EVALUATION OF THREE DECOMPOSITION MDO ALGORITHMS

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Abstract

Three decomposition MDO algorithms, Collaborative Optimization (CO), Concurrent Subspace Optimization (CSSO) and Bi-Level Integrated System Synthesis (BLISS) have been evaluated based on two typical applications. It shows that the BLISS is efficient for both applications. The CO is suggested to be more suitable for systems with little coupling between disciplines. The efficiency of the CSSO mainly depends on the construction of the simulation models.

1 Introduction

Multidisciplinary Design Optimization (MDO) has developed through the following three stages: 1.) direct integration of multiple disciplinary analyses and optimizations, 2.) integrated system design with distributed analysis, and 3.) MDO Strategies for distributed design optimization.

The first stage is usually efficient and suitable for small design systems, but it does require fairly stringent feasible constraints. As computational capabilities have grown rapidly in the past several years, the system design may involve many disciplines, which in most cases are coupled to each other. Distributed analysis systems could utilize multiple computers and increase the practical scale of MDO problems. But the reliance on a central optimizer as a decision-maker on all matters is not a practical approach to large-scale system design. This leads to the development of the strategy study for distributed design optimization, which is to decompose the entire system into individual disciplines, with appropriate coupling routines. The next step is to determine the `best' design through specific optimization procedures and system strategy design. In aerospace applications, for example, it is easy to visualize how such disciplines as aerodynamics, structures, and dynamic and control can be suitably optimized to provide the best design configuration.

In the field of the MDO strategy development, the concurrent operation for disciplines is also an important issue. The conventional approach, based on sequential processing of the disciplines, becomes inefficient, as large numbers of iterative loops might be required for the analysis to incorporate the couplings. The entire process of collecting all the feedback from different groups can also result in a long design cycle. As different from sequential processing, concurrent operation allows the expertise in each discipline to perform its design autonomously, and then in turn, collects updates from all disciplines and executes system level optimization with appropriate feedback to individual disciplines. This type of concurrent operation renders the MDO process more efficient and relatively easy to manage.

In this paper, three decomposition and concurrent MDO methods are evaluated. These include Collaborative Optimization (CO) [1], Concurrent Subspace Optimization (CSSO) [2], and Bi-Level Integrated System Synthesis (BLISS) [3]. The evaluations are performed based on two system designs. One is an explicit analytical function with two fully coupled disciplines. The other is the optimization of a conceptual supersonic business jet design, which involves four disciplines: structure, aerodynamics, propulsion and range performance.

2 Algorithms

The CO, CSSO and BLISS are all decomposition MDO algorithms and can be operated concurrently. For these three approaches, a complex problem can be hierarchically decomposed along disciplinary boundaries into a number of sub-problems. With the use of local subspace optimizers, each discipline is given control over its own design variables and is charged with satisfying its own disciplinary constraints.

In the CO approach, the goal of each local optimizer is to conform to other groups on specific values of the multidisciplinary variables by introducing a new defined objective function in the subsystem level, while a system-level optimizer provides coordination and minimizes the overall objective. For example, the objective function in the subsystem level in Ref. [1] is postulated as:

$$r_i = \sum\left(z_i - z_i^*\right)^2 + \sum\left(y_{ij} - y_{ij}^*\right)^2 + \left(f - f^*\right)^2 \qquad (1)$$

where z represents a set of design variables, y refers to the coupling variables and f stands for the objective function. The asterisk denotes the variable values assigned from the system. At the beginning of the optimization process, these variables should be guessed as the estimated optimal values. The optimization of each subsystem is to find its own subsystem variables to match these values by minimizing the discrepancy function r_i. The system optimization is then executed upon the system variables, as well as the coupling variables, treated as design variables to converge towards its optimal objective function. The discrepancy function between the variables, optimized by subsystems and the system design variables, is now served as the system constraints.

