THE DESIGN OF AN AUTOMATIC FLIGHT CONTROL SYSTEM FOR A HYPERSONIC TRANSPORT AIRCRAFT

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Abstract

The design of an automatic flight control system for a hypersonic transport aircraft (HST) is presented. It uses a mathematical model of the longitudinal motion of an HST flying at Mach 8.0 and at a height of 85000 feet. Included in the model were the non-linear dynamics associated with the propulsion system. Since the basic aircraft is dynamically unstable, closed-loop stabilization was required. Normally, the required degree of closed-loop stability is determined from published flying qualities specifications which are unavailable, however, for this type of aircraft. Consequently, a set of dynamic parameters for the closed-loop stabilization system was proposed which corresponded with notionally acceptable flying qualities. Using these parameters it was possible to use an analytical procedure to determine a corresponding state weighting matrix to be used in a quadratic performance index which, when minimized, provided a feedback control law that ensured the controlled HST had the specified flying qualities.

The effectiveness of the system is demonstrated from results obtained by digital simulation, including the propulsion system.

1 Introduction

Current subsonic transport aircraft are capable of flying very long distances at cruise Mach numbers up to about Mach 0.85. The Boeing 747-400, for example, has a maximum range of over 7000 nautical miles at Mach 0.84, a range of about one-third of the Earth's circumference. Flights of this distance currently require between 13–15 hours in the air. Longer distances, such as from New York to Sydney, Australia, for example, would take over 18 hours. Most passengers often consider these journeys too long, but with an HST aircraft, this problem of extended flight times can be solved. Business travellers are known to be especially dissatisfied with the lengthy transit times associated with long haul flights such as those between Europe and the Far East, or South America, or the west coast of Canada and the USA, or from the USA to Japan and the Pacific rim countries. Since most long haul flights take place on those routes and the number of passengers travelling on those routes is reliably predicted to double from today's figures by the year 2005 [1], [2], considerable attention is being given now in the USA, Europe and Japan to the development of HST aircraft. The big advantage of future HST aircraft is their potential for reducing long-range flight times and, in turn, increasing aircraft productivity, passenger comfort and convenience.

This paper presents information in relation to the design of a closed-loop, automatic flight control system (AFCS) which will provide such HST aircraft with desirable flying qualities which they do not possess in the absence of closed-loop control.

From an initial study of available HST dynamics, it was found that these aircraft were highly unstable statically and dynamically. Since the level of instability possessed by such aircraft was high, a robust control technique was required to take into account the many uncertainties of the mathematical model. Hence, Linear Quadratic Regulator (LQR) theory was chosen to design the stabilising control system. One of the disadvantages regularly complained of in the literature when using LQR theory is that the choice of weighting matrices, Q and G, on the state and control variables respectively, is restricted [3]. A method of systematically finding the state weighting matrix, Q, such that the aircraft displays closed-loop modes as specified by the designer is presented and discussed in this paper.

The mathematical model of the longitudinal motion of the HST aircraft uses three controls; the change in flap angle, $\Delta\delta_F$, the change in the ratio of the engine duct area, ΔA_D, and the change in the temperature across the engine combustor, ΔT_O. When the aircraft model was subjected to simulated disturbances, it was found that the flaps were the most active control in stabilizing the aircraft. This is a cause for great concern because at hypersonic speeds, the flaps become less effective and aerodynamic heating on the surface of the flaps can cause structural damage. It was decided that the engine of the aircraft must be used more actively and effectively as a means of controlling the aircraft which required a better representation of the hypersonic propulsion system. This mathematical model involved the use of variables such as the specific impulse, full-eigenvalue ratio and thrust.

The results obtained from a digital simulation of the AFCS indicate that the design of the AFCS was effective.

2 Mathematical Models

2.1 Longitudinal Motion of the HST

The A sketch of the HST, showing approximate overall dimensions and location of the control inputs, is shown in Figure 1. The mathematical model used was developed from an initial model based on that of Chavez and Schmidt [4]. However, the model of Chavez and Schmidt had to be modified to account for a number of errors,[1] and to allow the use of more appropriate physical units for the variables involved. The data given by Chavez and Schmidt for the stability and control derivatives were in degrees, not radians, so that care is needed in using the published data in any resulting mathematical model if consistency is to be achieved.

