A New Effective Multidisciplinary Design Optimization Algorithm

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Abstract:

Multidisciplinary Design Optimization [MDO] is an approach that can solve the optimization problems of complex and highly coupled engineering systems. Many algorithms, such as Concurrent Subspace Optimization [CSSO] and Collaborative Optimization [CO], are developed to solve these problems. Either derivative based approximation method or response surface approximation method is used as primary approximation strategy in these algorithms. Because of the inaccuracy of using derivatives and the low efficiency of using response surface, we developed a new MDO algorithm called Subspace Approximation Optimization [SAO]. This algorithm is similar to CO but use a set of linear constraints in the system level optimization to approximate the discipline level constraints. It has been proved that this method is very suitable for distributed computation and can robustly converge to the optimum very fast.

1 Introduction

Aircraft design is a complicated engineering system incorporating many disciplines, such as aerodynamics, flight dynamics, structural mechanics, computer science, system engineering and propulsion theory et al. The disciplines in aircraft design often have different requirements and couple with each other. So trade off and de-coupling analysis for multi-disciplines are needed during the process of aircraft design. A single-disciplinary optimum design is usually not a satisfactory design.

Multidisciplinary Design Optimization is a kind of effective integrated aircraft design algorithm. It can solve the problem of trade off and de-coupling among disciplines, and at the same time, an optimum of the whole system can be found.

There exist several MDO algorithms. But most of them, in our opinion, are non-efficient. In this paper we devised an algorithm called Subspace Approximation Optimization. In this SAO algorithm the whole system is decomposed into a system-level optimization and several discipline-level optimizations. The optimums of design variables in discipline level correspond to the design point that is the nearest to the optimums of design variables in system level. The linear constraints as the approximate feasible region boundary in discipline levels can be constructed using the optimums of design variables. The whole system would be improved under these linear constraints in system level. The coupling and coordination problem among disciplines would be solved by the iteratively renewed constraints and optimums of design variables. The frame of SAO is shown in Fig1.

The examples show that this algorithm not only converges to the optimum very fast, but also is fairly robust. The MDO test problem of a Micro Air Vehicle[MAV] is presented in this paper. The result shows that the SAO algorithm can solve complicated engineering problem and is very practical.

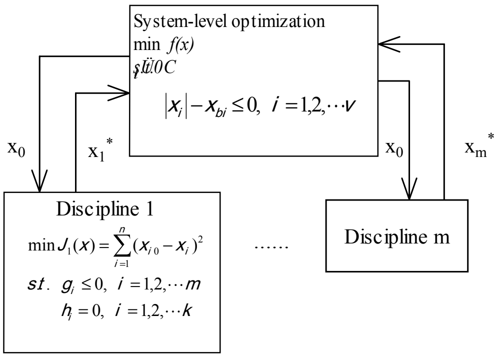


Fig.1 Frame of SAO

2 The description of Subspace Approximation Optimization algorithm

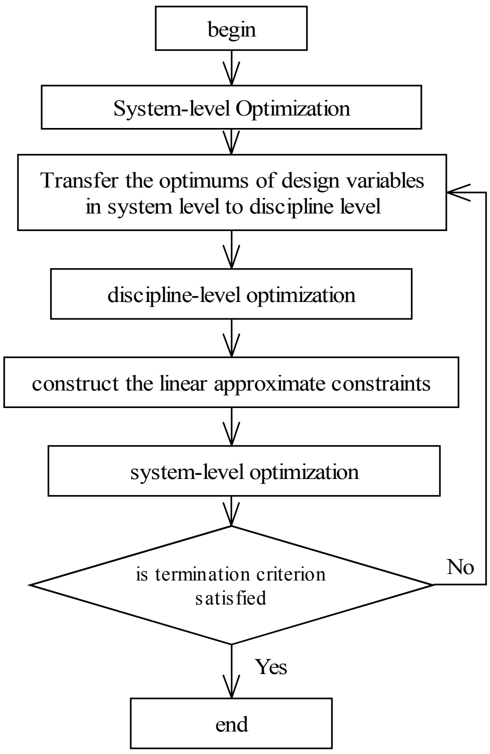


Fig.2 Flow chart of SAO

In SAO, the optimization of a complicated engineering problem with many constraints is also decomposed as a system level optimization and several discipline-level optimizations. In discipline level only the related constraints and the design and analysis models are concerned. The objective function is that the design point(a set of design variables) would be as close as possible to the design point transferred from system level. In system level the linear constraints, representing approximate feasible region boundary of discipline level, are constructed by means of the discipline-level optimums. It is the tangent plane of the original constraint boundary through the discipline-level optimums. With these linear approximate constraints the optimums of design variables in system level are obtained. The optimization process is an iterative one from system level to discipline levels until the termination criterion is satisfied. The whole system design will be improved and the coupling and coordination problems among disciplines will also be resolved with iteration. The termination criterion is that the difference of design result between the adjacent iterations is small enough within the specified tolerance. The flow chart is shown in fig2.

