

WING SHAPE OPTIMIZATION USING A CONSTRAINED NURBS SURFACE GEOMETRICAL REPRESENTATION

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Abstract

This paper describes techniques to construct NURBS based parametric geometric models for wing geometric modelling. The surface obtained intrinsically respects several geometric constraints as it consists in a mixture of ruled and cubic surfaces represented with a single NURBS model. The model is expected to provide a good framework for wing aerodynamic optimization based on CFD tools and formal mathematical programming.

Nomenclature

u	parameter in the chordwise direction
v	parameter in the spanwise direction
$N_{i,p}(u)$	B-Spline function of order p
$P_{i,j}$	control point
$\omega_{i,j}$	weight of control point
$R_{i,j}(u, v)$	piecewise rational basis function
$C(u)$	NURBS curve
$S(u, v)$	NURBS surface
x, y, z	Cartesian body axes
\bar{U}	knot vector U for the parameter u
\bar{V}	knot vector V for the parameter v
CAD	Computer Aided Design
CAGD	Computer Aided Geometric Design

1 Introduction

Owing to the continuous progress of the computational algorithms and computational power,

aerodynamic analysis using CFD are now routinely performed using Navier-Stokes solver on three-dimensional wing configurations. It is now tempting to use these powerful technologies in a computational-based design environment, coupling optimization software with CFD solvers based on appropriate geometric parametrization and cost function. Recent literature reports many attempts in this direction which are illustrating the state-of-the-art of the technology.

Our contribution to this domain lies in the construction of parametric geometric modeling specialized for wing design. Our methodology exploits the power and the flexibility of NURBS¹ interpolation curves to construct wing surface having special characteristics mostly related to the curvature field. Several authors ([8, 10, 11, 14, 16]) have shown that curvature is a main factor in determining the quality of a geometric representation. The objective of this work is to obtain, within a parametric model, a family of surfaces, controlled through a very small set of parameters, having strict respect of a set of geometrical constraints found on real wings. This will then insure that wings produced using this parametrization will intrinsically respect the constraints, without having to treat them explicitly in the optimizer. Also, the treatment of the geometric constraints at the geometric model level generally results in a reduction of the number of

¹NURBS computations presented in this article have been made using the C++ class library Pirate:
<http://www.cerca.umontreal.ca/pirate/>

free parameters in the model, thus reducing the number of design variables for the optimization phase.

In the present work, we describe three strategies which have been developed and implemented in order to construct the wing surface with special curvature requirements. The next section will describe the required wing surface characteristics while section 3 will present main definition and features of NURBS curves and surfaces. In section 5, we will describe the three modifications strategies developed and we will present results of their implementation. Section 6.2 will present the conclusion of our work.

2 Wing Surface Requirements

A plane view of the wings constructed in the present study is shown in Fig. 1.

The wing planform is fully described by the sweep angle α , and the chord of the wing sections at the wing root, break and tip. The slats and flaps cover a constant percentage of the chord length at the leading edge and trailing edge respectively. The three wing sections at the wing root, break and tip are airfoil shapes defined independently. The geometric constraints considered in the present work are related to the construction of the three dimensional surface. In fact, in the slat and flap regions, the surface must be a piecewise linear interpolation between the root and break sections and between the break and tip sections respectively. These patches of the surface are thus “ruled” surface; this constraint coming from manufacturing requirements. The inboard and outboard central surfaces have more freedom and a third degree NURBS interpolation approaching conic sections in the spanwise direction is sought. However, the Gaussian curvature must always be constant in sign, usually negative on the extrados, positive on a patch parallel to the flaps and negative on a patch parallel to the leading edge on the intrados. The presence of an inflection point at mid-chordwise position induces this inversion in sign of the Gaussian curvature. Controlling the sign of the Gaussian curvature on the different parts of the wing avoids oscillations

in the final surface.

3 Definition of NURBS Curves and Surfaces

In computer aided design, the most common geometric representation method use Bézier curves, B-splines and NURBS. It is not surprising that Bézier curves are often used because this was the first interpolation method developed to use control points ([2]). Furthermore, the method is simple to implement, and moving the control points allows for easy, and visual, shape modification. Nevertheless, Bézier curves have two major disadvantages: first, they cannot represent conics exactly; and, second, their parameterization is global, which means that when a control point is moved the whole curve is modified. B-splines use the concept of control points introduced by Bézier, but the interpolation functions are more complex, with local characteristics such that the displacement of a control point induces only a local modification of the curve([7, 15, 13]). Though conics cannot be represented exactly using B-splines, this method has often been used. NURBS are an extension of B-splines, using fractions of the same interpolation functions. Those fractions are weighted by a coefficient (weight) assigned to each control point. So, the family of curves that can be represented with NURBS is much wider than that with B-splines or Bézier curves, and also includes conics. The algorithms associated with NURBS are easier to implement with the definitions introduced by Cox ([6]) and De Boor ([4, 5]). It has been established that these algorithms, as evaluations of positions or derivatives, are stable and fast ([1, 3]).