The variables chosen for the aircraft's state vector were:

$$\underline{x}' = [u\ \alpha\ q\ \theta\ h\ \eta\ \dot{\eta}] \tag{1}$$

where

- u denotes the change in forward speed (in ft/s)
- α denotes the change in angle of attack (in radians)
- q denotes the change in pitch rate (rad/s)
- θ denotes the change in pitch attitude (rad)
- h denotes the change in height (ft)
- η denotes the change in displacement of the fundamental bending mode (ft)
- $\dot{\eta}$ denotes the rate of change of the displacement of the fundamental bending mode (ft/s).

The variables used as elements of the control vector, $\underline{u}$, were

$$\underline{u} = \begin{bmatrix} \delta_F \\ A_D \\ T_O \end{bmatrix} \tag{2}$$

The state equation governing the longitudinal motion was, therefore:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{3}$$

The matrices, A, of order [7 x 7], and B, of order [7 x 3], are shown in Table 1.

The corresponding eigenvalues of the matrix, A, are presented in Table 2. One of the eigenvalues (λ_6) has a positive real root which

[1] There are errors in eq. (22), the matrix T_1, the equation for $\dfrac{\partial M_e}{\partial M_3}$ and eq. (12a) of the original paper.

means that, without a stability augmentation system (SAS), the HST is highly unstable.

The response of the HST's angle of attack and pitch rate to a sudden disturbance is shown in Figure 2. It is obvious that the HST is dynamically unstable.

2.2 The Propulsion System

The mathematical model of the engine dynamics was based on the work of Raney [5] and is represented in the block diagram of Figure 3. The command input to the system is the fuel flow rate command, $w_{fuel_{comm}}$, and the output is net thrust. There are involved, apart from the limiters, four functional non-linearities:

(a) Fuel Equivalence Ratio (η)

The commanded fuel flow rate, $w_{f_{comm}}$, was used, in association with aircraft Mach number, M, and dynamic pressure, q_∞, to compute the fuel eigenvalue ratio, η. When η was used, in the range $0 < \eta \leq 1$, with an interpolation table, the specific impulse I_{sp}, could be evaluated. If $\eta > 1.0$ a penalty is applied by limiting the upper limit for I_{sp}. For any air-breathing engine, I_{sp} is a direct measure of fuel efficiency. The best efficiency for hypersonic propulsion occurs at about a fuel eigenvalue ratio of unity. Should the ratio be adjusted above 1.0, the fuel efficiency will decline and there will be an attendant decrease in η. A correction factor is used to limit I_{sp} to reflect the lower value of η. The product (I_{sp} x w_f) determines the net thrust, T_h. The fuel flow rate, w_f, depends upon the fuel flow rate commanded:

$$\frac{w_f(s)}{w_{f_{comm}}(s)} = \frac{1}{(1+T_E s)} \tag{4}$$

where the engine time constant, T_E, lies in the range 0.4 – 1.2 sec (T_E was taken as 0.5s in this paper).

The ideal flow rate corresponds to the fuel flow rate when η = 1.0, i.e.

$$w_f\Big|_{\eta=1} = q_\infty e^{f_1(M)} \tag{5}$$

However, the functional relationship between f_1 and M is non-linear and is shown in Figure 5. The fuel equivalence ratio, η, can then be found from:

$$\eta = \frac{w_f}{w_f\Big|_{\eta=1}} \tag{6}$$

At a specified Mach number, say 8.0, the specific impulse, I_{sp}, corresponding to η = 1.0, is shown in Figure 5. When $\eta \neq 1.0$ another non-linear specific impulse function (see Figure 6) had to be used.

(b) Maximum Specific Impulse

The maximum specific impulse was given by

$$I_{sp_{max}} = f_2(M)/\eta \tag{7}$$

and the net thrust from the engine was

$$T_h = I_{sp}\, w_f \tag{8}$$

The mathematical model of the engine dynamics was tested by digital simulation over a wide range of conditions. A typical engine response for a step change in fuel flow command for a range of Mach numbers is shown in Figure 7. The effect of Mach number of the steady-state thrust delivered can easily be discerned.