2. 1 System-level optimization

The mathematical expression is as follows:

min $f(x)$

$$\text{s.t. } c_i = \sum_{j=i}^{n_i}(x_{j0} - x_j^*)(x_j - x_j^*) \le 0 \qquad i=1,u$$

$$|x_j| - x_{bj} \le 0 \qquad j=1,v \qquad (1)$$

where

$f(x)$: objective function of whole system

u: number of disciplines

v: number of design variables

x_{bj}: upper limit of absolute value of design variable

c: approximate constraint equation

x_{j0}:optimum transferred to discipline level for last iteration

x_j^*: discipline level optimum i.e. the design point nearest to x_{j0}

x_j: design variable 171.2

n_i: number of design variables in discipline i.

2.2 Discipline-level optimization

The mathematical expression is as follows

$$\min J_i(x) = \sum_{j=i}^{n_i}(x_{j0} - x_j)^2$$

s.t. $g_j \leq 0 \quad j=1,p$

141.2

$$h_j \leq 0 \quad j=1,l \tag{2}$$

where

x_{j0}: optimum of design variable transferred from system level

x_j: design variable

3. General test problems

3.1 Test problem 1

This test problem is extracted from [1]. The optimization problem is as follows:

$$\min \quad f(x) = x_1^2 + x_2^2$$

$$s.t. \quad c_1 = x_1 + \beta x_2 \leq 4 \qquad c_2 = \beta x_1 + x_2 \geq 2 \tag{3}$$

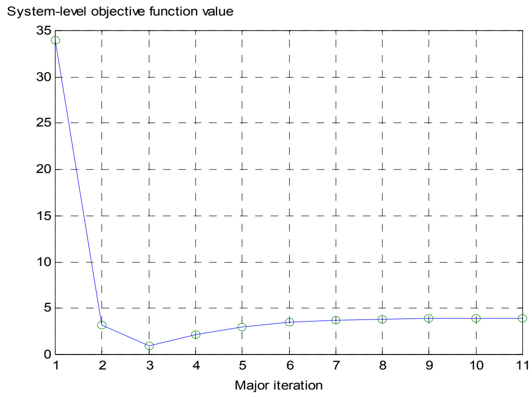


Fig 3 Iteration history of CO

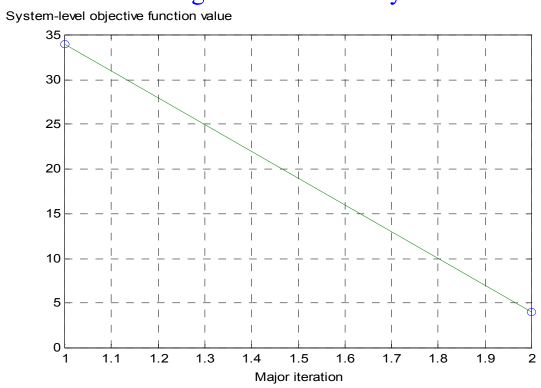


Fig 4 Iteration history of SAO

From [1],if β =0.1 the optimum is 3.959604 at (0.198, 1.98). The optimum found by CO is 3.9455 at (0.1977, 1.9768) and the iteration history is shown in Fig3. In this paper the optimum found by SAO is 3.9549 at (0.1979, 1.9788) and the iteration history is shown in Fig4. Only two iterations are required. It can be seen that SAO is more efficient than CO.