For the CSSO method in each subsystem operation, the system objective function is minimized, subject to the local constraints. The design variables include the ones from the system and those unique to the subsystem. The information needed from other subsystems is calculated by simulation models. The system design, which provides the full coupling of the subsystems, is then performed with all design variables defined for both system and subsystems. All the disciplinary analyses in the system level use simulation models.

In the BLISS method, the system objective function is strongly related to the subsystem objective functions. The system objective function may be a single output from one of the subsystems. The postulates of the subsystem objective functions are connected to the system through a solution of the Global Sensitivity Equations [4].

Each iteration cycle in BLISS mainly involves two important steps. The first deals with the subsystem optimization with the design variables X, where the system variables Z are held constant. The second step proceeds toward the system level optimization with the design variables Z. The system design is improved after each iteration cycle with the updated design variables X and Z.

The sensitivity analyses are performed, based on the algorithm described by Barthelemy and Sobieszczanski-Sobieski [4], in which the Lagrange multipliers may be interpreted as the prices for constraint changes by incrementing the design variables Z. In this report, these derivatives are given by:

$$\begin{aligned} D\left(y_{1,i}, Z\right)^T &= D\left(y_{1,i}, Z\right)_0^T \\ &-\left[\left(L^T d\left(G_0, Z\right)\right)_1 + \left(L^T d\left(G_0, Z\right)\right)_2 + \left(L^T d\left(G_0, Z\right)\right)_3 + \ldots\right] \\ &-\left[\left(L^T d\left(G_0, Y\right)\right)_1 + \left(L^T d\left(G_0, Y\right)\right)_2 + \left(L^T d\left(G_0, Y\right)\right)_3 + \ldots\right] \\ &\times\left(D(Y, Z)\right) \end{aligned} \qquad (2)$$

where L is the vector of Lagrange multipliers.

For the detailed descriptions of these three methods, please refer to the references [1], [2] and [3].

3 Applications and Discussions

3.1 Application 1

The first application involves the optimization of analytical formulae, which were chosen from Chan's report [5]. It is stated as:

$$
\begin{array}{lll}
\text{Minimize:} & f = x_2^2 + x_3 + y_1 + e^{-y_2} & \\
\text{Satisfy:} & g_1 = \left(\dfrac{y_1}{3.16}\right) - 1 \geq 0 & \\
 & g_2 = 1 - \left(\dfrac{y_2}{24}\right) \geq 0 & \\
\text{and} & -10 \leq x_1 \leq 10 & (3) \\
 & 0 \leq x_2 \leq 10 & \\
 & 0 \leq x_3 \leq 10 & \\
\text{where:} & y_1 = x_1^2 + x_2 + x_3 - 0.2 y_2 & \\
 & y_2 = \sqrt{y_1} + x_1 + x_3 &
\end{array}
$$

The problem is simple in form, but has two fully coupled subsystems, y_1 and y_2.

The optimizer used for Application 1 for the three algorithms is the Sequential Quadratic Programming (SQP) method in MATLAB Version 6.

3.1.1 Collaborative Optimization

In this approach, the system is decomposed into two subsystems (y_1 and y_2). Coordination of various procedures is a very important step in solving a MDO problem. The design variables x_1, x_2, x_3 are all included in the system design variables, as they are needed for evaluating the objective function f in the system and subsystem levels. The objective functions in the subsystems measure the discrepancy between the design parameters and the corresponding variables assigned from the system level. The minimal values of the functions indicate the best possible matches to the system values. The same philosophy is expressed in the system level constraints imposed by the assigned values from the subsystems. Reference 6 describes the detailed coordination for this application.

Fig. 1 shows the convergence histories for the design variables: x_1, x_2 and x_3, and Fig. 2 is for the system objective function f. When the convergence criteria is set to 10^{-3} for $\dfrac{\left|f_{opt} - f_{All-in-One}\right|}{f_{opt}}$, the computation takes *1936* iteration cycles. The optimized results are listed in Table 1. Note that the optimal results of the system and the subsystems are slightly different for all variables. This is one of the weaknesses of the CO method, but it represents the reality in practical system designs.