3 Automatic Flight Control System (AFCS)

An AFCS was required:

(a) to provide closed-loop stability;
(b) to provide satisfactory flying qualities from the closed-loop system.

If the feedback control law could also provide some degree of robustness that would be a practical advantage. An obvious control design method to apply, which provides these requirements, is the Linear Quadratic Regulator (LQR) theory. Bryson [3] has summed up why LQR theory is deemed to be the most suitable candidate theory for the design of any system to control an aircraft. He claimed that LQR designs produce co-ordinated controls which yield graceful flight paths comparable to those produced by expert pilots. They represent the best performance the control engineer can expect, given accurate sensors and a low-noise environment. However, a disadvantage of using this method is that there is apparently no systematic way of obtaining the weighting matrices, Q and G, used in the performance index, J, which has to be minimized viz:

$$J = \frac{1}{2}\int_{o}^{\alpha} (\underline{x}' Q \, \underline{x} + \underline{u}' G \, \underline{u}') dt \qquad (9)$$

What is required is a systematic approach for obtaining the weighting matrices, which would result in the dynamics of the HST responding to disturbances in a manner specified by the flying quality criteria. The flying qualities parameters published by all the world's aviation authorities are based on low level mathematical models corresponding to particular modes of flight. These modes can be identified from the eigenvalues of the coefficient matrix, A, for open-loop systems and the eigenvalues of (A+B∗K) for closed-loop systems.

A method proposed by Luo and Lan [6] provides a systematic determination of the state weighting matrix corresponding to a set of closed-loop eigenvalues specified by the designer. The control weighting matrix, G, was designed according to Bryson's rule [7]. The desired set of closed-loop eigenvalues were derived from the flying qualities achieved with the SR-71 aeroplane flying at Mach 3.5 at a height of 70000 ft. It is possible that the resulting matrix, Q, obtained using Luo and Lan's technique, might be negative definite. In that event the authors recommended that the negative elements of the matrix be forced to zero to satisfy the positive semidefinite requirement stated in most of the literature on the subject. Should the closed-loop eigenvalues become too different from the desired eigenvalues as a result of doing this, the authors recommended that the designer choose a different set of eigenvalues for the system. An example in Luo and Lan demonstrated that by using a positive semidefinite matrix, Q, after forcing the negative elements of the matrix, Q, to be zero, the authors managed to obtain a set of desired closed-loop eigenvalues which were not identical, but were close, to those specified.

From the LQR theory, a canonical equation is obtained viz

$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{\underline{\psi}} \end{bmatrix} = \begin{bmatrix} A & -BG`B' \\ -Q & -A' \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{\psi} \end{bmatrix} = A_{CANON} \begin{bmatrix} \underline{x} \\ \underline{\psi} \end{bmatrix} \qquad (10)$$

where $\underline{\psi}$ is the co-state vector. The 2n eigenvalues of the canonical matrix A_{CANON} satisfy the equation

$$[\sigma \; I - A_{CANON}] = 0 \qquad (11)$$

Assuming that the matrix, Q, is diagonal with elements

$$\left.\begin{aligned} Q_{ii} &= q_i \\ Q_{ij} &= 0 \quad i \neq j \end{aligned}\right\} \qquad (12)$$

then, if each specified closed-loop eigenvalue

$$\sigma_i = \mu_i + j\, w_i \qquad (13)$$

eq. (11) will provide one equation which can be solved for q_i. Thus,

$$\begin{aligned} f(q_1, q_2, \cdots q_n) &= det\,[\,(\mu_i + j\, w_i)\, I - A_{CANON}\,] \\ &= 0 \\ & \quad i = 1, 2, \cdots n \end{aligned} \qquad (14)$$

These n algebraic equations can be solved for the unknown elements, q_i. How the technique was used in the design of the AFCS for the HST is discussed next.

It was shown earlier that the uncontrolled HST was dynamically unstable. Here, a specified set of stable closed-loop eigenvalues with corresponding damping ratios and natural frequencies are shown in Table 3.

