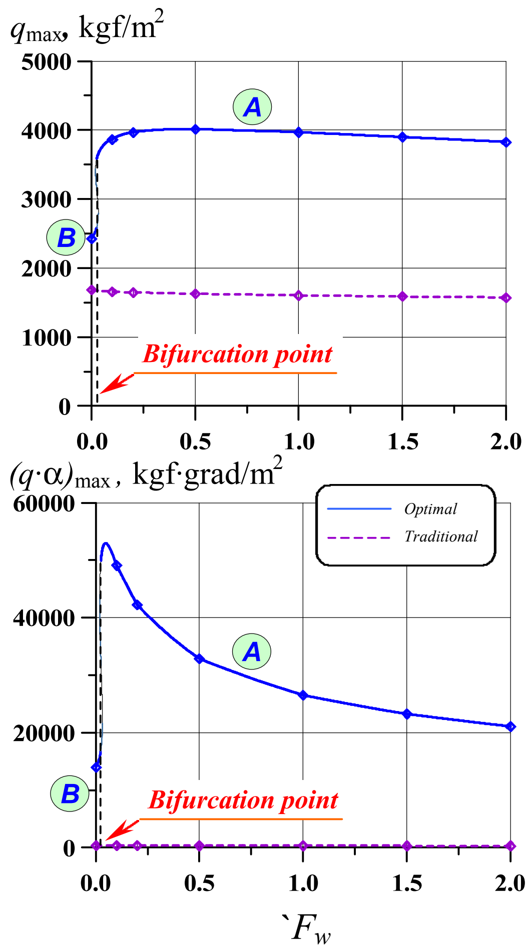

Fig. 9 Maximum dynamic pressure q_{max} and factor $(q\cdot a)_{max}$ versus the relative outer wing area `F_w on the optimum injection trajectories.

Moreover, if the maximum admissible loads q_{adm} and $(q\alpha)_{adm}$ are smaller than the maximum values q_{max} and $(q\alpha)_{max}$ on the optimal trajectories, the losses of the injected mass are small values of higher order than variations $\delta q_{adm} = q_{max} - q_{adm}$ and $\delta(q\alpha)_{adm} = (q\alpha)_{max} - (q\alpha)_{adm}$:

$$\delta \bar{m}_f = \bar{o}(\delta q_{adm}, \delta(q\alpha)_{adm}).$$

The typical relation of the maximum injected mass to $(q\alpha)_{adm}$ is shown in Fig. 10 for the outer wing which area equals to the cross section one ($\bar{F}_w = 1$). According to Fig. 10 the $(q\alpha)_{adm}$ drop on 30 % results in loss of target functional $\bar{m}_f$ of only 0.2%.

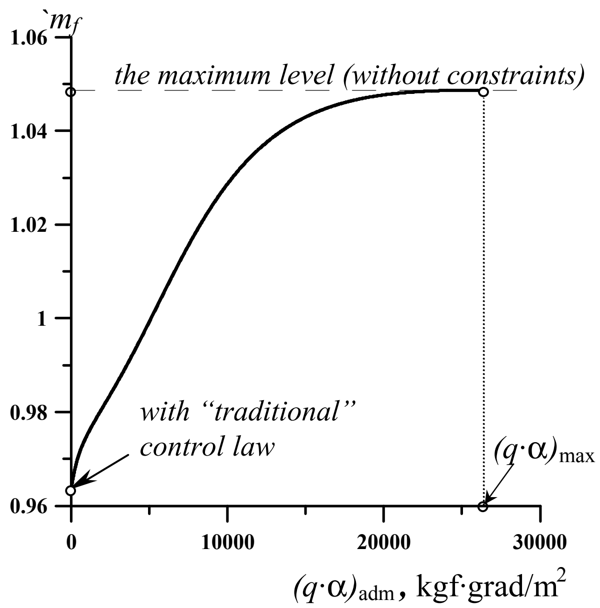

Fig. 10 The relative injected mass $\bar{m}_f$ versus $(q{\cdot}\alpha)_{adm}$ for the winged cylindrical vehicle, $\bar{F}_w = 1$.

5 Conclusions

The developed LDC approach to the integrated optimization of multiregime aircraft parameters by a target criterion allows to find effectively the optimal solution in the cases of complex functional relations including bifurcations. Potentially, it would be a reliable tool for objective analysis of aircraft marginal possibilities in any applications.

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