NUMERICAL OPTIMIZATION DESIGN OF WINGS BY SOLVING ADJOINT EQUATIONS†

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Abstract

A numerical optimization method of wing design by solving adjoint equations based on Euler equations is studied for better considering performance requirements of wing design in this paper. The three dimensional adjoint equations are derived according the given requirements of aerodynamic performance and the given desired pressure distribution respectively. Some improvements have been made, such as far field conditions using characteristics theory, smoothing the co-state variable and cost function by using artificial dissipation method and using the vector flux boundary conditions for better convergence performance and using Hicks-Henne function for smoothing the surface variation caused by perturbation of design variables. Some numerical tests have been made. The test results show that the present method is very effective for wing design of both subsonic and transonic case using Euler equations.

1 Introduction

For a long time, aerodynamic design has had to rely upon a trial-and-error process of design, analysis, test, and redesign according to test results in wind tunnel. In the recent years, CFD(Computational Fluid Dynamics) has rapidly become a key tool of aircraft design process as wind tunnel.

Airfoil and wing design methods can be categorized as either inverse or direct procedures. Inverse methods involve the specification of a desired pressure distribution and the calculation of the corresponding airfoil. In direct design methods a numerical optimization algorithm is coupled with a suitable aerodynamic analysis method for specific performance requirements.

In the inverse design approach, one problem is hard to find desirable (or target) pressure distribution for highly three-dimensional flows. Reference[1] pointed out the inverse method is not good for the design of practical wing with nacelles, because the interference caused by the presence of large nacelles results in a local highly three dimensional flow field. If one insists on imposing the straight isobar pattern the resulting wing surface geometry will be highly twisted and will be difficult to manufacture. And in this case a smooth geometry will give a highly distorted local flow field. Another problem is consideration of off design performance, because the inverse method is inherently a single point design process. The third problem is that unless the pressure distribution satisfies certain constraints a physical realizable shape may not necessarily exist. So far the existence of the solutions for general type of flow, especially for three-dimensional flow has not been proved.

These difficulties lead to the desire to develop an optimization methodology for aerodynamic design. These methods may help engineers to achieve quickly a good compromise between aerodynamic objectives and the constraints imposed on the geometry by other design requirements such as manufacturing and structures. It will also allow considering off design conditions and even multi-disciplinary optimization (MDO) design applications.

In order to make newly developed CFD approaches integrate directly into future multi-disciplinary optimization applications, the direct design methods are studied by most design researchers recently due to its ability to use any new CFD methods, any numerical optimization algorithms and to avoid the limitations and difficulties of traditional inverse methods. But the traditional direct methods have problems for their inherent computational efficiency, they are computationally expensive because of the large number of flow solutions needed to determine the gradient information for a necessary number of design variables.

By solving the adjoint equation of Euler equations the gradient of cost function can be obtained with roughly the computation costs of two flow solutions, independent of number of design variables. The method has been applied to fluid dynamics problem by Jameson[2], Reuther[3], Pironneau[10], and Ta'asan[8] and applied to three dimensional wing-design problems by Jameson[2], Reuther[3] and Qiao[4] recent years. In Jameson method a wing is a device to produce lift by controlling the flow, and its design can be regarded as a problem in the optimal control of the flow equations by variation of the shape of the boundary. Using techniques of control theory, the gradient of cost function can be determined by solving adjoint equations, and it is independent of number of design variables.

A numerical optimization method of wing design by solving adjoint equations based on Euler equations is studied for better considering performance requirements of wing design in this paper. The three dimensional adjoint equations are derived according the given requirements of aerodynamic performance and the given desired pressure distribution respectively. And some improvements have been made, such as far field conditions using characteristics theory, smoothing the co-state variable and cost function by using artificial dissipation method, using the vector flux boundary conditions for better convergence performance and using Hicks-Henne function for smoothing the surface variation caused by perturbation of design variables. Some numerical tests have been made for airfoil and wing design. The test results show that the present method is very effective for wing design of both subsonic and transonic case using Euler equations.