These eigenvalues are arbitrary, but all are stable. Note that the damping ratio of the short period motion has been set very low for demonstration purposes. If this specific short period damping does not satisfy the flying quality criteria, then a new damping ratio can be easily specified. To solve the corresponding eq. (14) required the use of Newton's method. The symbolic, numerical software package Maple was used. The resulting matrix, *Q*, corresponding to a choice of identity-matrix, *G*, was found to be:

$$Q = diag\begin{bmatrix} 0.00004 & -511.95 & 15.0 & -682.4 & \cdots & & \\ & & \cdots & 2.7\times10^{-8} & 37387.0 & -123.32 \end{bmatrix} \quad (15)$$

Thus, the matrix obtained was negative definite. The corresponding feedback gain matrix was:

$$K = \begin{bmatrix} 0.006 & -6.24 & -4.2 & 3.93 & 0.000038 & 22.75 & -0.153 \\ 0.0024 & 5.44 & -1.27 & -13.67 & 0.000011 & -53.65 & -0.78 \\ 0 & 0.00037 & 0.0001 & -0.0002 & 0 & 0.0015 & 0.00003 \end{bmatrix} \quad (16)$$

The closed-loop eigenvalues obtained were identical to the specified set. The corresponding step response from the controlled HST is shown in Figure 8. This figure shows the effect of the short period mode on all the step responses. The short period oscillation can be seen clearly in the first 50 seconds after the command was initiated; however, the magnitude of oscillation is not large.

Another choice of closed-loop eigenvalues such as that shown in Table 4.

Note that a higher value of damping ratio for the short period motion has been chosen compared to that in Table 3. By choosing this higher damping ratio, the effect of the short period oscillation on the step responses will be reduced.

Using the same matrix, *G*, the matrix, *Q*, was found to be:

$$Q = diag\begin{bmatrix} 0.035 & -26082.0 & -153.55 & 6575.7 & \cdots & & \\ & & \cdots & 8.4\times10^{-7} & 3796100.0 & 2491.0 \end{bmatrix} \quad (17)$$

which is negative definite.

The resulting feedback gain matrix was found to be:

$$K = \begin{bmatrix} 0.18 & -130.0 & -16.28 & -35.85 & 0.00075 & 465.9 & -20.36 \\ -0.02 & 198.2 & 0-.92 & -245.45 & -0.0006 & -1584.7 & -75.66 \\ 0.0 & 0.034 & -0.002 & -0.047 & 0.0 & 0.036 & 0.00023 \end{bmatrix} \quad (18)$$

Using this matrix, *K*, the closed-loop eigenvalues were found to be identical to those specified in Table 4. The large values of gain were a consequence of choosing a very high value of damping ratio for the bending motion. Figure 9 illustrates the response of the aircraft subjected to commanded step change in height of 1000 ft.

The method of finding the required state-weighting matrix, *Q*, proposed by Luo and Lan, provides AFCS designers with sufficient freedom to find an appropriate matrix, *Q*, and hence a corresponding feedback gain matrix, *K*, to obtain any specified stable set of closed-loop eigenvalues. The choice of control weighting matrix, *G*, is restricted only to a rule which requires that the matrix be positive definite. The biggest problem found during this work was how to find an efficient solution to the set of non-linear algebraic equations. Using Maple, the equations were solved using Newton's method, but involved much iteration, and took a considerable time to converge to a satisfactory solution.

The most unusual aspect of using the method of Luo and Lan is that the resulting *Q* matrix is often negative definite, even though much of the literature on optimal control insists that to obtain an optimal feedback control law (which will guarantee closed-loop stability) the state weighting matrix must be at least positive semidefinite. Notwithstanding the statements in the literature, however, it is possible to secure stable, optimal closed-loop response with the use of a negative definite Q matrix. There are occasions when the use of such a matrix will fail and no stabilizing feedback control can be found which means that the set of desired eigenvalues cannot be achieved using linear full state variable feedback. The condition which has to be satisfied is that the performance index should be positive, i.e.

$$\underline{x}' Q \underline{x} + \underline{u}' G \underline{u} > 0$$

Thus a negative definite Q matrix can be used provided $\underline{x}' Q \underline{x} < \underline{u}' G \underline{u}$.