3.1.2 Concurrent Subspace Optimization

In the CSSO method, simulation models have to be used to create a data bank. There are two popular methods to create simulation models in MDO. One is the neural network technology, and the other is the response surface. For this application, the quadratic response surface is used,

$$f(z) = C_0 + \sum_{i}^{n} C_l z_i + \sum_{1 \leq i \leq j \leq n}^{n} C_{ij} z_i z_j \tag{4}$$

The training histories of the design variables, x_2, for the two subsystems and the main system are displayed in Fig. 3. At the beginning of the process, the x_2 values are quite distanced from each other. With the improvement of the accuracy of the simulation models, the difference becomes smaller. Similar observations can be seen for the objective function (Fig. 4).

From the methodology description of the CSSO, it can be seen that the CSSO is essentially an

ALL-in-One method in the system-level optimization. The difference is that the All-in-One method uses the analysis models from the subsystems directly, while the CSSO uses simulation models to perform these analyses. The subsystem-level optimizations are actually utilized as the training tools to improve the accuracy of the simulation models. The CSSO is designed to guide the training process to follow the optimal domains close to the system optimal results. Therefore, the CSSO performs more analyses than the All-in-One method (Table 1). However, this doesn't mean that the All-in-One can replace the CSSO. For this simple MDO problem, it is possible to carry out the All-in-One process. But for most practical designs, there is no explicit formula to conduct the analyses. Performing all the subsystem analyses directly in the system-level might be impossible. Even if it was possible, it could still be a big task to put all complex subsystem analyses in one platform. Therefore, the efficiency and accuracy of the CSSO method depends on the efficiency and accuracy of the construction of the simulation models.

Note that in Table 1, for the CSSO method the optimal results of the system and subsystems are not the same. One simulation model was used in the other subsystem analysis, but in its own discipline, it used direct formula analyses. The discrepancies of the optimal results are caused by the errors of the simulation models. As long as these errors meet the accuracy requirement, the results are acceptable.

3.1.3 Bi-Level Integrated System Synthesis

In order to apply the BLISS algorithm to the above system design, besides the two explicit subsystems in Eq. (3), the objective function f is taken as the third subsystem:

$$y_3 = x_2^2 + x_3 + y_1 + e^{-y_2} \tag{5}$$

The system objective function is now chosen as the state variable y_3 of Subsystem 3,

$$f = y_3 \tag{6}$$

Since all the design variables, x_1, x_2 and x_3, are shared by two or three subsystems in this optimization problem, they should all be catalogued to the system design variables,

$$Z = [x_1 \quad x_2 \quad x_3]\,. \tag{7}$$

For this particular case, there is no single design variable specified to the subsystem, therefore, no subsystem optimization is involved.
Since the state equations for the three subsystems are explicit and simple, the derivative calculation in the sensitivity analyses can be expressed by the following analytical formulae:

$$\frac{DY}{DZ} = A/\frac{dY}{dZ} = \begin{bmatrix} 1 & 0.2 & 0 \\ -\dfrac{1}{2\sqrt{y_1}} & 1 & 0 \\ -1 & e^{-y_2} & 1 \end{bmatrix} / \begin{bmatrix} 2x_1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2x_2 & 1 \end{bmatrix}$$
$$y_1 = x_1^2 + x_2 + x_3 - 0.2y_2$$
$$y_2 = \sqrt{y_1} + x_1 + x_3 \tag{8}$$

Figures 5 and 6 display the convergence histories of the objective function, the design variables and various coupling variables. With the prescribed accuracy of 10^{-3} for the objective function f, and based on the initial guess for the design variables listed in Table 1, the optimal solutions for the main system are obtained after 40 iteration cycles,

From Table 1, it can be seen that all three methods produced very close results to the All-in-One solutions. The CSSO method used the least iteration cycles. But the BLISS method performed fewer analyses than the other two methods. Therefore, the BLISS method is the most efficient of the three for Application 1.