NURBS curves are defined by

$$C(u) = \sum_{i=0}^n R_{i,p}(u)P_i \quad (1)$$

$$\text{with } R_{i,p}(u) = \frac{N_{i,p}(u)\omega_i}{\sum_{j=0}^n N_{j,p}(u)\omega_j} \quad (2)$$

where P_i are the control point coordinates, ω_i their respective weights, $N_{i,p}$ the p-th degree B-spline basis functions and $C(u)$ the position of a

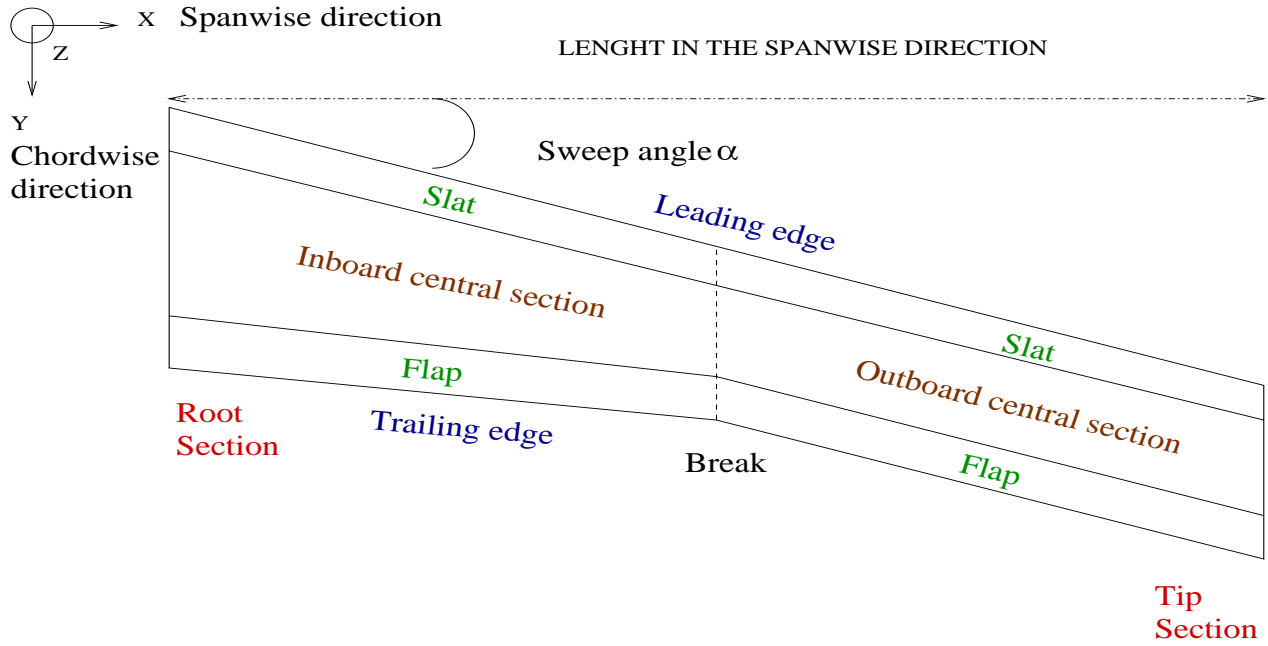


Fig. 1 Descriptive wing planform.

point on the curve. The basis functions are obtained through a *knot vector* of the form

$$\underbrace{\{0, \dots, 0\}}_p, u_{p+1}, \dots, u_{m-p+1}, \underbrace{\{1, \dots, 1\}}_p \quad (3)$$

which defines the functions' break points. The basis functions are equal to zero everywhere except on an interval delimited by $(m+1) - 2 \cdot p$ knots. This interval defines the area of influence of the corresponding control point. NURBS curves have the visual aspect of Bézier curves and the local properties of B-splines. They are suitable for use in a very wide range of applications and are likely to produce naturally smooth profiles. The tensorial notation of a NURBS surface induce properties corresponding to the univariate basis functions defining NURBS curves. A NURBS surface of degree p in the u direction and of degree q in the v direction is a bivariate vector-valued piecewise rational function of the form:

$$S(u, v) = \sum_{i=0}^n \sum_{j=0}^m R_{i,j}(u, v) P_{i,j} \quad 0 \leq u, v \leq 1 \quad (4)$$

$$\text{with } R_{i,j}(u, v) = \frac{N_{i,p}(u) N_{j,q}(v) \omega_{i,j}}{\sum_{k=0}^n \sum_{l=0}^m N_{k,p}(u) N_{l,q}(v) \omega_{k,l}} \quad (5)$$

where the $P_{i,j}$ form a bidirectional control net, $\omega_{i,j}$ their respective weights, $N_{i,p}(u)$ and $N_{j,q}(v)$ are the non-rational B-Spline basis functions defined respectively on *knot vectors* of the form

$$\underbrace{\{0, \dots, 0\}}_{p+1}, u_{p+1}, \dots, u_{r-p-1}, \underbrace{\{1, \dots, 1\}}_{p+1} \quad (6)$$

$$\underbrace{\{0, \dots, 0\}}_{q+1}, v_{q+1}, \dots, v_{s-q-1}, \underbrace{\{1, \dots, 1\}}_{q+1} \quad (7)$$

where $r = n + p + 1$ and $s = m + q + 1$. The weights $\omega_{i,j}$ considered in this formulation are positive in order to keep the rational function $R_{i,j}(u, v)$ also positive. The important properties of the functions $R_{i,j}(u, v)$ are the same as the non-rational basis functions $N_{i,p}(u)$, $N_{j,q}(v)$. A brief enumeration of these properties is given here:

1. Non-negativity: $R_{i,j}(u, v) \geq 0$ for all i, j, u and v .
2. Partition of unity: $\sum_{i=0}^n \sum_{j=0}^m R_{i,j}(u, v) = 1$ for all $(u, v) \in [0, 1] \times [0, 1]$.
3. Local support: $R_{i,j}(u, v) = 0$ if (u, v) is outside the rectangle given by $[u_i, u_{i+p+1}] \times [v_j, v_{j+q+1}]$ and thus the corresponding

control point $P_{i,j}$ has no influence on the surface outside of this patch.

4. In any given rectangle of the form $[u_{i_0}, u_{i_0+1}] \times [v_{j_0}, v_{j_0+1}]$, at most $(p+1)(q+1)$ basis functions are nonzero, in particular the $R_{i,j}(u, v)$ for $i_0 - p \leq i \leq i_0$ and $j_0 - q \leq j \leq j_0$ are nonzero.

The properties of the $R_{i,j}(u, v)$ functions imply important geometric properties as:

1. Local modification: if $P_{i,j}$ is moved, or $\omega_{i,j}$ is changed, it affects the surface shape only in the rectangle $[u_i, u_{i+p+1}] \times [v_j, v_{j+q+1}]$.
2. Strong convex hull property: if $(u, v) \in [u_{i_0}, u_{i_0+1}] \times [v_{j_0}, v_{j_0+1}]$, then $S(u, v)$ is in the convex hull of the control points $P_{i,j}$, $i_0 - p \leq i \leq i_0$ and $j_0 - q \leq j \leq j_0$.

Complete definitions can be found in [12].

4 General Approach for the construction of the wing surface using NURBS

Our approach for the construction of a NURBS surface respecting the above described constraints starts with the construction of a bi-cubic interpolation surface combined with a special positioning of the control points in the spanwise direction. The first step is the interpolation of airfoil sections at the root, break and tip sections. This interpolation is illustrated in figure Fig. 2.

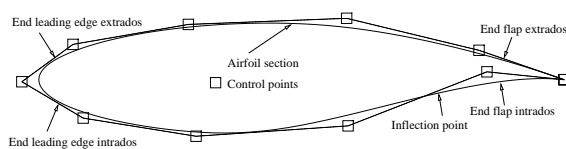


Fig. 2 Airfoil section represented by a cubic NURBS curve and its control polygon.

The airfoil is first given as a set of discrete points. The algorithm for the construction of the NURBS interpolation curve section comprises five steps:

1. reading the points defining the 2D normalized airfoil sections and the parameters fixing their position in 3D space. Usually there are around 200 definition points per section;
2. subdividing each set of data points in 5 subsets of data points belonging respectively to the flap intrados, inner center intrados, outer center intrados, center extrados and flap extrados;
3. computation and storage of the discrete curvature κ_i at the data points of each subset, serving to drive the selection of the points to be interpolated;
4. for every subset at each section, m_i ($0 \leq i \leq 4$) points to be interpolated are chosen among the initial subset of data points;
5. collecting all points to be interpolated for each section to construct a cubic B-spline curve interpolating these data points for each section.

The criteria used to produce the five subsets of data points at each section are the lengths of the flaps and the leading edge in the chordwise direction and the inflection point dividing the inner and outer central parts on the intrados. The continuity between each subset is achieved by selecting the first and last points of the subset defining the flaps and the leading edge. The inner parts contributes only by giving inner points. This process is repeated for the three sections at the root, break and tip. Based on these three interpolation curves, an interpolation surface is initially constructed using a tensorial NURBS surface with 26 interpolation points. The leading edge having higher variations in the curvature needs 8 interpolation points in the chordwise direction to be well fitted. Chord length parametrization is used in the chordwise and spanwise direction.

If one uses only a linear interpolation in the spanwise direction, the interpolation surface will be linearly varying in this direction. This will intrinsically respect the constraints in the flap and

the slat regions but will not provide a cubic interpolation in the central part, thus limiting the smoothness of the surface, especially at the break position.

If one simply uses three sections and increase the NURBS to a second degree, then the alignment of the trailing and leading edge will not be maintained as illustrated in figure Fig. 3. Depending on the parametrization, oscillations are probable in the spanwise direction.

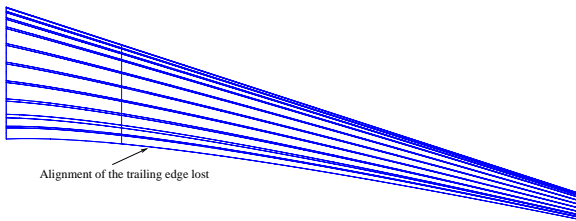


Fig. 3 NURBS surface of a wing of order $3 \otimes 3$.