$\underline{u}' G \underline{u}$ is always positive, since G is always positive definite, since it must be non-singular. In this work it was unnecessary therefore to adopt the procedure proposed by Luo and Lan that the negative elements of the Q matrix should be forced to zero to ensure that the Q matrix was positive semidefinite. The result of this procedure is to ensure that the eigenvalues of the closed-loop system do not match the defined set.

A concern with any AFCS design, but particularly for an HST, where the magnitude of aerodynamic control surface deflections should not be large, is the extent of the activity involved in the closed-loop control action. The response of δ_F, corresponding to the step response shown in Figure 9, is shown in Figure 10. The maximum negative value required using G, the identity control matrix, was 1.95º and the maximum positive value was 0.5º.

Neither value is unacceptably large. By altering the penalties on the use of the other controls, such as choosing as the control weighting matrix, G_3, where

$$G_3 = diag\,[1 \quad 5 \quad 9] \qquad (19)$$

there is a small effect on the increased use of δ_F. See Figure 10.

The AFCS has been designed on the basis of linear analysis, although the dynamics of the propulsion system have been shown in section 2.1 to be non-linear. To determine the effect of these non-linear dynamics on the performance of the linear AFCS the closed-loop system was simulated to incorporate the dynamics of the propulsion system characterized in Figures 3–6 inclusive. The arrangement is shown in Figure 11. As expected, there was no difference observed in the response of the AFCS for Mach 8.0 at height 85000 ft, since the fixed values resulted in the same situation. However, when the investigation was extended to consider other flight conditions, the effects of the engine dynamics became more evident, chiefly in the slowing of the overall dynamic response.

4 Conclusions

Although the basic flying qualities of any proposed HST will be unacceptable, since it will be both statically and dynamically unstable, it is possible to design an effective AFCS using all the available control inputs in a feedback control law. The optimal control theory known as LQR can be used with confidence since there is a procedure to obtain the necessary state and control weighting matrices which will ensure that the eigenvalues of the optimally-controlled HST will match a set defined by the designer to ensure that desirable flying qualities are provided. In this work, this set of eigenvalues was determined from the flight response of the SR-71 flying at above the same height, but at only half the Mach number. If the HST is to be studied for different flight conditions, involving different Mach numbers and/or heights, then the non-linear nature of the dynamics of the propulsion system needs to be modelled, as shown in section 2.2.

This paper has described a design which provides the designer with considerable freedom to adjust to account for different constraints on the flying qualities criteria or control input activity, and produces effective, stabilizing control.

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$$A_{long8} =$$

$$\begin{bmatrix}
-4.1857x10^{-3} & -3.5030x10^{1} & 4.2686x10^{-1} & -3.2200x10^{1} & 7.9938x10^{-4} & 1.8614x10^{1} & 4.3006x10^{-1} \\
-2.3158x10^{-6} & -5.8716x10^{-2} & 1.0002 & 0 & 4.4227x10^{-7} & -3.9534x10^{-2} & 2.1974x10^{-4} \\
-9.4647x10^{-6} & 4.3430 & -5.7885x10^{-2} & 0 & 1.8076x10^{-6} & 7.2990 & -5.2846x10^{-2} \\
0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\
0 & -7.8487x10^{3} & 0 & 7.8487x10^{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.0000 \\
1.4938x10^{-3} & 5.4953x10^{1} & -4.1812x10^{-1} & 0 & -2.8529x10^{-4} & -2.6905x10^{2} & 1.1340
\end{bmatrix}$$

$$B_{long8} =$$

$$\begin{bmatrix}
-1.1359x10^{2} & -1.7159x10^{2} & 1.3329x10^{-2} \\
-1.4513x10^{-2} & 4.7726x10^{-3} & -1.6720x10^{-7} \\
-2.3511 & -8.2859x10^{-1} & 6.9090x10^{-5} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & -9.8249x10^{-1} & 3.4421x10^{-5}
\end{bmatrix}$$

Table 1. The coefficient matrices, *A* and *B*, of the state equation for the HST aircraft flying at Mach 8.0 and at a height of 85000 ft