2 Basic Equations

Euler equations are adopted as the basic equations for flow analysis

$$\frac{\partial w}{\partial t}+\frac{\partial f_i}{\partial x_i}=0, \quad i=1,2,3 \quad in \ D' \tag{1}$$

Where

$$w=\begin{Bmatrix}\rho\\ \rho u_1\\ \rho u_2\\ \rho u_3\\ \rho E\end{Bmatrix}, \quad f_i=\begin{Bmatrix}\rho u_i\\ \rho u_i u_1+p\delta_{i1}\\ \rho u_i u_2+p\delta_{i2}\\ \rho u_i u_3+p\delta_{i3}\\ \rho u_i H\end{Bmatrix} \tag{2}$$

And δ_{ij} is Kronecker delta function,

$$p=(\gamma-1)\rho\left\{E-\tfrac{1}{2}(u_i^2)\right\} \tag{3}$$

$$\rho H=\rho E+p \tag{4}$$

Where γ is the ratio of specific heat. After transformation the equations from the physical coordinates (x_1,x_2,x_3) to computational coordinates, the Euler equations can be written as

$$\frac{\partial W}{\partial t}+\frac{\partial F_i}{\partial \xi_i}=0 \quad \text{in} \quad \text{D} \tag{5}$$

With

$$w=J\begin{Bmatrix}\rho\\ \rho u_1\\ \rho u_2\\ \rho u_3\\ \rho E\end{Bmatrix}, \quad F_i=J\begin{Bmatrix}\rho U_i\\ \rho U_i u_1+\frac{\partial \xi_i}{\partial x_1}p\\ \rho U_i u_2+\frac{\partial \xi_i}{\partial x_2}p\\ \rho U_i u_3+\frac{\partial \xi_i}{\partial x_3}p\\ \rho U_i H\end{Bmatrix} \tag{6}$$

Where

$$\begin{Bmatrix}U_1\\ U_2\\ U_3\end{Bmatrix}=K^{-1}\begin{Bmatrix}u_1\\ u_2\\ u_3\end{Bmatrix} \tag{7}$$

And

$$K=\{K_{ij}\}, \ K_{ij}=\frac{\partial x_i}{\partial \xi_j}, \ J=\det(K) \tag{8}$$

Now, in the new coordinate system, wing surface B_W is represented by $\xi_2=0$, and the boundary condition on the wing surface is

$$U_2=0 \quad on \ B_W \tag{9}$$

On the far field boundary B_F, the free stream conditions are specified for incoming waves and the outgoing waves are determined by the solution.

3 Adjoint Equations

The cost function of the aerodynamic design problem can be expressed with pressure p on the wing surface or aerodynamic performance such as drag or the ratio of drag to lift.

3.1 Wing design with the given desired pressure

The cost function is defined as follows,

$$I = \frac{1}{2}\iint_{B_W}(p-p_d)^2 d\xi_1 d\xi_3 \tag{10}$$

Where p_d is the desired pressure (or target pressure) on the wing. The design problem can be studied as a control problem choosing wing surface as the control function to minimize I subject to the constraints defined by the flow equations. The variation of the cost function caused by the variation of wing surface,

$$\delta I = \iint_{B_w}(p-p_d)\delta p d\xi_1 d\xi_3 \tag{11}$$

From Euler equations in steady state, we have

$$\frac{\partial}{\partial \xi_i}(\delta F_i) = 0 \tag{12}$$

Where
$$\delta F_i = C_i \delta w + \delta\left(J\frac{\partial \xi_i}{\partial x_j}\right) f_j \tag{13}$$

$$C_i = JK_{ij}^{-1}A_j\ ,\ \ A_j = \frac{\partial f_j}{\partial w} \tag{14}$$

Multiplying by a vector co-state variable ψ and integrating over the domain, equation (12) becomes

$$\int_D \psi_j^T \left(\frac{\partial \delta F_i}{\partial \xi_i}\right) d\xi_j = 0 \tag{15}$$

If ψ is differentiable, the equation can be integrated by parts to give

$$\int_D \left(\frac{\partial \psi^T}{\partial \xi_i}\delta F_i\right) d\xi_j = \int_B \left(n_i \psi^T \delta F_i\right) d\xi_j \tag{16}$$

Where n_i are components of a unit vector normal to the boundary. Thus the variation in the cost function can be written as

$$\delta I = \iint_{B_W}(p-p_d)\delta p d\xi_1 d\xi_3 - \int_D \left(\frac{\partial \psi^T}{\partial \xi_i}\delta_{F_i}\right) d\xi_j + \int_B (n_i \psi^T \delta F_i) d\xi_j \tag{17}$$

And

$$\begin{aligned} n_1 = n_3 = 0 \quad on \quad B_W \\ n_1 = n_2 = 0 \quad on \quad B_S \end{aligned} \tag{18}$$

From the second integral of equation (17), to eliminate the term, which contains δw, the adjoint equation can be obtained

$$\frac{\partial \psi}{\partial t} - C_i^T \frac{\partial \psi}{\partial \xi_i} = 0 \qquad \text{in} \quad D \tag{19}$$