In order to keep the ruled surfaces in the slat and the flap region, we simply introduce two additional spanwise control sections all precisely located initially at the break section. This results in a line of triple control points at the break section. With these triple points, the surface can be interpolated using third degree NURBS blending functions but will remain linear because of the tripling of the points. This surface is illustrated in Fig. 4.

At this point, our approach investigate two ways to modify this net of control points in order to produce cubic interpolation in the central sections. The first approach enables the cubic interpolation in the inboard central section only while the second approach enables a cubic interpolation in both inboard and outboard central sections.

5 Method I: cubic inboard central section

5.1 Spreading out the tripled control points by setting linear displacements

The method to obtain a cubic interpolation surface in the inboard central region involves the insertion of an additional line of control points at the mid-distance between the root and the break

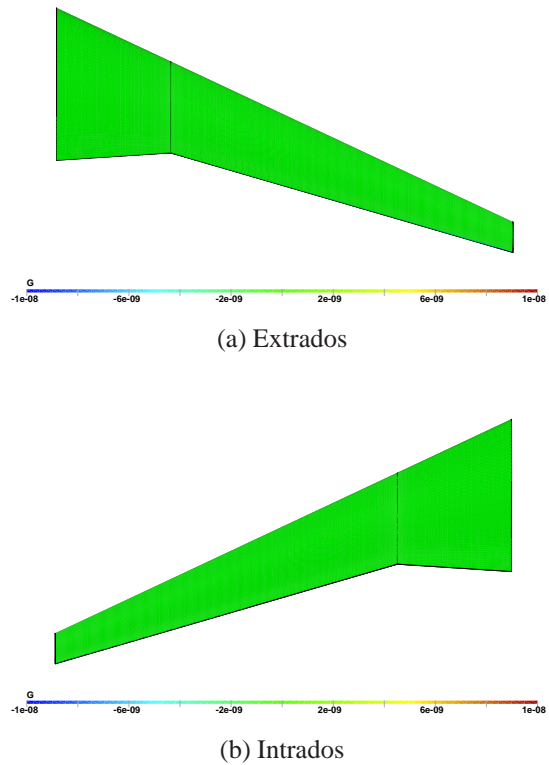


Fig. 4 Null Gaussian curvature identifying linearity in the spanwise direction.

sections. Applying a knot insertion algorithm modifies the knot vector in the spanwise direction, which results in a new line of control points as a linear interpolation of the control points at the break and root sections. This does not modify the surface, which remains linearly interpolated in the spanwise direction. This procedure is illustrated in Fig. 5.

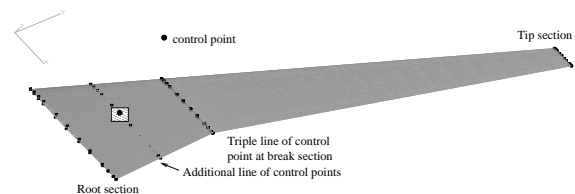


Fig. 5 NURBS surface of a wing of order $4 \otimes 4$ with a triple line of control points at break section and an additional line of control points between the root and the break sections

The next step is then to consider the z dis-

placement of the control point marked with a lined square. In Fig. 5 modification of this degree of freedom will then allow us to generate a family of surfaces which will become cubic interpolation surfaces in the spanwise direction. Since only one point is allowed to move, the region influenced by the movement can be adjusted to span the only central inboard region through a proper selection of the knot vector. Typical results obtained using this method will be presented in the form of section cuts along the chordwise and spanwise directions.

Fig. 6 illustrates how the movement of the control point in the z direction has been parameterized. The position noted $\alpha = 0$ corresponds to the initial position of the control point and leads to a linear interpolation surface in the spanwise direction. The position $\alpha = 1$ corresponds to a displacement which will generate a $C^{(1)}$ continuity of the interpolation surface at the break section in the spanwise direction.

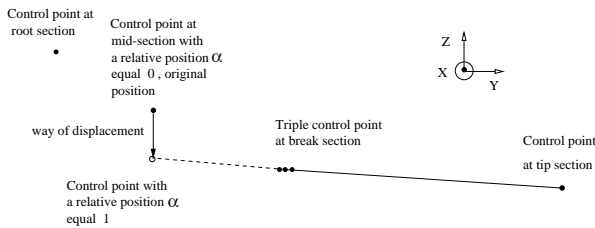


Fig. 6 Description of the linear displacement of a control point at mid-distance from the root and the break sections.

Results are presented for a wing with a planform as described in figure Fig. 1 and airfoils section with thickness to chord ratios $\frac{t}{c}$ of 15%, 10% and 8% at the root, break and tip sections respectively. The airfoils sections, with the required ratio $\frac{t}{c}$, are initially NACA64015, NACA64010 and NACA64008 airfoils. Results are shown in figures Figs 7 to 12. Deformation of airfoil sections become unacceptable for values of relative position exceeding 0.2 as shown in figures Figs 7 and 8. In figure 9, oscillations are negligible. The corresponding ISO-vcurve lies at the limit of influence of the control point being displaced. Gaps between original ISO-ucurves (straight lines) and the altered surface are pre-

sented in figures Figs 10, 11 and 12, for relative positions inferior or equal to 0.2. The resulting ISO-ucurves have unfortunately no special intrinsic geometric property at this stage. The next paragraph explains how the conic curves in the spanwise direction are obtained.