3.2 Application 2

Application 2 is a conceptual system design for a supersonic business jet. The system involves four subsystems. Except for the range performance (Subsystem 4), the other three subsystems, Structures (Subsystem 1), Aerodynamics (Subsystem 2) and Propulsion (Subsystem 3) are coupled to each other. The system has 10 design variables and there are 9 coupling variables between the subsystems. The design variables are defined as:

Subsystem design variables:

$$X_1 = [\lambda, x] \quad X_2 = [C_f] \quad X_3 = [T] \tag{9}$$

System design variables:

$$Z = \left[\frac{t}{c}, h, M, AR, \Lambda, S_{ref}\right] \tag{10}$$

State variables:

$$\begin{aligned} Y_1 &= [W_T, W_F, \Theta] \\ Y_2 &= [L, D, \frac{L}{D}] \\ Y_3 &= [SFC, W_E, ESF] \\ Y_4 &= [Range] \end{aligned} \tag{11}$$

The state variable *Range* was chosen as the system objective function:

$$\Phi = Range \tag{12}$$

The calculations of all the state variables are cited from Ref. [3] with some corrections. There exist complex coupling relationships between the four subsystems for this aircraft system design. Many empirical formulae are used to simplify the calculations. For example, the range is calculated by the Breguet range equation:

$$Range = \frac{M \frac{L}{D} 661 \sqrt{\theta}}{SFC} \ln\left(\frac{W_T}{W_T - W_F}\right). \tag{13}$$

In real design, the state variable analyses might involve large-scale computations performed by experts from the individual disciplines.

3.2.1 Collaborative Optimization

For Application 2, there are a number of design variables commonly shared among the four disciplines. For this kind of complex system, proper coordination is just as critical as the means of handling the information exchanges between the disciplines. In Table 2, it is shown that there are 16 design variables in the system level. Compared with the design variables defined in Eq. (9), the CO added 10 more design variables. Reference 6 lists the detailed coordination for this system design.

Based on the initial guess in Table 3, the optimized results using the CO method were obtained at iteration cycle 1176. Note that the solutions are again not identical between the individual subsystem and the system designs. This is the inherent nature of the CO method. Compared with the results of Application 1, the discrepancies are bigger. The objective in the subsystem designs is to reduce these discrepancies to a minimum in a Euclidean measure. Thus the design variables have a limited freedom to adjust themselves. This freedom is system dependent. For Application 1, the system definition is quite precise, thus high accuracy can be reached. For Application 2, the system definition is quite complex. One subsystem optimization may be limited severely from other subsystems. To minimize this limitation, the Genetic Algorithm (GA) was used as an optimizer for this application in order to reach the global optimal values in each subsystem optimization and system optimization. But some discrepancies (for example, W_F) are not acceptable. So the CO is not suitable for this particular application.

The above two evaluations for applications 1 and 2 are related to highly coupled systems. Owning to certain drawbacks in communications between subsystems, the CO method was found to be the slowest of the three approaches. The CO may be more efficient to system designs with little boundary interactions between subsystems.

3.2.2 Concurrent Subspace Optimization

When the CSSO method is applied to this aircraft design, the range calculation is taken as the objective function for all subsystems and the system. The subsystems are structure, aerodynamics and propulsion. The quadratic response surface (Eq. (4)) is also used in this application for the construction of the simulation models.

For this application, the CSSO method is not efficient. By monitoring the training procedure, it is found that the search for feasible solutions in the subsystem optimizations is very difficult. This is a common situation when the system design is complex and involves many design variables. In this case, the optimized values from one discipline may not fall into the feasible domain of the other discipline analysis. This results in a large number of analyses for each discipline.

Also, because the system has a large number of variables, the time used for the construction of the simulation models is quite large. It negates the initial purpose of using simulation models in the optimization, which is to save system design time. Therefore, CSSO is not suitable for the large-scale system design, or it is too restrictive for feasible solutions.

