FAILURE ANALYSIS OF COMPOSITE LAMINATES WITH AN OPEN HOLE UNDER BI-AXIAL COMPRESSION-TENSION LOADING

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Abstract

An approximate solution for the normal stress distribution adjacent to a circular hole in an orthotropic laminate is employed with a recently developed fracture mechanics model to predict its failure strength under compression-tensionl loading. Using the independently measured laminate parameters of unnotched strength and in-plane fracture toughness, the model successfully predicts the damage initiation, growth and final fracture of notched multidirectional laminates under various biaxiality ratios. British Crown Copyright © 2000 Published by ICAS.

1 Introduction

The uniaxial tensile or compressive behaviour of composite laminates with an open hole have been given considerable attention in the literature [1, 2]. However, very few studies [3, 4] have been reported for the evaluation of notched strength of laminates under biaxial loading. Strength prediction methods have been almost entirely limited to uniaxial loading. Daniel [5] extended the average stress failure criterion, developed by Whitney and Nuismer [6] for uniaxial tensile loading, to solve the problem of a quasi-isotropic notched laminate under equi-biaxial tensile stress; the method was complex and did not result in a simple solution suitable for use in design.

In the present paper, the Soutis-Fleck model [7] is modified to predict the notched strength of carbon fibre reinforced plastic (CFRP) laminates under tension-compression loading, where final failure is due to 0° fibre microbuckling; the 0° plies are parallel to the compression load. The model is based on the stress intensity factor, K_I, for cracks emanating symmetrically from the edge of the hole and the stress distribution adjacent to the hole. The latter can be calculated by using either analytical or computational (Finite Element) methods. The stress distribution near a circular hole, in orthotropic plates under bi-axial in-plane loading, Figure 1, has been examined analytically [8] and is based on the complex variable mapping approach [9, 10]. However, this solution is cumbersome to apply without the use of an electronic computer. Here, a simple polynomial expression for the stress distribution developed in previous work by Filiou and Soutis [8] is employed with the Soutis-Fleck fracture model [7]. It is based on the limiting characteristics of the exact solution and is an extension of the polynomial expression developed by Konish and Whitney [11] for the uniaxial loading case.

Strength data of XAS/914 carbon fibre/epoxy laminates under bi-axial compressive dominated loading are compared to experimental data.

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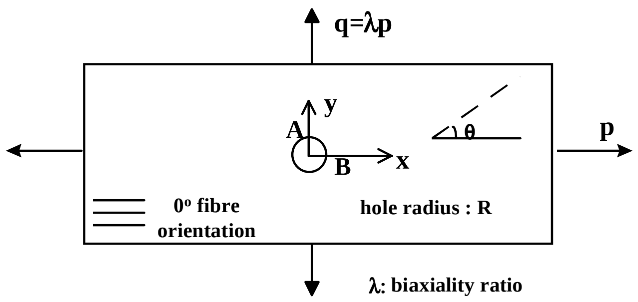

Figure 1: Orthotropic open-hole plate subjected to biaxial loading

2 Fracture toughness model [7]

The original model [7] considers a multi-directional composite laminate with an open hole subjected to uniaxial compression; it employs the stress distribution at the edge of the hole and linear elastic fracture mechanics concepts. Compressive failure of such laminates is mainly due to 0° fibre buckling (fibre kinking) from the hole edges, accompanied by fibre/matrix debonding, matrix yielding and delamination [12, 13]. The microbuckle propagates initially in a stable manner along the transverse direction under increasing load. At a critical stress level it grows rapidly and catastrophic failure occurs. On a penetrant-enhanced X-ray radiograph the microbuckled zone resembles a fatigue crack in metals. Due to its crack-like appearance the authors [7] modelled the damage zone at the edges of the hole as a through-thickness crack (line-crack) with no traction on the crack surfaces.