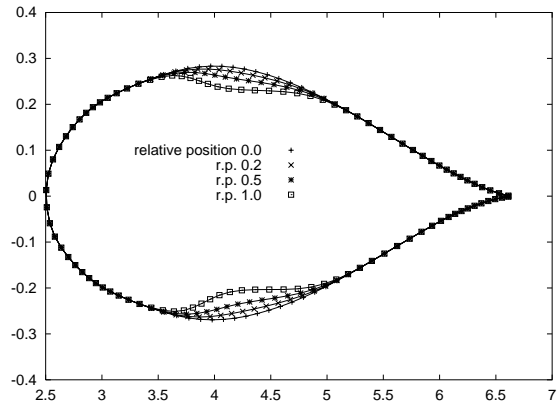


Fig. 7 ISO-parametric curves ($\nu = 0.33$) for different values of the relative position: inflexions appear for values exceeding 0.2.

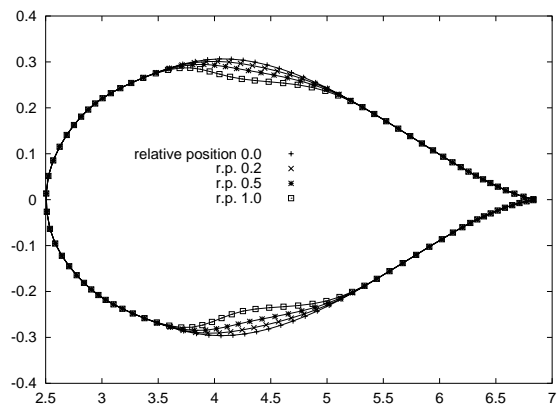


Fig. 8 ISO-parametric curves ($\nu = 0.26$) for different values of the relative position.

6 Methods II and III: cubic inboard and outboard central sections

To better understand the geometric modelling possibilities offered by spreading tripled control points, two other displacement methods are now considered. Modification method II constrains the displacement of all tripled control points to

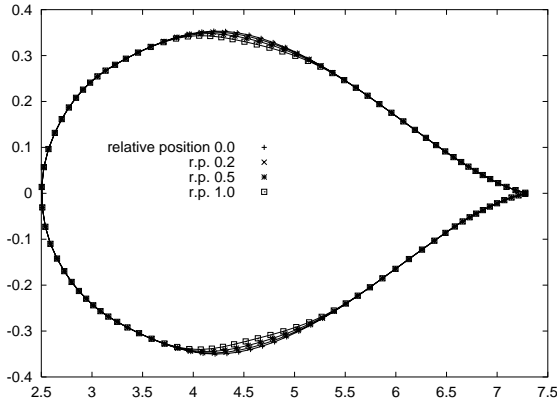


Fig. 9 ISO-parametric curves ($v = 0.13$) for different values of the relative position near the break section.

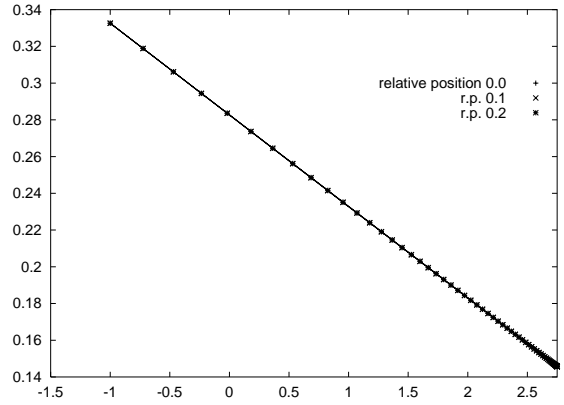


Fig. 12 ISO-parametric curves ($u = 0.66$), projection on plane (YZ), for values of the relative position not inducing inflexions.

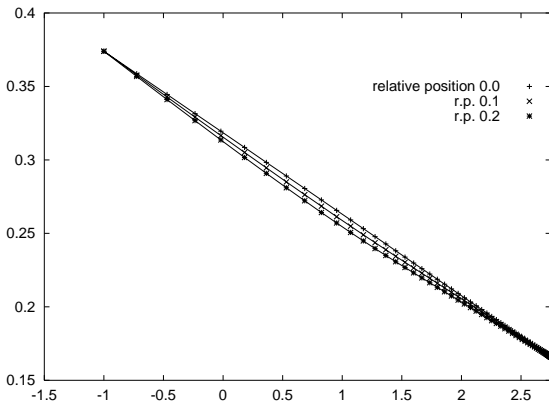


Fig. 10 ISO-parametric curves ($u = 0.72$), projection on plane (YZ), for values of the relative position not inducing inflexions.