3.2.3 Bi-Level Integrated System Synthesis

For this complex application case, it is impossible to perform the direct analytical calculations for the sensitivity information used in the BLISS. The first order finite difference method was used for the derivative calculations of the BLISS algorithm. The optimizer used for the system and all the subsystems is the SQP method.

The BLISS algorithm is efficient for this aircraft application. It took only 9 iteration cycles to reach a converged solution (Fig. 7). Fig. 8 displays the contributions of the individual subsystems and the main system for the range optimization. It is observed that Subsystem 1 (Structures) and the main system are the main contributing parts. Subsystem 3 (Propulsion) contributes moderately in the early process and Subsystem 2 (Aerodynamics) plays a very small role in this particular optimization case.

The optimal results for the range, the design variables and the state variables are shown in Table 4. For the BLISS method, the optimal results of the subsystems and system are always consistent.

Note that Application 1 took more iteration cycles to get close to its optimal value than the more complex system Application 2. Since there was no subsystem optimization involved in Application 1, the optimization objective was only upgraded by the main system contribution. For Application 2, the three subsystems and the main system contributed to the objective function in every iteration cycle. It speeded up the optimization process in a more economic way.

For Application 2, BLISS is the only method that produced consistent and feasible solutions.

4. Summary and future work

Three MDO methods have been applied to two system-design applications. Considering the complexity, the CO method is the simplest among these three in programming. And it is also so far the most direct and autonomous MDO method. However, owing to lack of communication between the subsystems, the CO convergence is slow. This method is not suitable for systems with many coupled disciplines. Also, because the coordination problem combines the system optimization with the system analysis, one may have to be confronted with a large number of design variables. Table 2 lists the number of design variables for the three methods. At both the system and the subsystem levels, the CO and the CSSO methods involved catering for a lot more design variables than the BLISS method.

For the CSSO approach, the process is fully explicit. The simulation models, which pass information among system and subsystems, have to be constructed. The efficiency of constructing the simulation models is directly related to the efficiency of this method. For the larger systems, the construction of the simulation models added extra workloads for the optimization process. For some cases, it may not be possible to find feasible solutions to construct the simulation models.

The BLISS performs an explicit system behaviour and sensitivity analysis using global sensitivity equations. The coordination problem engages only a relatively small number of design variables. However, it does involve derivation calculations. For Application 1, these derivatives can be easily expressed by formulae (Eq. (8)), but for Application 2, the process is very tedious.

In summary, the CO is recommended for systems with little interaction between disciplines. For highly coupled system, the BLISS method is more suitable. The CSSO method is only efficient for small-scale systems. For large-scale systems, the CSSO method is not efficient.

The evaluation has been performed for two system designs. As noted in this report, the two systems contain highly coupled subsystem disciplines. For the CO method, the present case studies may not be the most appropriate, as the method would be directly suitable for systems with weak couplings.

The aircraft design is more realistic, but it involves too many empirical formulae. While comparing the performance of the CSSO and CO methods, it is difficult to assess their efficiency, as the use of curve-fitting polynomials or other empirical formulae are not the best tests for optimization studies.

Acknowledgements

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References

[1]. Braun, R. D. and Kroo, I. M. (1995), "Development and Application of the Collaborative Optimization Architecture in a Multidisciplinary Design Environment", SIAM Journal, pp. 98-116.

[2]. Sell, R. S., Batill, S. M. and Renaud, J. E. (1996), "Response Surface Based Concurrent Subspace Optimization for Multidisciplinary System Design", AIAA Paper 96-0714.

[3]. Sobieszczanski-Sobieski, J., Agte, J. S. and Sandusky R. R. (1998), "Bi-Level Integrated System Synthesis (BLISS)", NASA /TM-1998-208715.

[4]. Barthelemy, J-F and Sobieszczanski-Sobieski, J., "Optimum Sensitivity Derivatives of Objective Functions in Nonlinear Programming", AIAA Journal, Vol. 21, No. 6, p.910-915, June 1983.