		Eigenvalues
$\lambda_{1,2}$	=	-0.00189 $\pm$ j0.0578
$\lambda_{3,4}$	=	-0.55 $\pm$ j16.4
λ_5	=	-2.49
λ_6	=	2.33
λ_7	$\rightarrow$	0

Table 2. Eigenvalues of the HST at Mach 8.0 and at a height of 85000 ft

Desired eigenvalues	Natural frequency (rad/s)	Damping ratio	Motion represented
$\sigma_{1,2}$ = -0.9±j17.9775	18.0	0.05	Structural bending
$\sigma_{3,4}$ = -0.06±j1.91	1.91	0.0314	Short period
$\sigma_{5,6}$ = -0.04±j0.012	0.0418	0.958	Phugoid
σ_{7} = -10.0	–	–	Height

Table 3. Desired eigenvalues for HST aircraft flying at Mach 8 and at a height of 85000 ft

Desired eigenvalues	Natural frequency (rad/s)	Damping ratio	Motion represented
$\sigma_{1,2}$ = -5±j18	18.7	0.268	Structural bending
$\sigma_{3,4}$ = -40±j12	41.8	0.958	Short period
$\sigma_{5,6}$ = -0.04±j0.012	0.0418	0.958	Phugoid
σ_{7} = -10	–	–	Height