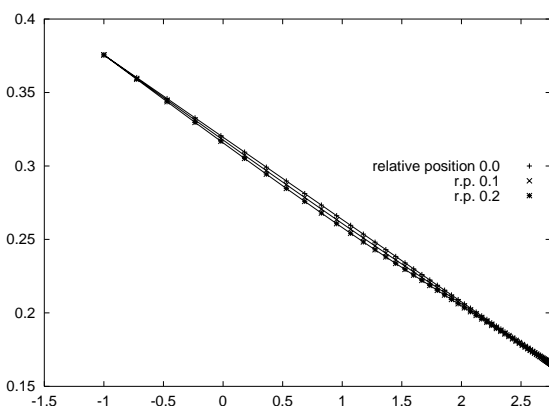


Fig. 11 ISO-parametric curves ($u = 0.7$), projection on plane (YZ), for values of the relative position not inducing inflexions.

lie in a plane, while method *III* uses a modified cubic interpolation scheme to compute displacement values.

6.1 Method II- planar displacement of tripled control points

The plane of displacement is defined in Fig. 13. Unit vectors are first defined in order to have parameters controlling the displacements. The unit vector e_{inner} is on the line joining control points at the break section and tip section, oriented towards the inboard central section. The second unit vector $normal_{inner}$ is obtained by rotating counterclockwise by 90^0 . The displacement plane is thus fixed with these two vectors. Given two parameters, inner deviation angle and inner relative position, a control point is then moved aside the tripled control points towards inboard central region. The relative position parameter is the position of the displaced control point along the normal defined by the e_{inner} vector. Similarly unit vectors e_{outer} and $normal_{outer}$ are defined for displacements in the outboard central region. The unit vector e_{outer} is simply computed by a 180^0 counterclockwise rotation of the unit vector e_{inner} . Numerous attempts force us to conclude that only very small deviation angle avoid oscillations in airfoil sections. Figure 14, shows that while deviation angles are maintained to zero, no effec-

tive improvements are recorded in comparison with the preceding method. Furthermore Figs. 15 and 16 show that using this method the break section is no longer interpolated, one of the requirements of the industry. A more efficient algorithm needed to be implemented at this stage of development. Next section introduce a successful method based on a modified interpolation scheme.

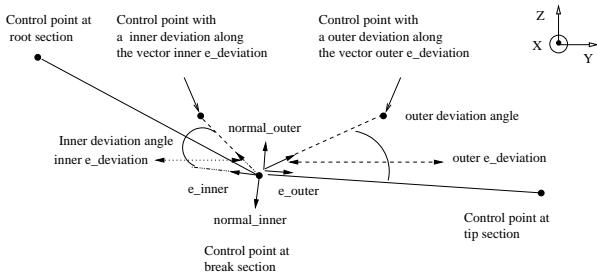


Fig. 13 Description of the plane displacements of control points at mid-distance from the root and the break sections.

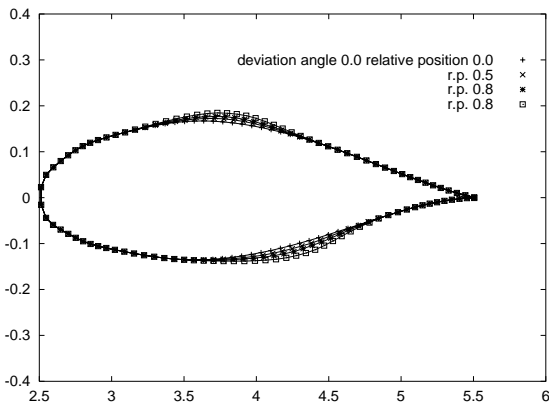


Fig. 14 Intersection of the wing surface and the plane ($Y = 2.75$) for different deviations (inner and outer) on the extrados and the intrados. Deviation angles are maintained to zero.

6.2 Method III-Spreading out the tripled control points by modified interpolation scheme

From the previous studies, the final wing surface must interpolate the three sections of definition. Based on this requirement a major feature of the

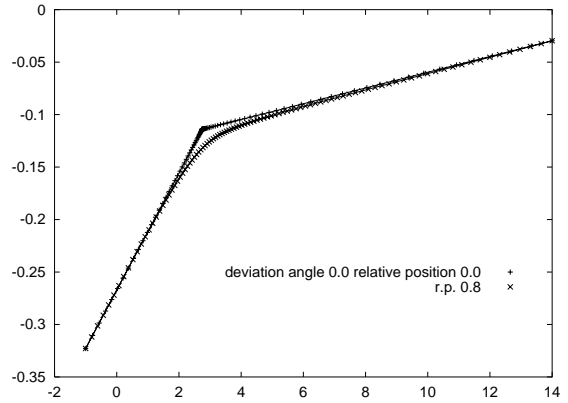


Fig. 15 Trace of the ISO-parametric curve ($u = 0.22$) on the plane (YZ), for extreme relative position values on the intrados without oscillation.