[5]. Chan, L. (2001), "Evaluation of Two Concurrent Design Approaches in Multidisciplinary Design Optimization", NRC report LM-A-077.

[6]. Chen, S., Zhang, F. and Khalid, M. (2002), "A Preliminary Evaluation of Three Decomposition MDO Algorithms", LM-AL-2002-0051

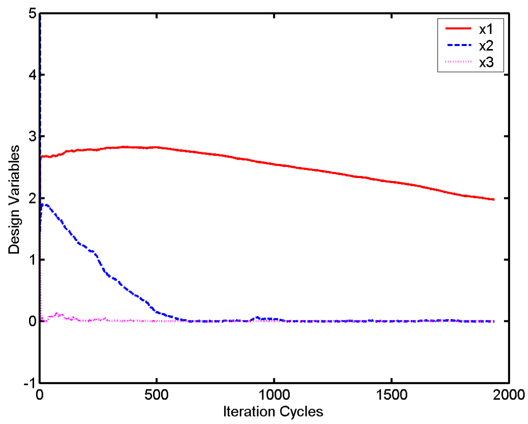

Fig. 1 CO: Convergent histories for the design variables (Appl.1)

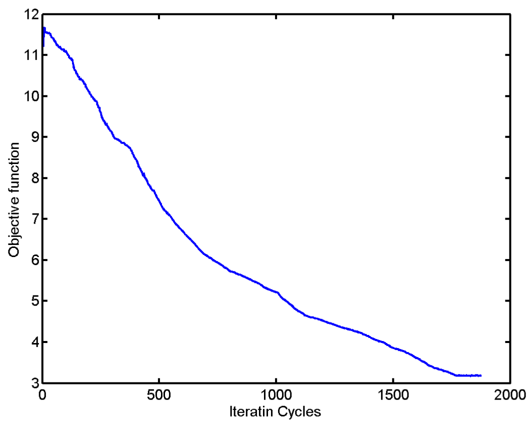

Fig. 2 CO: Convergent history for the objective function (Appl.1)

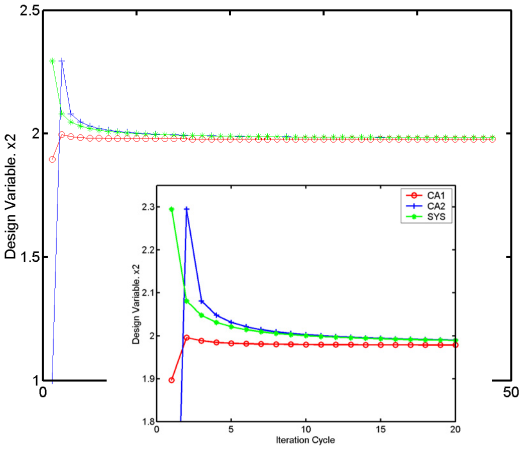

Fig. 3 CSSO: Convergent histories for the design variable x_2 (Appl.1)

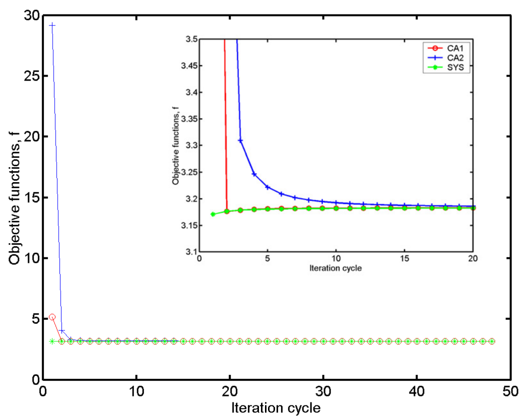

Fig. 4 CSSO: Convergent histories for the objective function (Appl.1)