Table 4. Another set of specified closed-loop eigenvalues for HST aircraft

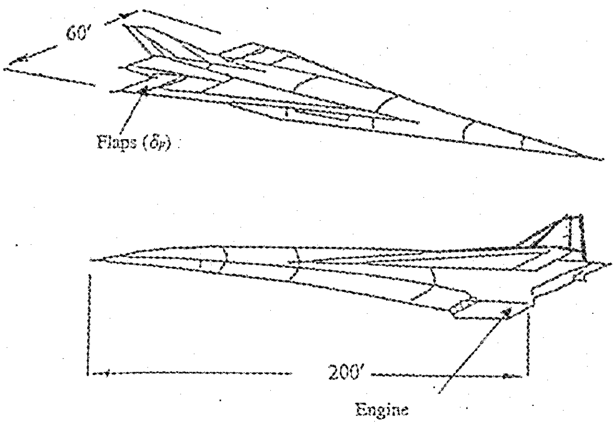

Figure 1. The Hypersonic Aircraft

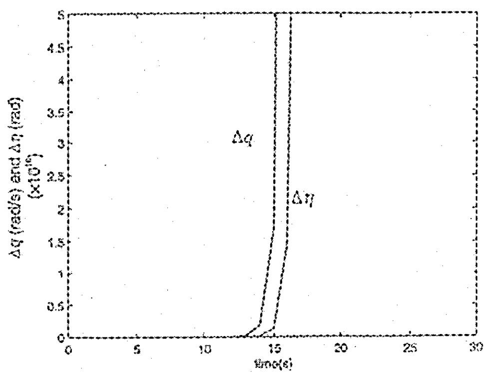

Figure 2. Dynamic Response of Hyperion without Control

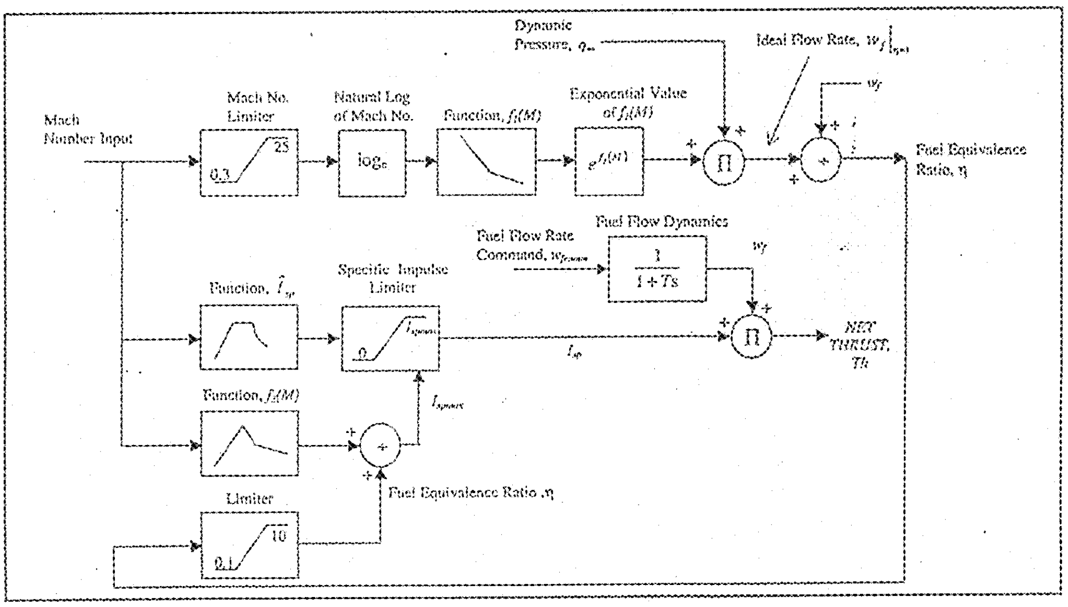

Figure 3. Block Diagram of Engine Dynamics for Hyperion

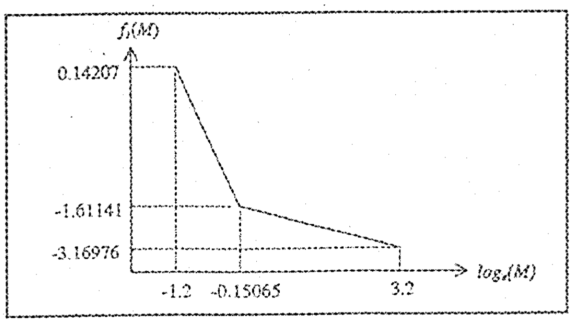

Figure 4. Function of Mach Number

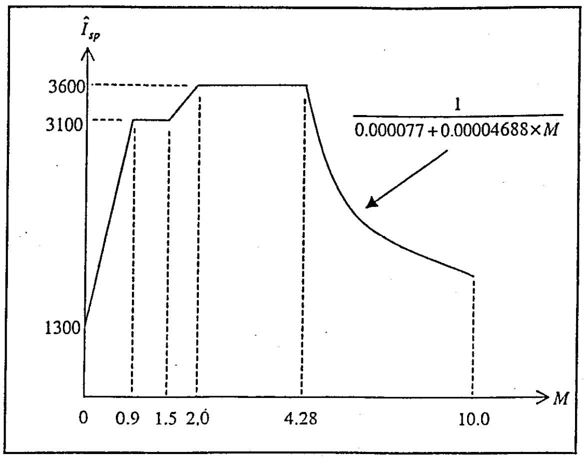

Figure 5. Specific Impulse Function when Fuel Equivalence Ratio = 1

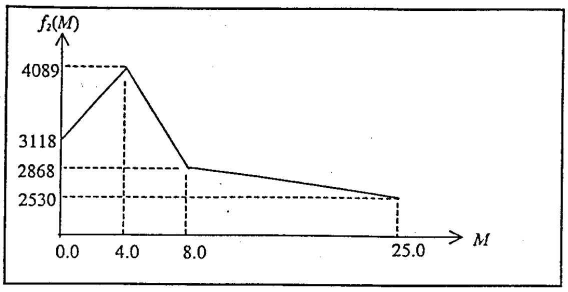

Figure 6. Specific Impulse Function Limit as a Function of Mach Number when Fuel Equivalence Ratio ≠ 1