3.2 Test Problem2

This problem is defined as follows:

$$\min f(x) = (x_1-10)^2 + 5(x_2-12)^2 + x_3^4 + 3(x_4-11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7$$

st :

$$127 - 2x_1^2 - 3x_2^4 - x_3 - 4x_4^2 - 5x_5 \geq 0$$
$$282 - 7x_1 - 3x_2 - 10x_3^2 - x_4 + x_5 \geq 0$$
$$196 - 23x_1 - x_2^2 - 6x_6^2 + 8x_7 \geq 0$$
$$-4x_1^2 - x_2^2 + 3x_1x_2 - 2x_3^2 - 5x_6 + 11x_7 \geq 0$$
$$-10.0 \leq x_i \leq 10.0\ ,\ i = 1,2,......7 \tag{4}$$

This test problem is taken from [5]. The optimum of the objective function is $f(x) = 680.6300$, the optimum of design variables is:

$$x^* = (2.330499, 1.951372, -0.4775414, 4.365726, -0.6244870, 1.0381831, 1.594227)$$

When using SAO algorithm, the optimization is accomplished within 9 iterations, see Fig.5. The optimum obtained by SAO is

$$x^* = (2.3166, 1.9446, -0.4414, 4.3885, -0.6290, 1.0906, 1.6068)$$

the optimum of objective function is $f(x) = 680.6037$. This result is very close to that in [5].

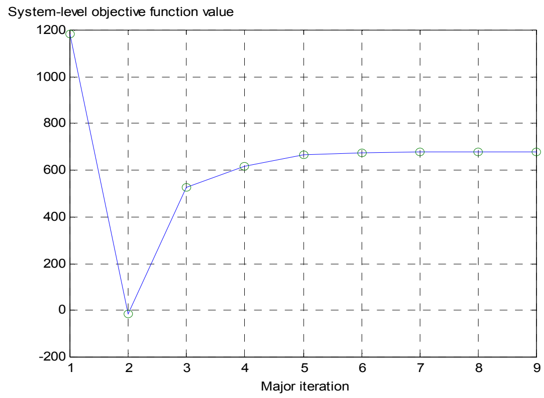


Fig.5 Iteration history of SAO

3.3 Test Probelm3

This test problem contains two equality constraints that simulate the coupling relationships between different disciplines. The definition of this problem is:

$$
\begin{aligned}
&\min f(x) = x_1^2 + x_3 + x_4 + e^{-x_5} \\
&St:\ g_1 = x_4 - x_1^2 - x_2 - x_3 + 0.2x_5 = 0 \\
&\quad g_2 = x_5 - x_4 - x_1 - x_3 = 0 \\
&\quad g_3 = -x_4 + 1 \le 0 \\
&\quad g_4 = x_5 - 1 \le 0 \\
&\quad g_5 = x_1^2 - 100 \le 0 \\
&\quad g_6 = -x_2 \le 0 \quad g_7 = x_2 - 10 \le 0 \\
&\quad g_8 = -x_3 \le 0 \quad g_9 = x_3 - 10 \le 0
\end{aligned}
\tag{5}
$$

When using Sequential Quadratic Programming[SQP] without decomposition, the optimums of design variables are $x^* = (-0.0100, 1.2187, 0.0100, 1.0000, 1.0000)$, and the optimum of objective function is 1.3780. When using SAO algorithm, the optimums of design variables are x* =(-0.0066， 1.1960， 0.0100， 0.9928， 1.0060)，and the optimum of objective function is 1.3685. The iteration history is shown in Fig.6.

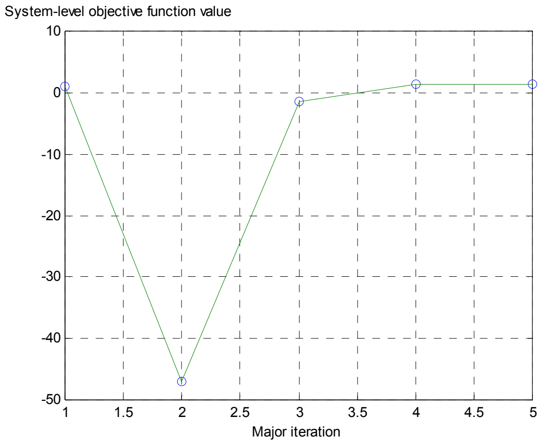


Fig.6 Iteration history of SAO

The difference of the optimums between using SAO and SQP(without decomposition) is just 0.0094, which shows that the accuracy of SAO is satisfied.