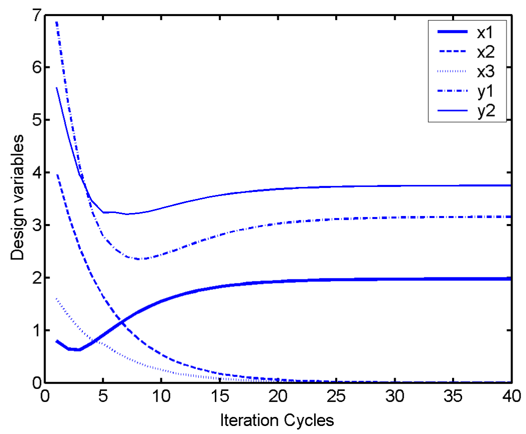

Fig. 5 BLISS: Convergent histories for the design variables (Appl.1)

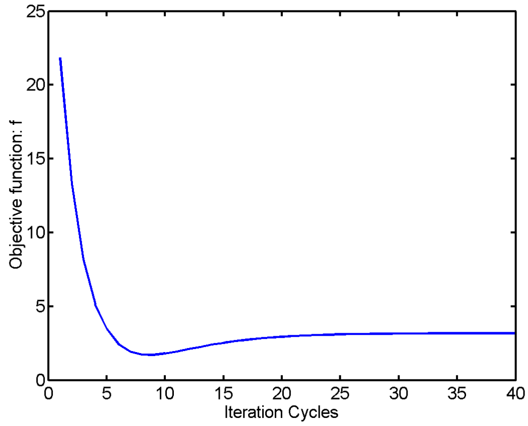

Fig. 6 BLISS: Convergent history for the objective function (Appl .1)

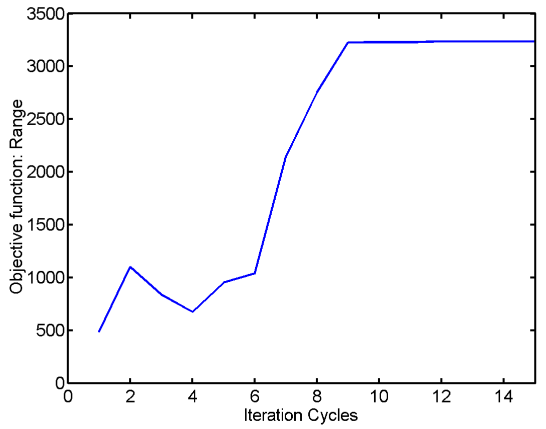

Fig. 7 BLISS: Convergent history for the objective (Appl.2)

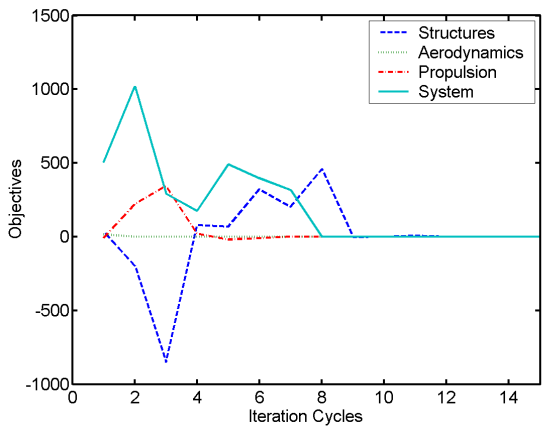

Fig. 8 Contributions of subsystems and system to range optimization (Appl. 2)