If the coordinate transformation is such that $\delta(JK^{-1})$ is negligible in far field, from the boundary integral of equation (17) by letting ψ satisfy the boundary conditions,

$$J\left(\psi_2\frac{\partial \xi_2}{\partial x_1} + \psi_3\frac{\partial \xi_2}{\partial x_2} + \psi_4\frac{\partial \xi_2}{\partial x_3}\right) = p - p_d \quad on\ B_W \tag{20}$$

$$J\left(\psi_2\frac{\partial \xi_3}{\partial x_1} + \psi_3\frac{\partial \xi_3}{\partial x_2} + \psi_4\frac{\partial \xi_3}{\partial x_3}\right) = 0 \quad on\ B_S \tag{21}$$

Finally

$$\begin{aligned} \delta I = &-\int_D \frac{\partial \psi^T}{\partial \xi_i}\delta\left(J\frac{\partial \xi_i}{\partial x_j}\right) f_j d\xi_k \\ &-\iint_{Bw}\left\{\psi_2\delta\left(J\frac{\partial \xi_2}{\partial x_1}\right)+\psi_3\delta\left(J\frac{\partial \xi_2}{\partial x_2}\right)+\psi_4\delta\left(J\frac{\partial \xi_2}{\partial x_3}\right)\right\} p d\xi_1 d\xi_3 \\ &-\iint_{Bs}\left\{\psi_2\delta\left(J\frac{\partial \xi_3}{\partial x_1}\right)+\psi_3\delta\left(J\frac{\partial \xi_3}{\partial x_2}\right)+\psi_4\delta\left(J\frac{\partial \xi_3}{\partial x_3}\right)\right\} p d\xi_1 d\xi_3 \end{aligned} \tag{22}$$

The gradient can then be defined with respect to the design variable b_i.

$$G(b_i) = \frac{\delta I}{\delta b_i} \tag{23}$$

3.2 Wing design with the given requirements of aerodynamic performance

The cost function can be written as:

$$I = w_1 \tfrac{1}{2} C_D^2 + w_2 \tfrac{1}{2}(C_L - C_{Ld})^2 \tag{24}$$

Where w_1, w_2 are weight coefficients, C_{Ld} is requirement value of lift coefficient. Considered the change of cost function caused by the change of shape and angle of attack, from Euler equations in steady state, Multiplying by a vector co-state variable ψ and integrating over the domain, then the variation of cost function can be written as

$$\delta I = \Omega_1 \delta C_D' + \Omega_2 \delta C_L' + [\Omega_1 \frac{\partial C_D}{\partial \alpha} + \Omega_2 \frac{\partial C_L}{\partial \alpha}]\delta\alpha - \int_B (n_i \psi^T \delta F_i) d\xi_j + \int_D \left(\frac{\partial \psi^T}{\partial \xi_i} \delta F_i \right) d\xi_j \quad (25)$$

Where $\Omega_1 = w_1 C_D$, $\Omega_2 = w_2 (C_L - C_{Ld})$, $\delta C_D'$ and $\delta C_L'$ are caused by the change of wing surface shape.

From the fifth term of above δI expression, the adjoint equations can be derived

$$J \frac{\partial \psi^T}{\partial x_j} A_j = 0 \quad (26)$$

or written as unsteady form for numerical computation

$$\frac{\partial \psi}{\partial t} - A_j^T \frac{\partial \psi}{\partial x_j} = 0 \quad (27)$$

Boundary conditions can be derived from the fourth term of δI expression

$$\psi_2 n_x + \psi_3 n_y + \psi_4 n_z = 2/(\gamma M_\infty^2 S_{ref}) \{ \Omega_1 [n_X \cos\alpha + n_y \sin\alpha] + \Omega_2 [n_y \cos\alpha - n_x \sin\alpha]\} \quad on \quad B_w \quad (28)$$

$$\psi_4 = 0.0 \quad \text{on} \quad B_S \quad (29)$$

Finally

$$\delta I = -2/(\gamma M_\infty^2 S_{ref}) \int_{B_W} (p/p_\infty - 1)(\Omega_1 [\delta(n_x |S_2|)\cos\alpha + \delta(n_y |S_2|)\sin\alpha] + \Omega_2 [\delta(n_y |S_2|)\cos\alpha - \delta(n_x |S_2|)\sin\alpha]) d\xi_1 d\xi_3 + \left[\Omega_1 \frac{\partial C_D}{\partial \alpha} + \Omega_2 \frac{\partial C_L}{\partial \alpha} \right] \delta\alpha + \int_D \frac{\partial \psi^T}{\partial \xi_i} \delta(J \frac{\partial \xi_i}{\partial x_j}) f_j d\xi_j + \int_{B_W} \left[\psi_2 \delta(J \frac{\partial \xi_2}{\partial x_1}) + \psi_3 \delta(J \frac{\partial \xi_2}{\partial x_2}) + \psi_4 \delta(J \frac{\partial \xi_2}{\partial x_3}) \right] p d\xi_1 d\xi_3 \quad (30)$$