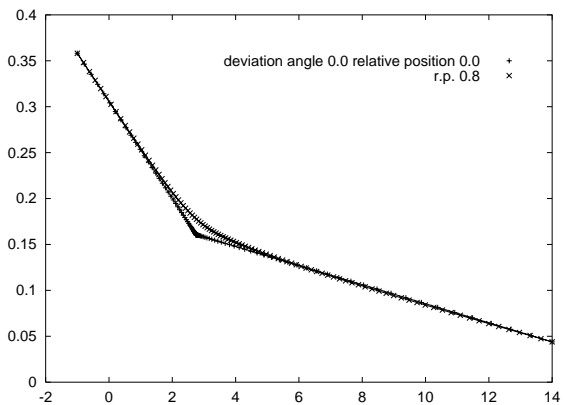


Fig. 16 Trace of the ISO-parametric curve ($u = 0.72$) on the plane (YZ), for extreme relative position values on the extrados without oscillation.

surface must consider the control points resulting from the approximation of the airfoil sections as interpolation points in the spanwise direction. The tensorial nature of the NURBS surface allows such an assumption. So central parts of the wing must be described using five control points. Initially, we have three spanwise control points at our disposal. Conic curves are easily obtained by the quadratic rational Bézier arc with three control points represented in a local coordinate system as shown in Fig. 17. By keeping constant the weights of control points P_0 and P_2 and by modifying the weight of the middle control points P_1 from 0 to infinity a family of conic curves can be generated. We can force the conic curve to inter-

cept a middle point and we can thus generate an infinity of *conic* curves interpolating three points namely P_0 , a middle point P and P_2 . The two additional control points needed to close our system of unknowns are extracted from intrinsic properties of the conic curve being considered, i.e. first and second derivatives at the middle point. The knot corresponding to the middle point is the inner knot v_4 of a cubic curve with five control points. We then solve a system of five equations with five unknown represented by the matrix:

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 0. \\ 0. & N_{1,3}(v_4) & N_{2,3}(v_4) & N_{3,3}(v_4) & 0. \\ 0. & N_{1,2}(v_4) & N_{2,2}(v_4) & N_{3,2}(v_4) & 0. \\ 0. & 6. & -12. & 6. & 0. \\ 0. & 0. & 0. & 0. & 1. \end{pmatrix}$$

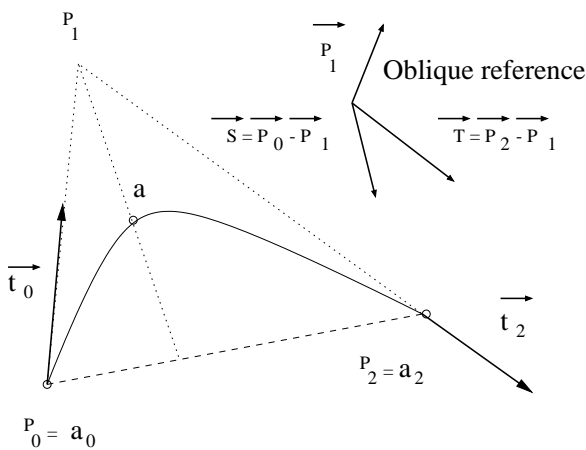


Fig. 17 Local coordinate system for rational quadratic Bézier curve.

The resulting displacement is shown in the figure Fig. 18.

Isoparametric curves are presented in figures Figs. 19, 20, 21 and 22. One can observe that the break section is interpolated on the intrados and the extrados, a major advantage in comparison with the two preceding methods. And also, the ISO-parametric curves approach in that case a *conic* curve in the spanwise direction. A better visualization of the overall final wing surface involves the use of the Gaussian curvature. Gaussian curvature is a powerful tool to characterize any local modification on the wing surface. In

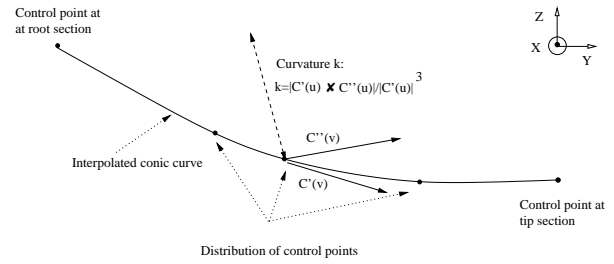


Fig. 18 Description of the *conic* displacements of control points at mid-distance from the root and tip sections.

the field of visualization in CAD/CAGD, it is often used ([9]). Ruled surfaces have in general a negative or nil Gaussian curvature. In the particular case of the flap and slat regions the Gaussian curvature is identically equal to zero. The Gaussian curvature of the central parts are related to the *conic* curve used during the modified interpolation scheme. The extrados is by construction a hyperbolic surface. The intrados due to the existence of an inflection point presents a hyperbolic patch and an elliptic patch. This visualization identifying with appropriate colors each part of the wing insures a $C^{(2)}$ surface without oscillations.

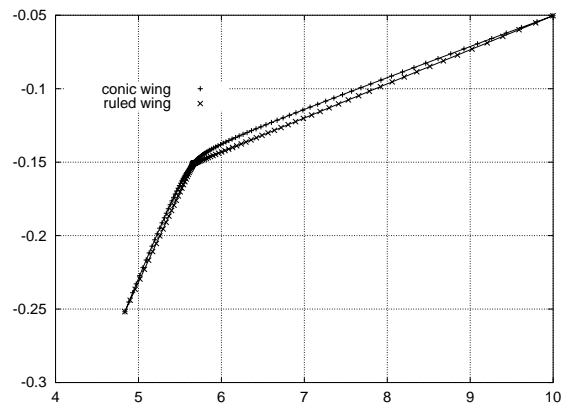


Fig. 19 Trace of the ISO-parametric curve ($u = 0.28$) on the plane (XZ) on the intrados.