Table 1 Optimization Results for Application 1

$\frac{\lvert f_{opt} - f_{All-in-One} \rvert}{f_{opt}} = 10^{-3}$	x_1	x_2	x_3	y_1	y_2	f	Call No.	Iteration Cycles
Initial value	*1*	*5*	*2*	*10*	*4*	*10*		
All-in-One	*1.978*	*0*	*0*	*3.16*	*3.7553*	*3.1834*	*62*	
CO (1)	*1.9771*	*0.0020*	*0*	*3.1600*	*3.7538*	*3.1834*	*30812*	
CO (2)	*1.9775*	*0.0020*	*0.0001*	*3.1600*	*3.7541*	*3.1835*	*18733*	*1936*
CO (S)	*1.9754*	*0.0012*	*0*	*3.1531*	*3.7518*	*3.1850*		
CSSO-RS (1)	*1.9786*	*-0.0012*		*3.1602*	*3.7675*	*3.1831*	*528*	
CSSO-RS (2)			*0*	*3.1184*	*3.7445*	*3.1864*	*234*	*20*
CSSO-RS (S)	*1.9894*	*0*	*0*	*3.1184*	*3.7675*	*3.1828*		
BLISS	*1.9770*	*0.0007*	*0.0003*	*3.1583*	*3.7544*	*3.1804*	*95*	*40*

Table 2. Comparison of the Number of Design Variables for the Three Methods

		CO	CSSO	BLISS
Application 1	System	*5*	*3*	*3*
	Subsystem 1	*4*	*2*	
	Subsystem 2	*4*	*1*	
Application 2	System	*16*	*10*	*6*
	Subsystem 1	*8*	*8*	*2*
	Subsystem 2	*10*	*7*	*1*
	Subsystem 3	*4*	*7*	*1*
	Subsystem 4	*6*		

Table 3 Optimization Results of the CO Method for Application 2

Design Variables	R (nm)	λ	x	C_f	T	t/c	h (ft)	M	AR	Λ (deg.)	S_{ref} (ft^2)
Initial Value	3378	0.25	1.0	1.0	0.5	0.05	45000	1.6	5.5	55	1000
Optimal Value (Structure)		0.119	0.99			0.089			5.031	49.087	1066.3
Optimal Value (Aerodynamics)				1.176		0.0726	58521	1.457	5.668	55.998	1027.7
Optimal Value (Propulsion)					0.127		57656	1.400			
Optimal Value (Performance.)	3901.4						51036	1.471			
Optimal Value (System.)	3990.0					0.0837	55442	1.512	5.534	56.333	1047.8
Coupling Variable	W_T (lb)	W_F (lb)	Θ	L (lb)	D (lb)	L/D	SFC.	W_E (lb)	ESF		
Initial Value	41195	11254	1.0285	46231	5264	9.5	0.8818	6550	0.536		
Optimal Value (Structure)	47912	12867	1.0000	40000				7251			
Optimal Value (Aerodynamics)	40641		1.9726	40641	5627	7.223			0.983		
Optimal Value (Propulsion)					4371		1.622	1343	0.114		
Optimal Value (Performance)	51319	27733				5.634	0.947				
Optimal Value (System)	45207	37708	1.9927	38839	4313	5.813	0.954	7882	0.949		

Table 4 Optimization Results of the Three Method for Application 2

Design Variables		R (nm)	λ	x	C_f	T	t/c	h (ft)	M	AR	Λ (deg.)	S_{ref} (ft^2)
Initial Value		3378	0.25	1.0	1.0	0.5	0.05	45000	1.6	5.5	55	1000
Optimal Value	CO	3990	0.12	0.99	1.18	0.127	0.08	55442	1.5	5.5	56	1047
	CSSO-RS	3435	0.4	0.84	0.99	0.208	0.081	59154	1.7	3.6	44.7	1208
	BLISS	3235	0.4	0.75	0.75	0.156	0.06	60000	1.4	2.5	70	1500
Coupling Variable		W_T (lb)	W_F (lb)	Θ	L (lb)	D (lb)	L/D	SFC.	W_E (lb)	ESF		
Initial Value		41195	11254	1.0285	46231	5264	9.5	0.8818	6550	0.536		
Optimal Value	CO	45207	37708	1.9927	38839	4313	5.8	0.954	7882	0.949		
	CSSO-RS	46828	16241	1.0641	46828	5332	8.783	1.1451	6739	0.530		
	BLISS	51411	7306	1.0002	51411	13478	3.814	1.1075	7058	0.556		