From the δI expression the gradient with respect to the design variables b_i can be obtained as

$$G(b_i) = \frac{\delta I}{\delta b_i} \quad (31)$$

4 Solution of the Adjoint Equation

4.1 Coordinate transformation

To use finite volume method for solving the adjoint equations (19) we transform the equations (19) and boundary conditions (20), (21) from computational space into physical space.

$$\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial x_i}(A_i^T \psi) = 0 \qquad \text{in} \quad D' \quad (32)$$

$$\psi_2 s_x + \psi_3 s_y + \psi_4 s_z = p - p_d \quad on \quad B_W$$
$$\psi_2 s_x + \psi_3 s_y + \psi_4 s_z = 0 \quad on \quad B_S \quad (33)$$

4.2 Far field boundary condition using characteristics theory

From the adjont equations, introducing transformation

$$P^{-1}(A_i n_i) P = \Lambda \quad (34)$$

$$\overline{\psi} = (P^{-1})^T \psi \quad (35)$$

Where P is a transformation matrix that can be derived with the method like the case of flow equations. Thus we obtain

$$\frac{\partial \overline{\psi}}{\partial t} + \Lambda^T \frac{\partial \overline{\psi}}{\partial n} = 0 \quad (36)$$

$$\Lambda = \begin{bmatrix} q_n + c & & & & \\ & q_n & & & \\ & & q_n & & \\ & & & q_n & \\ & & & & q_n - c \end{bmatrix} \quad (37)$$

$$\psi = P^T \overline{\psi} \quad (38)$$

4.3 Vector flux boundary conditions

Boundary conditions (33) can be transformed into vector flux boundary conditions.

The semi-discretized adjoint equation of equation (32) can be written as

$$\frac{d(\Omega\psi)_{ijk}}{dt} - R_{ijk} = 0 \quad (39)$$

$$R_{ijk} = \tilde{F}_{i+\frac{1}{2}jk} - \tilde{F}_{i-\frac{1}{2}jk} + \tilde{F}_{ij+\frac{1}{2}k} - \tilde{F}_{ij-\frac{1}{2}k} + \tilde{F}_{ijk+\frac{1}{2}} - \tilde{F}_{ijk-\frac{1}{2}} \quad (40)$$

Where Ω_{ijk} are cell volume, and the vector flux

$$\tilde{F}_i = n_i A_i^T \psi \cdot \vec{s} \quad (41)$$

Where

$$A_i = \frac{\partial f_i}{\partial w} \quad (42)$$

Using the expression of A_i, the vector flux $\tilde{F}$ on the wing surface B_W can be derived as follows,

$$\begin{bmatrix} \tilde{F}_1 \\ \tilde{F}_2 \\ \tilde{F}_3 \\ \tilde{F}_4 \\ \tilde{F}_5 \end{bmatrix} = \begin{bmatrix} \frac{\gamma-1}{2}(p-p_d)q^2 \\ Es_x - u(\gamma-1)(p-p_d) \\ Es_y - v(\gamma-1)(p-p_d) \\ Es_z - w(\gamma-1)(p-p_d) \\ (\gamma-1)(p-p_d) \end{bmatrix} \quad (43)$$

Numerical test shows that using the derived vector flux boundary condition (43) much better convergence performance can be obtained than using boundary condition (33) directly.

4.4 Artificial dissipation

Introducing the artificial dissipation term to the equation (39) is crucially important to avoid discontinuities and to keep co-state vector variable ψ is differentiable. In present paper, dissipation term can be written as

$$\bar{d}_{i+\frac{1}{2}jk} = \alpha_{i+\frac{1}{2}jk}\left[\varepsilon^{(2)}_{i+\frac{1}{2}jk}\left(\psi_{i+1jk} - \psi_{ijk}\right)\right] - \varepsilon^{(4)}_{i+\frac{1}{2}jk}\left(\psi_{i+2jk} - 3\psi_{i+1jk} + 3\psi_{ijk} - \psi_{i-1jk}\right) \quad (44)$$

$$\varepsilon^{(2)}_{i+\frac{1}{2}jk} = k^{(2)}\max(\gamma_{i+1jk}, \gamma_{ijk}) \quad (45)$$

$$\varepsilon^{(4)}_{i+\frac{1}{2}jk} = \max\ (0, k^{(4)} - \varepsilon^{(2)}_{i+\frac{1}{2}jk}) \quad (46)$$

$$\gamma_{ijk} = \left|\frac{l_{i+1jk} - 2l_{ijk} + l_{i-1jk}}{l_{i+1jk} + 2l_{ijk} + l_{i-1jk}}\right| \quad (47)$$