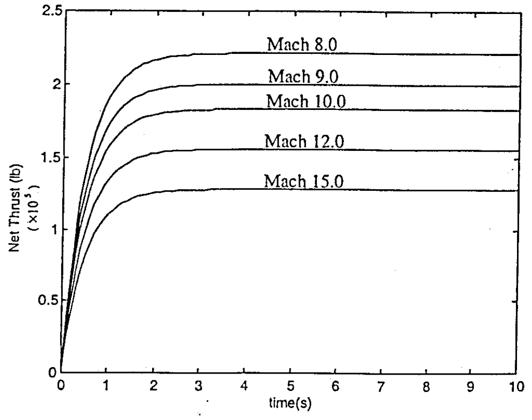

Figure 7. The Response of Thrust with Time at various Mach Numbers

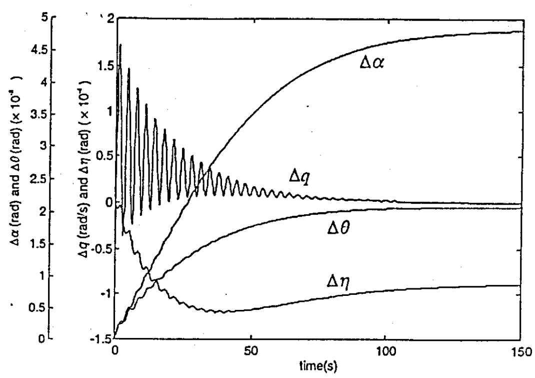

Figure 8. Hyperion Response to a Step Command for Height Change

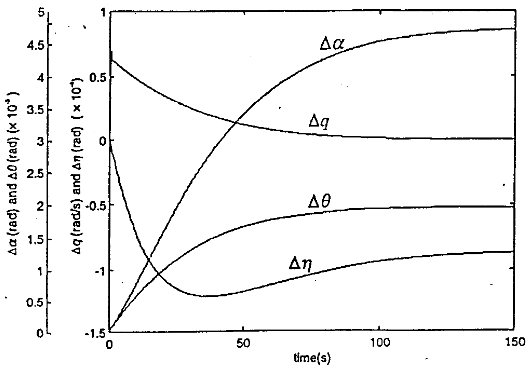

Figure 9. HST Response to a Step Command of Height Change

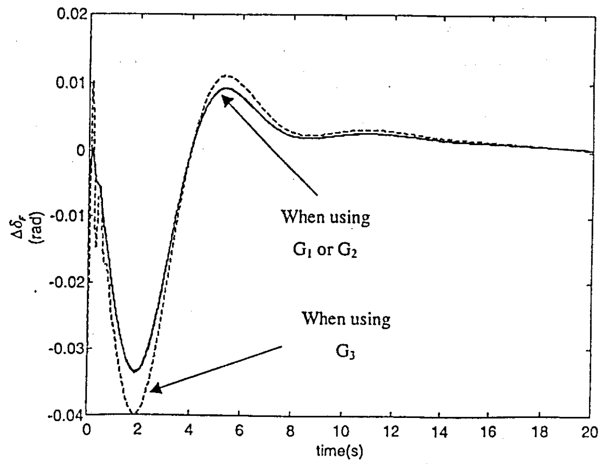

Figure 10. The Response of the Flaps in the first 20 seconds

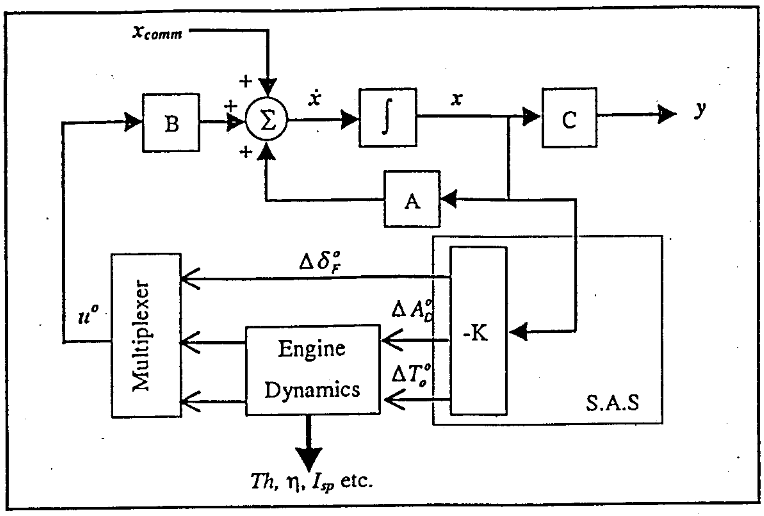

Figure 11. Block Diagram of AFCS with Engine Dynamics incorporated