A NOVEL EIGENSTRUCTURE ASSIGNMENT METHOD FOR STOCHASTIC DYNAMIC SYSTEMS

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Abstract

In this paper, we present a method that has the capability of exact assignment of eigenstructure with the stochastic robustness for LTI(Linear Time-Invariant) systems. The stochastic robustness of LTI systems is determined by the probability distributions of closed-loop eigenvalues.

1 Introduction

Eigenstructure assignment provides the advantage of allowing great flexibility in placing the closed-loop system response by allowing specification of closed-loop eigenvalues and eigenvectors[1], and has been shown to be a useful tool for a flight control design[2-3]. But, the eigenstructure assignment has a disadvantage that both stability robustness and frequency domain properties are not directly considered in design procedure[4]. Especially, stochastic uncertainties in plant parameters could not be considered in the eigenstructure assignment framework.

In this paper, the stochastic parameter variations are considered as the perturbed terms in the system, and the probabilistic stability region, which is based upon a relationship between perturbed eigenvalues and nominal closed-loop eigenvalues, is presented. Monte Carlo simulations were conducted to evaluate the eigenvalues of the perturbed system, and thus they can be used to develop the concept of the probability of instability[5].

2 Probability of Instability

Consider the following linear, time-invariant system subject to parameter uncertainty

$$\dot{x}(t) = A(p)x(t) + B(p)u(t) \quad (1)$$
$$u(t) = -Kx(t) \quad (2)$$

where $x(t), u(t)$ and p are state, control and parameter vectors of dimension n, m, and r, respectively.

Since stability requires all the roots to be in the open left-half s plane, while instability results from even a single right-half s plane root, as follows:

$$\mathbf{P}(\text{instability}) = 1 - \int_{-\infty}^{0} \text{pr}(\sigma + \Delta\sigma) \text{d}\sigma, \quad (3)$$

where $\sigma + \Delta\sigma$ is an n-vector of the real parts of the systems' eigenvalues, $\text{pr}(\sigma + \Delta\sigma)$ is the joint probability density function of $\sigma + \Delta\sigma$(unknown statically), and the integral that defines the probability of stability is evaluated over the space of individual components of $\sigma + \Delta\sigma$.

Denoting the probability density function of p as $\text{pr}(p)$, (3) is evaluated ε times with each element of $p_j, j = 1$ to ε, specified by a random-number generator whose individual outputs are shaped by $\text{pr}(\sigma + \Delta\sigma)$. This Monte Carlo evaluation of the probability of stability becomes increasingly precise as ε becomes large, then

$$\int_{-\infty}^{0} pr(\sigma + \Delta\sigma) d\sigma = \lim_{\varepsilon \to \infty} \frac{N(\max(\sigma + \Delta\sigma) \leq 0)}{\varepsilon}, \quad (4)$$

where, $N(\cdot)$ is the number of cases for which all elements of $\sigma + \Delta\sigma$ are less than or equal to zero, that is, for which $\sigma_{max} \leq 0$, where σ_{max} is the maximum real eigenvalue component in $(\sigma + \Delta\sigma)$.

To reduce the instability, the controller should be designed to minimize $\mathbf{P}$. Since eigenstructure assignment can place eigenvalues arbitrary and exactly at desired positions, the design specifications of proposed controller can be determined to render $\mathbf{P} = 0$.

3 Eigenstructure Assignment Control

3.1 The Probabilistic Region

In order to analyze the stability of LTI Systems with stochastic parameter variations, we separate the given system (1) into the nominal terms and the perturbed terms as follows:

$$\dot{x} = (A + \Delta A(p))x(t) + (B + \Delta B(p))u(t), \quad (5)$$

where $A = \mathrm{E}[A(p)], B = \mathrm{E}[B(p)]$. $\mathrm{E}[\cdot]$ is the expected value of variations for each matrix element '$\cdot$'.

A relationship between the perturbed eigenvalues and the nominal closed-loop eigenvalues is as follows:

$$\begin{aligned} \left|\hat{\lambda} - \lambda_i\right| &\leq \|\Phi\| \left\|\Phi^{-1}\right\| \|E(p)\| \\ &= k(\Phi) \|E(p)\|. \end{aligned} \quad (6)$$

Let λ_i be the center of the disk in the s plane, the perturbed eigenvalues $\hat{\lambda}$ of the closed-loop system $A_c + E(p)$ are contained in the disk (6). By the viewpoint of the probabilistic stability region, we can show that the radius $K(\Phi) \|E(p)\|$ of the disk can be increased by stochastic uncertainties $\|E(p)\|$.

When the closed-loop system is unstable for stochastic uncertainties, we can move the probabilistic stability region into the left-half s plane using the eigenstructure assignment, and design the system more robust against stochastic uncertainties.

3.2 Desired Eigenvalues Update

If the right eigenvectors of the closed-loop system are achieved by the procedure given in section 3.1, the probabilistic stability region is guaranteed to be the minimum. But, either the closed-loop eigenvalues are quite near to the complex axis or the probabilistic stability region contains the complex axis may cause the increase of the probability of instability. Using the relationship between the probabilistic stability region and the desired closed-loop eigenvalues, the desired closed-loop eigenvalues should be newly updated in order to stabilize the closed-loop system by using the following procedure. Define

$$\delta s = -\min \left| k(\Phi) \|E(p)\| - \mathrm{real}(\hat{\lambda}) \right|, \quad (7)$$

where $\min|\cdot|$ is the smallest absolute value of '$\cdot$', $\mathrm{real}(\cdot)$ is the real value of '$\cdot$', and δs is the update variable, that is used to determine the desired eigenvalues.

When $\mathrm{real}(\hat{\lambda})$ is greater than $k(\Phi) \|E(p)\|$, δs is not updated because the probability of instability is 0. Otherwise, when $\mathrm{real}(\hat{\lambda})$ is smaller than $k(\Phi) \|E(p)\|$, δs must be updated by

$$\lambda^{nd} = \lambda^{od} + \delta s, \quad (8)$$

where λ^{nd} and λ^{od} denote the updated eigenvalue and the previously desired eigenvalue, respectively. The updated eigenvalues make the probability of instability to be minimum, the probabilistic stability region is moved as much as δs to the left.

Conclusions

In this paper, the eigenstructure assignment for a system with stochastic parameter variations is characterized in terms of both minimizing the eigenvector sensitivity via a probabilistic stability region, and guaranteeing the stochastic robustness via an eigenvalue update algorithm.

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