After comparing following definitions of the γ_{ijk},

$$l = \psi_1 \quad (48)$$

$$l = \sqrt{\sum_{n=1}^{5}\psi_n^2} \quad (49)$$

$$l_n = \psi_n \qquad n=1,2,3,4,5 \quad (50)$$

$$l = p \quad (51)$$

Among the above definitions, $l = p$ is better than others.

5 Numerical Tests For Wing Design

5.1 Wing Design with Given Desired Pressure

In the test Design case, wing planform was fixed while the wing sections were free to be changed arbitrarily in design process. In the design example the wing has a unit root section chord, with aspect ratio of 6, with 20-degree leading edge sweep and 25-degree trailing edge sweep. The initial wing sections were based on NACA 0012 airfoil, the target pressure is obtained from RAE2822 airfoil at Mach number 0.80 with zero angle of attack.

The design results are given in Fig.1 and Fig.2.

5.2 Wing Design with the given requirements of aerodynamic performance

Sweep angle of the leading edge is 40 degree for inner wing and 34.4 degree for outer wing respectively. The aspect ratio is 9.02. A linear twist distribution was imposed on the geometry such that the incident angle varies from +4.0° at the root to –1.0° at the tip. In initial wing 9 design profiles and 26 design variables for each profile are used. The wing planform is fixed, but the angle of attack and twist angle of each profile can be changed. The design Mach number is 0.839. Design lift coefficient is 0.5. The design results show that the lift to drag ratio is increased from 2.99 to 3.84. The details can be found from Fig.3-Fig.4.

The above test cases show that the method is very effective for airfoil and wing design in subsonic and transonic flow using Euler equations.

Acknowledgments

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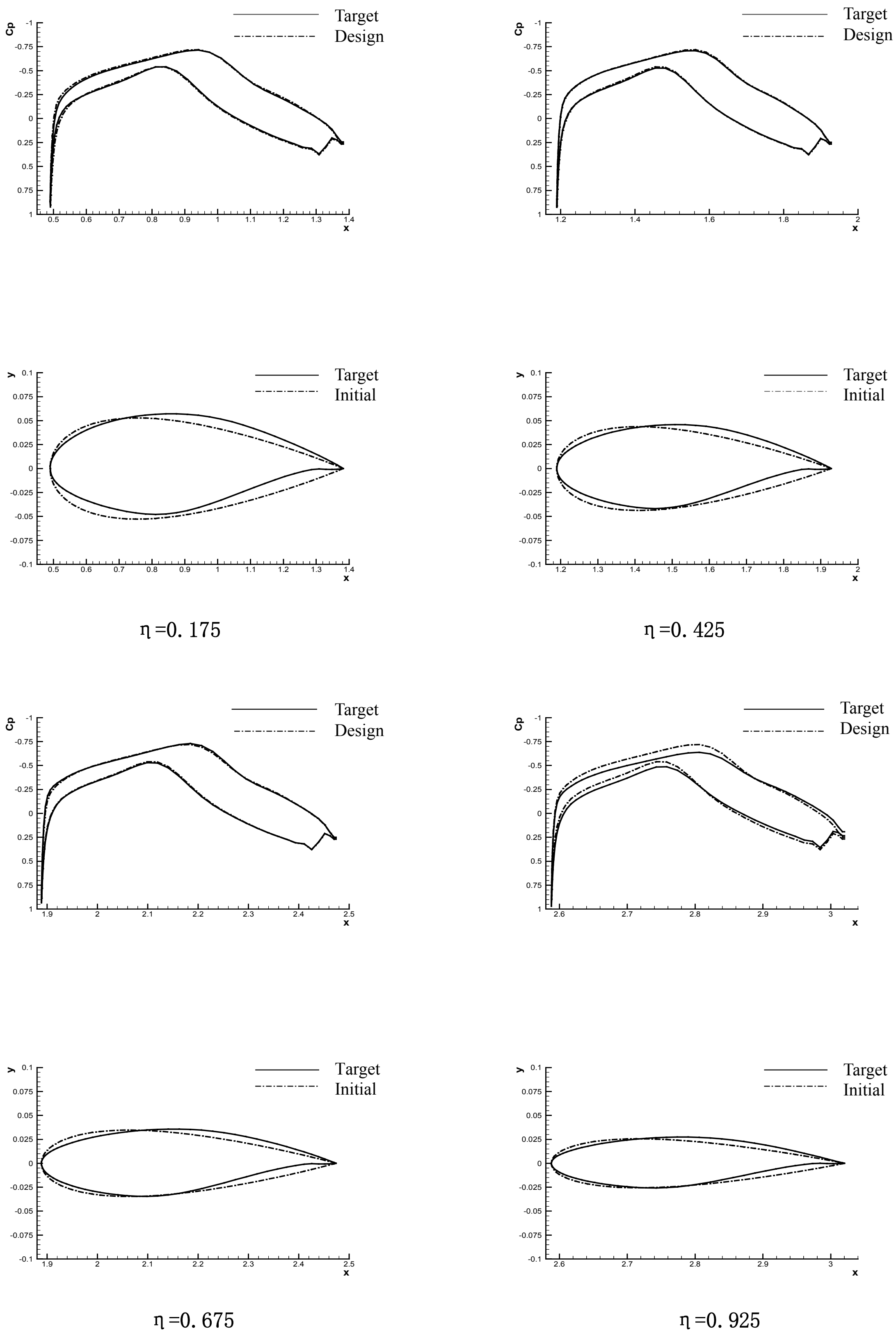