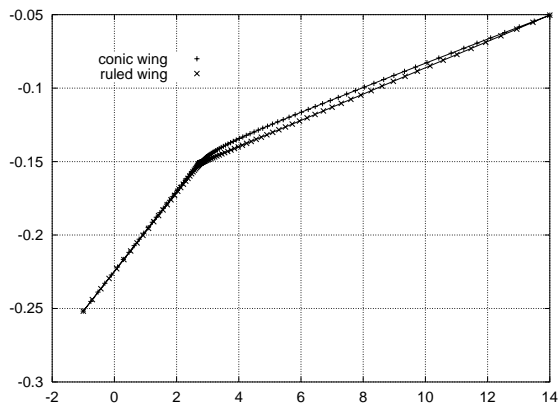


Fig. 20 Trace of the ISO-parametric curve ($u = 0.28$) on the plane (YZ) on the intrados.

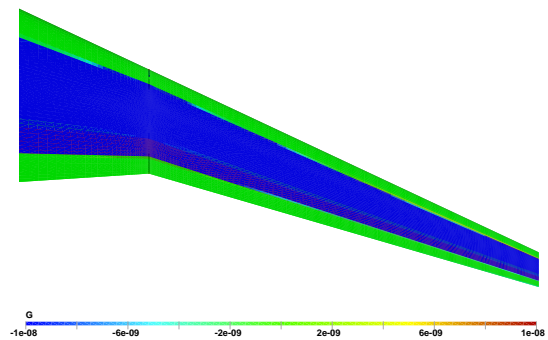


Fig. 23 Extrados of one break wing surface: null Gaussian curvature on ruled parts (green color) and negative on hyperbolic part (blue color).

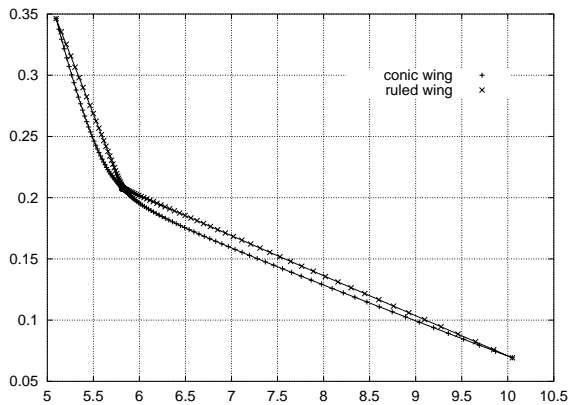


Fig. 21 Trace of the ISO-parametric curve ($u = 0.77$) on the plane (XZ) on the extrados.

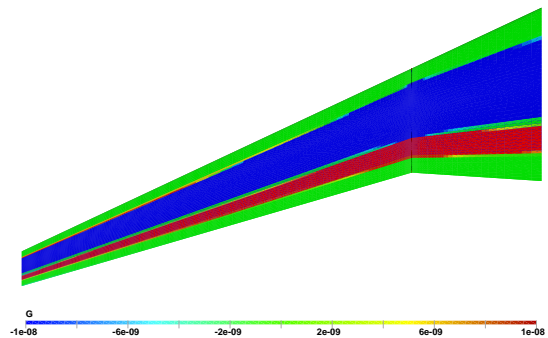


Fig. 24 Intrados of one break wing surface: null Gaussian curvature on ruled parts (green color), negative on hyperbolic part (blue color) and positive on elliptic part (red color). Existence of an inflection point responsible for the positive values.

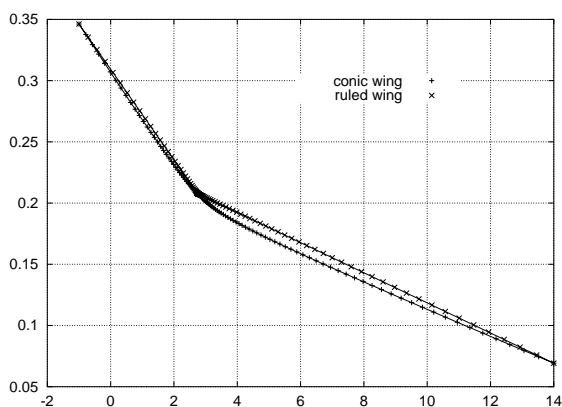


Fig. 22 Trace of the ISO-parametric curve ($u = 0.77$) on the plane (YZ) on the extrados.

Conclusion

In this work, we have discussed how to introduce geometrical constraints in a wing represented as a NURBS surface: alignment and positioning of the trailing edge and the leading edge, ruled surfaces and central parts described as *conic* sections. This work has already been extended to a wing with two breaks and three slats not aligned with the breaks in the spanwise direction. At this stage two problems attract our attention: improving the parametrization in the chordwise direc-

tion in order to fix the length of the break and the slats and the automatization of the construction. This will greatly help for shape optimization of practical aeronautical applications.

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