4. Micro Air Vehicle design

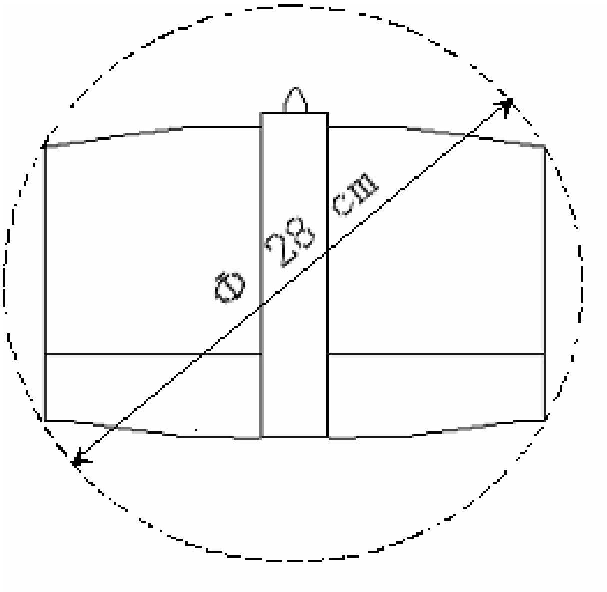
This test problem presents the application of SAO in integrated aircraft design optimization. The objective of optimization is to find the wing chord length distribution and wing spar thickness distributions that can minimize the size of MAV. The constraints involve strength constraint and range constraint. This MAV design problem is defined as follows:

$$
\begin{aligned}
&\text{Find: } \{at_i\},\ \{ac_i\} \\
&\min b^2 + ac_1^2 \\
&\text{s.t.: } \sigma_{\max g} \le 1.5\ [\sigma] \\
&\qquad R_{\max} \ge 600m
\end{aligned}
\tag{6}
$$

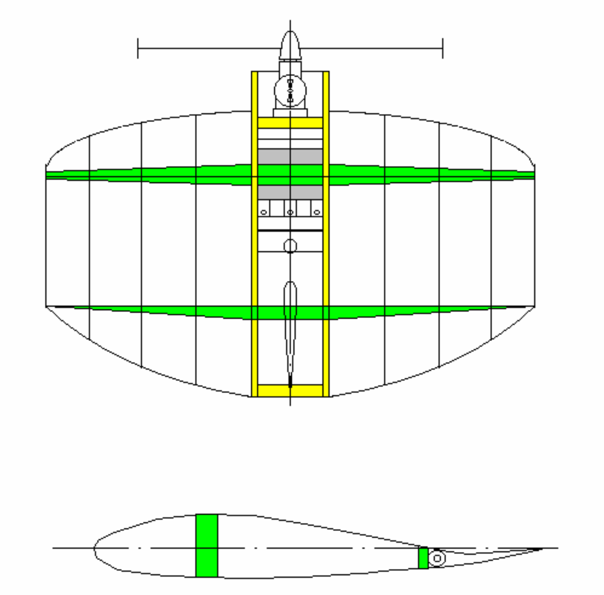
Where b is the wing span, ac_1 is the root chord length, $\{ac_i\}$ ($i = 1,2,...5$) is the distribution of wing chord length. $\{at_i\}$ ($i = 1,2,...5$) is the distribution of wing spar thickness. The size of MAV is measured by the square of the radius of the smallest sphere that can contain the MAV.

For SAO algorithm, we decompose the MAV design problem into system-level optimization and two discipline-level optimizations (Aerodynamics and Structure). The design variables are the same both in system-level and discipline-level optimizations. The objective of system-level optimization is the size of MAV. Constraints of system-level optimization are linear constraints, which are the approximation of feasible region boundary for discipline levels. Constraints of aerodynamics discipline are composed of range constraint and the related coordination or limit constraints. Constraints of the structure discipline are composed of strength constraint and the related coordination or limit constraints. The objectives of discipline-level optimization are to minimize the difference of design variables between the value transferred from system level and the current one in discipline level.

The vortex panel method is used to calculate lift and moment coefficients[6]. The result of optimization makes the MAV fitted into a circle with diameter of 28cm. The wing span is 19.39cm, wing root chord length 20.36, wing tip chord length 19.19cm. The structural weight of MAV is 32.95g. see Fig7.



a)



b)

Fig.7 MDO result for MAV

5. Conclusions

For the Subspace Approximation Optimization the design variables are the same both in system level and discipline-level optimization. The linear constrains i.e. approximate feasible region boundary are used in system-level optimization, therefore the coupling and coordination problems among disciplines are solved while the optimization process is performed.

The examples show that this algorithm not only converges to the optimum very fast, but also is fairly robust. Because of its high efficiency and robustness, SAO would be a promising algorithm that can be widely used in the field of MDO.

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