Fig. 1 The example of wing design $Ma = 0.8 \quad \alpha = 0.0$

Pressure and section comparison of the initial、the target and the designed wing

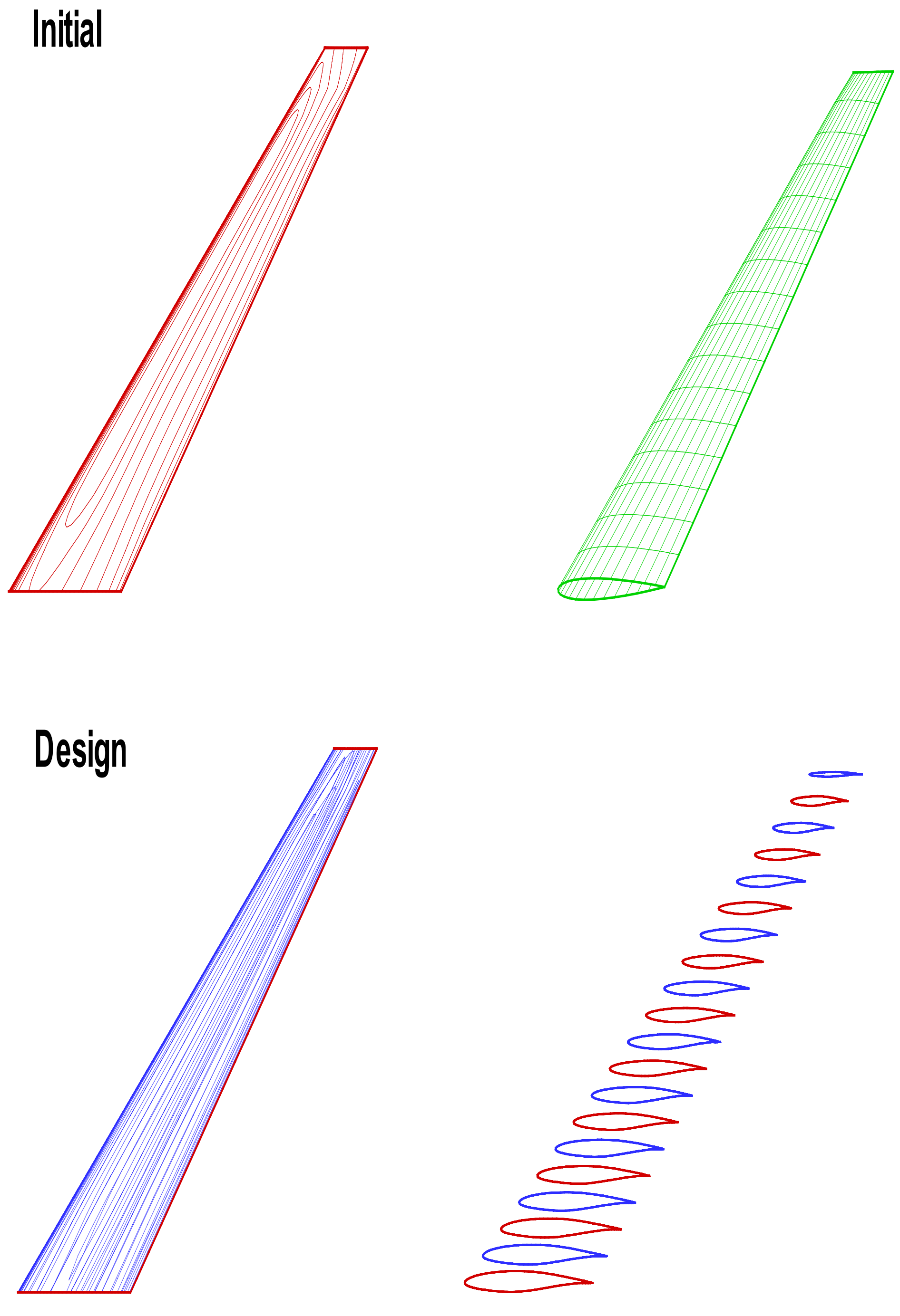