The model, applied to biaxial loading, is based on the two following criteria:

i) Stable crack growth: Prediction of stable crack growth is based on a critical value and the stress distribution adjacent to the circular hole. It is postulated that microbuckling occurs over a distance ℓ from the hole when the average stress over this distance reaches the critical stress of the unnotched laminate, σ_{un}, Fig.2.

$$\sigma_{un} = \frac{1}{\ell}\int_R^{R+\ell} \sigma_{xx}(0,y)dy \tag{1}$$

where the stress distribution $\sigma_{xx}(0,y)$ is given by [8].

$$\frac{\sigma_{xx}{}^{orth}(0,y)}{p} \cong 1 + \frac{(\lambda+1)}{2}\left(\frac{R}{y}\right)^2 + \frac{3(1-\lambda)}{2}\left(\frac{R}{y}\right)^4 - (3-\lambda)\frac{[H_A - 1]}{2}\left[5\left(\frac{R}{y}\right)^6 - 7\left(\frac{R}{y}\right)^8\right] \tag{2}$$

λ is the biaxiality ratio and $H_A = \frac{K_A{}^{orth}}{K_A{}^{iso}}$ measures the magnitude of the orthotropic effect. K_A^{orth} is the orthotropic stress concentration factor at point A of the hole boundary, Figure 1.

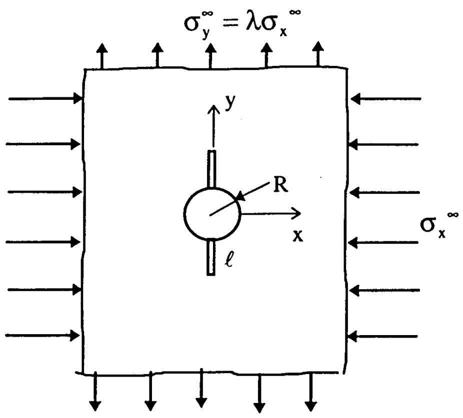

Figure 2: Schematic representation of microbuckling (line crack) growing along the y-axis under compression-tension loading

After integration and for a quasi-isotropic laminate (H_A=1), have

$$\sigma_{xx}^{\infty} = \frac{2(1-\zeta)\sigma_{un}}{(\lambda+1)(1-\zeta^2)+(1-\lambda)(1-\zeta^4)} = \\ = \sigma_{un} f(R,\ell,\lambda) \tag{3a}$$

where $\zeta = R/(R+\ell)$. Microbuckling begins when $\ell = 0$ at a stress σ_i^{∞} given by eqn.(3a).

Experimental evidence [2, 12] shows that in unnotched multidirectional laminates loaded in compression failure is always by 0° fibre microbuckling and the failure strain is almost independent of lay-up and comparable to the failure strain of unidirectional plates. Therefore, by using the laminate plate theory, the unnotched strength, σ_{un}, in eqn.(3a) could be replaced by the unidirectional strength, σ_c^0, i.e.,

$$\sigma_{xx}^{\infty} = \sigma_c^0 g(R,\ell,E,\lambda) \tag{3b}$$

where g is a function of the hole radius R, the crack length ℓ, the laminate and 0° lamina stiffness properties (represented by the variable E) and the biaxiality ratio λ.

ii) Unstable crack growth: The microbuckle at the edge of the hole is assumed to behave as a crack of the same length, with no traction on the crack surfaces. Then the stress intensity factor at the tip of the crack of length ℓ from the hole edge, Fig.3, is expressed as

$$K_I = \sigma_{xx}^{\infty}\sqrt{\pi(R+\ell)}\,Y(R,\ell,\lambda) \tag{4}$$

The length ℓ is small but finite; R is the hole radius and σ_{xx}^{∞} is the remote applied stress. The parameter Y is a correction factor, which depends on the geometry (ℓ, R), the laminate properties (orthotropy) and the biaxiality ratio. The Y-factor can be obtained from a finite element analysis or can be derived from an analytical solution obtained by Nweman [14], for an isotropic open hole cracked plate under biaxial loading.