Fig. 2 Shape and isobar comparison between the initial and the designed wing
$M_a = 0.8$ α=0.0

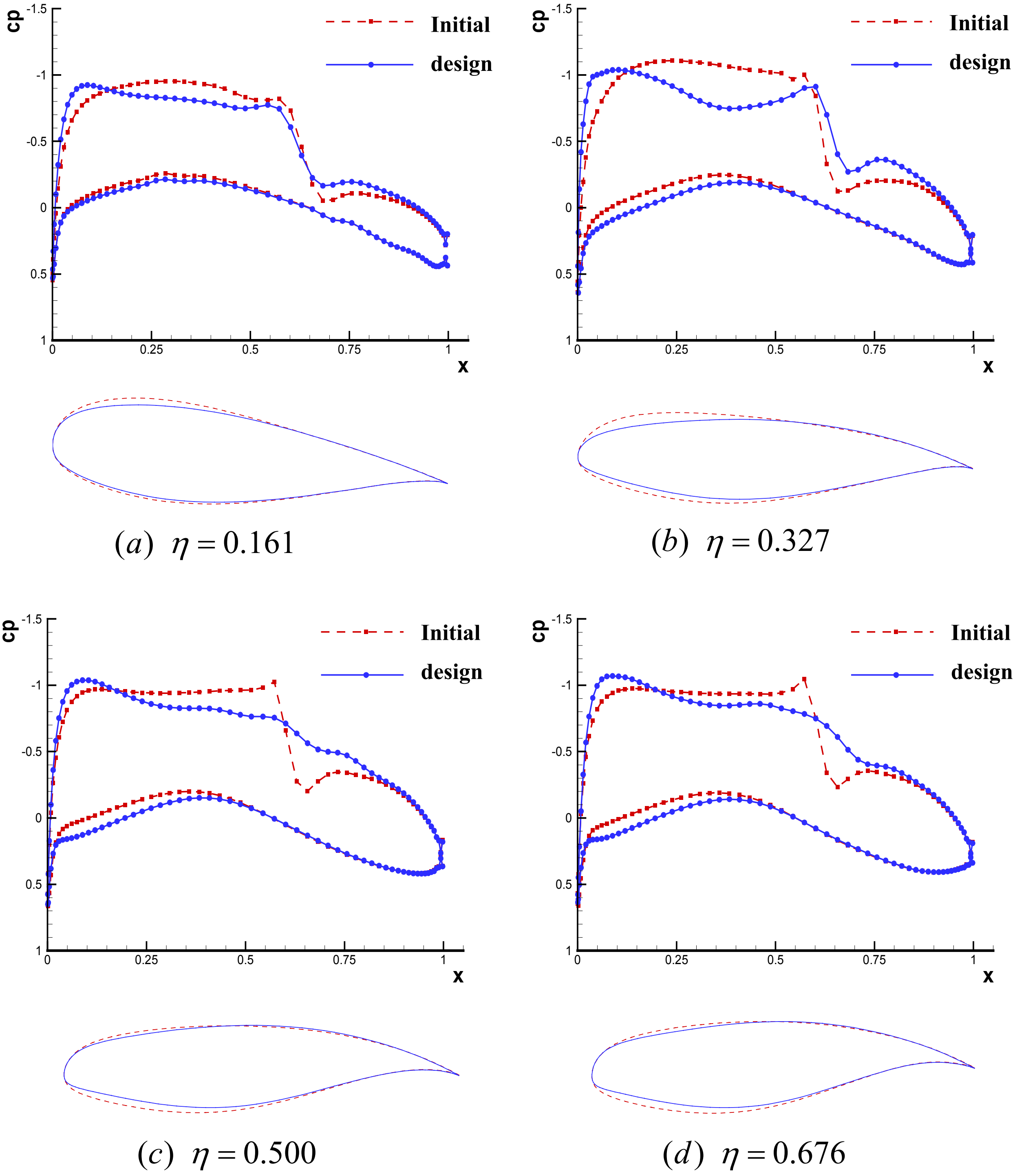
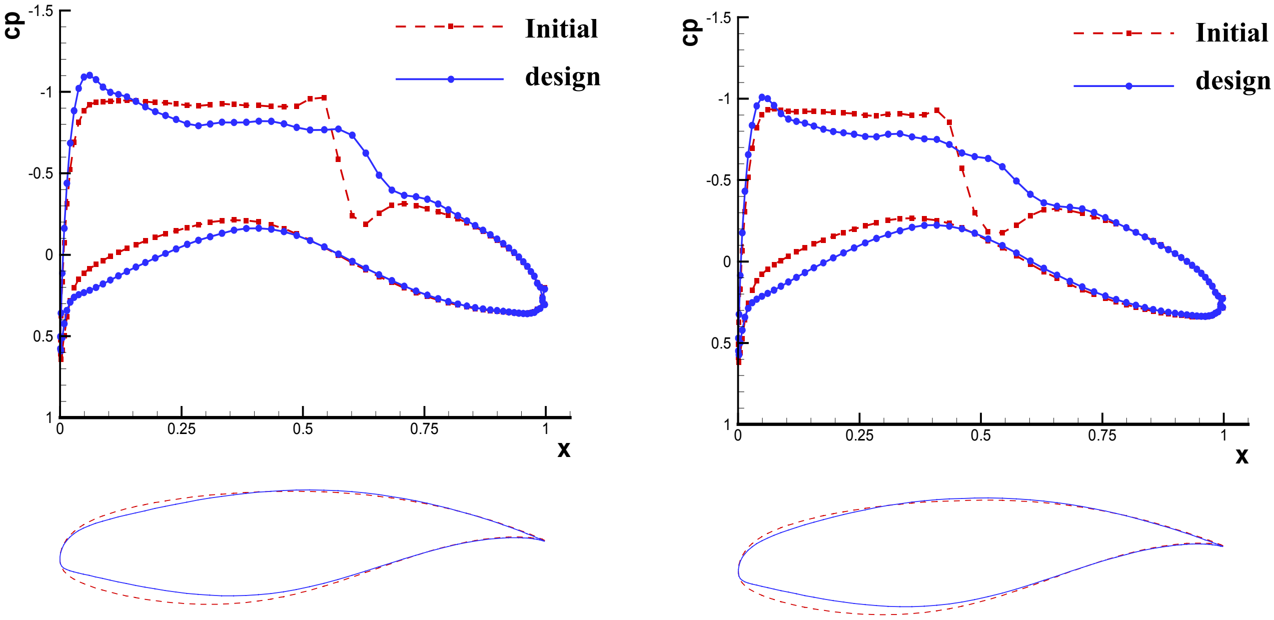


Fig. 3 Pressure and section comparison of the initial and designed wing

initial: $Ma = 0.839 \quad C_L = 0.472 \quad C_D = 0.01577 \quad \alpha = 0.787^0$

design: $Ma = 0.839 \quad C_L = 0.492 \quad C_D = 0.01281 \quad \alpha = 0.926^0$



(e) $\eta = 0.842$ (f) $\eta = 0.935$

Fig. 3 Pressure and section comparison of the initial and designed wing (Continued)

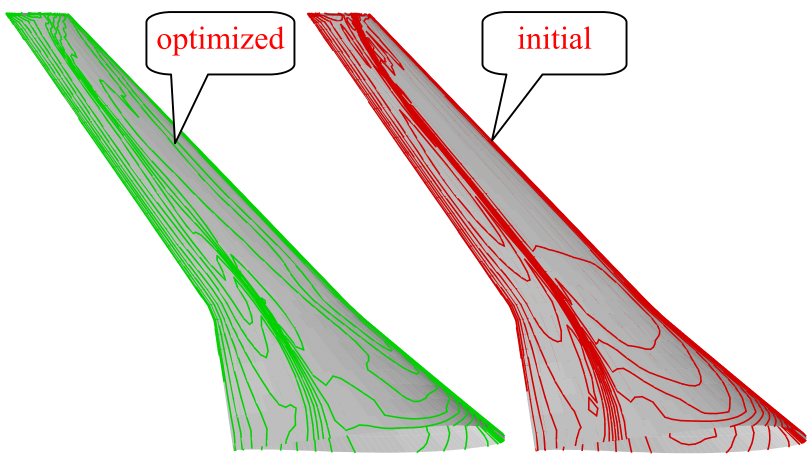


Fig. 4 Comparison of isobar on the initial and optimized wing