$$Y = \left\{ \frac{\left[(1-\lambda) f_0\left(\frac{R}{R+\ell}\right) + \lambda f_1\left(\frac{R}{R+\ell}\right)\right]}{\sqrt{1-\left(\frac{R}{R+\ell}\right)}} \right\} \tag{5}$$

where

$$f_0 = 1 + 0.35\left(\frac{R}{R+\ell}\right) + 1.425\left(\frac{R}{R+\ell}\right)^2 - \\ -1.578\left(\frac{R}{R+\ell}\right)^3 + 2.156\left(\frac{R}{R+\ell}\right)^4 \tag{6}$$

and

$$f_1 = 1 + 0.4577\left(\frac{R}{R+\ell}\right) + 0.7518\left(\frac{R}{R+\ell}\right)^2 - \\ -0.8175\left(\frac{R}{R+\ell}\right)^3 + 0.8429\left(\frac{R}{R+\ell}\right)^4 \tag{7}$$

It is noted that even though Newman [14] derived these expressions for the tension-tension loading case, the solution is still applicable to the compression-tension case, since they are based on geometric parameters only.

The model then assumes that unstable crack growth occurs when the stress intensity factor at the crack tip is equal to the laminate in-plane fracture toughness K_{IC}. The remote stress is then expressed as

$$\sigma_{xx}^{\infty} = \frac{K_{IC}}{\sqrt{\pi\ell}\,Y(R,\ell,\lambda)} \tag{8}$$

Using equations (3) and (8), the remote stress σ_{xx}^{∞} is plotted as a function of the microbuckle length ℓ. Then the failure strength, σ_n^{∞}, of the notched plate is obtained from the point where the two curves intersect, Fig.3. This point also provides the critical buckling length, ℓ_{cr}.

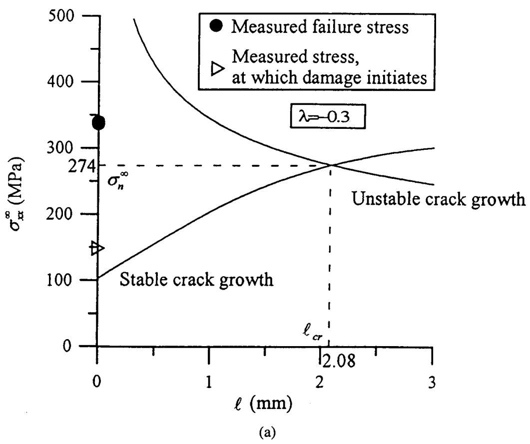

Figure 3a: strength predictions for an XAS/914C $(+45/0/-45/90)_{2s}$ plate under compression-tension loading, l=-0.3. (W=30 mm, R=3 mm and K_{IC}=40 MPa $m^{1/2}$)

The model is formulated by assuming that the plate is of infinite width. However, small-scale laboratory test results provide notched strength data on finite-width specimens, σ_n,. For proper comparisons between the experimental results and predictions, the test data should be corrected into σ_n^∞ by using an appropriate finite width correction factor [15].

The K approach is justified as the microbuckled zone resembles a crack and the damage zone associated with the microbuckle is small in extent compared with other specimen dimensions. The in-plane fracture toughness K_{IC} is considered as a laminate property and could be measured by performing a series of tests on centre-cracked compression specimens [2, 12]. Typical K_{IC} values for carbon fibre-epoxy systems are in the region of 40 to 50 MPa $m^{1/2}$, depending on lay-up. For theoretical predictions of the compressive fracture toughness and microbuckling propagation the reader should refer to some recent work by Fleck and co-workers [16, 17].

3 Applications

The fracture model was evaluated by the first author [7, 12] under uniaxial compression and concluded that the effects of hole size and lay-up upon the compressive strength can be obtained with reasonable accuracy; six T800/924C laminate stacking sequences were examined with circular holes of diameter 4 - 25 mm. The strength and critical microbuckle length predicted by the line-crack model were accurate to within 10% for the 0°-dominated laminates but less accurate for the laminate composed of mainly ±45 plies (83%). For the 45°-dominated lay-up, the model underestimated the strength by approximately 20%. The damage in this laminate is diffuse in nature, and an 'equivalent' crack representation of damage becomes inappropriate.

To illustrate the application of the current theory to notched laminates loaded in bi-axial compression-tension some results are presented in Figures 3a and 3b for an XAS/914C carbon/epoxy $(+45/0/-45/90)_{2s}$ quasi-isotropic laminate.

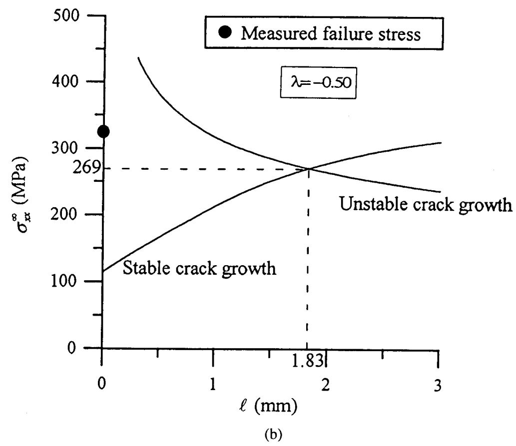

Figure 3b: strength predictions for an XAS/914C $(+45/0/-45/90)_{2s}$ plate under compression-tension loading, l=-0.3. (W=30 mm, R=3 mm and K_{IC}=40 MPa $m^{1/2}$)

The critical buckling length is about 2 mm long and the predicted strength values for $\lambda\approx$-0.3 and -0.5 are 15-20% lower than the measured data. In these calculations, the K_{IC} value was assumed equal to 40 MPa $m^{1/2}$; a higher value of K_{IC} would reduce the difference. The experimental results were obtained by testing a cruciform-type test specimen with a 30 mm square test section and a 6 mm hole diameter in a servo-hydraulic test machine. The damage initiation at the edge of the hole was observed by using penetrant-enhanced X-ray radiography. All specimens failed from the hole in a direction almost perpendicular to the compressive loading axis. Post failure examination revealed that fibre microbuckling in the 0° plies was the dominant failure mechanism. It initiated from the hole edges and propagated across the y-axis, Figure 2. Of course delamination was present but little damage occurred away from the hole supporting the theoretical approach. Fibre microbuckling (or fibre kinking) that forms near the cut-out in the 0° plies was also the critical failure mode observed by Khamseh and Waas [18] in composite plates loaded in biaxial compression.

The present model provides a useful predictive tool for design engineers, but it is recognised that further work is required to monitor the progressive damage development, evaluate in more detail the effect of biaxial loading on the fracture toughness and the accuracy of the model for other laminate stacking sequences.

4 Concluding Remarks

The complex nature of the exact solution for the stress distribution near the notch makes its application cumbersome, therefore a rather simple approximate solution for biaxial loading, has been introduced. This solution (extended isotropic solution) is based on the Konish & Whitney [11] polynomial expression developed for orthotropic plates under uniaxial loading. It yields the same stress values with the exact, at the hole boundary and both curves show the same general shape. Over a widely varying range of laminates and biaxiality ratios the agreement between the two solutions is acceptable [8]. The degree of accuracy is influenced by the lay-up and biaxiality ratio, and for lay-ups containing 0/±45 or 0/90/±45 layers the extended isotropic solution varies from the exact by less than 6%. When the approximate stress distribution is used with the modified Soutis-Fleck model the notched strength of an XAS/914C quasi-isotropic laminate, loaded in compression-tension, is predicted with reasonable accuracy; 15%-20% lower than the measured value. The input data required in the model calculations are the compressive unnotched strength of the unidirectional plies and the in-plane fracture toughness, K_{IC}, of the plate. The fracture mechanics approach is justified as the microbuckled zone resembles a crack, and damage ahead of the microbuckle is small in extent compared to other specimen dimensions. Further experimental data for different biaxiality ratios and lay-ups are needed for the model validation.

Acknowledgements

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