ROBUST CONTROL LAW SYNTHESIS FOR HIGH MANOEUVRABLE AIRCRAFT

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Abstract

The purpose of the authors is to present a new design example for solution of the LQG controller synthesis problem for the high performance fighter aircraft. The closed loop time domain and the frequency domain behavior were tested. For the solution of the controller synthesis problem and for the control system time domain and frequency domain analysis a new computer program has been created by the authors.

1 Optimal control law synthesis using LQG design method

During solution of the LQG controller synthesis problem there is considered the disturbed state-space model of the plant as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{\Gamma}\mathbf{w},\ \ \mathbf{y} = \mathbf{C}\mathbf{x} + \boldsymbol{\upsilon} \tag{1}$$

The synthesis of the LQG controller can be achieved using the so-called separation principle. The derived control law will minimize the following average 'cost'

$$J = \lim_{T\to\infty} E\left\{\int_0^T \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\,\mathbf{u}\right)dt\right\} \to Min \tag{2}$$

The Kalman filter state equation can be derived as given below

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}\left(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}\right) \tag{3}$$

The static gain of the optimal state observer **L** can be found by equation given below

$$\mathbf{L} = \mathbf{\Sigma}\mathbf{C}^T\mathbf{R}_o^{-1} \tag{4}$$

In eq (4) **L** is the Kalman-filter static gain, $\mathbf{\Sigma}$ is a positive definite cost matrix and, $\mathbf{R_o}, \mathbf{Q_o}$ is the set of weighting matrices of the state and the input vectors, respectively. The cost matrix $\mathbf{\Sigma}$ can be derived solving the following equation:

$$\mathbf{A\Sigma} + \mathbf{\Sigma A}^T - \mathbf{\Sigma C}^T\mathbf{R}_o^{-1}\mathbf{C\Sigma} + \mathbf{\Gamma Q}_o\mathbf{\Gamma}^T = 0 \tag{5}$$

During solution of the LQG controller synthesis problem constant unity weighting matrices $\mathbf{Q}_0$ and **R**o are used for solution of the LQE design, weighting matrix **Q** is used for LQR design stage as tuning parameters. These matrices were found fully heuristically and they are listed below:

$$\mathbf{Q} = \begin{bmatrix} 10^4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 10 & 0 \\ 0 & 10^3 \end{bmatrix} \tag{6}$$

$$\mathbf{Q}_o = \mathbf{I}_3, \mathbf{R}_o = \mathbf{I}_3, \tag{7}$$

where: $\mathbf{I}_3$ is (3×3) identity matrix.

2 A numerical example

Let us consider the mathematical model of the lateral motion of the high performance fighter aircraft during approach as it given in textbook of D. McLean. The state equation of the perturbed lateral motion of the fighter aircraft is as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot\beta \\ \dot\omega_X \\ \dot\omega_Y \end{bmatrix} = \begin{bmatrix} -0.16 & 0.174 & -1 \\ -12.7 & -2.13 & 2.19 \\ 1.44 & 0.065 & -0.56 \end{bmatrix} \begin{bmatrix} \beta \\ \omega_X \\ \omega_Y \end{bmatrix} + \begin{bmatrix} -0.0016 & 0.033 \\ 4.38 & 1.1 \\ -0.21 & -1.2 \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_R \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \beta_w \tag{8}$$

In eq (8) β is the sideslip angle, ω_x is the roll rate, ω_y is the yaw rate, β_w is the angle of the crosswind disturbing motion of the aircraft, δ_A is angular deflection of the ailerons, δ_R is angular deflection of the rudder.

Find the dynamic controller for the aircraft, which will minimize average cost function defined by eq (2) and the closed loop dynamic performances are defined by their damping ratios grater than 0.7.

The full state-feedback gain matrix was found for **Q** and **R** as follows

$$K = \begin{bmatrix} 26{,}3669 & 0{,}9293 & -6{,}4573 \\ 0{,}4887 & 0{,}0148 & -0{,}1549 \end{bmatrix} \qquad (9)$$

For weighting matrices (7) the observer static matrix was found to be:

$$L = \begin{bmatrix} 0.3152 & -0.6423 & 0.0787 \\ -0.6423 & 2.2222 & -0.3187 \\ 0.0787 & -0.3187 & 0.0654 \end{bmatrix} \qquad (10)$$

After forming dynamic controller with 'reg.m' built-in file of the Control System Toolbox of MATLAB® computer package the time domain behavior of the closed loop system was tested. Results of the computer simulation can be seen in Figures 1, 2 and 3.

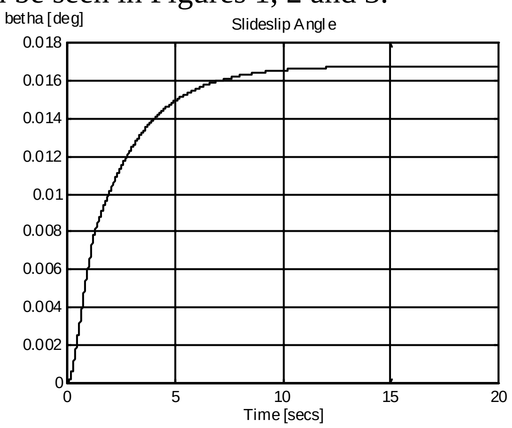

Figure 1 Sideslip angle time domain behavior

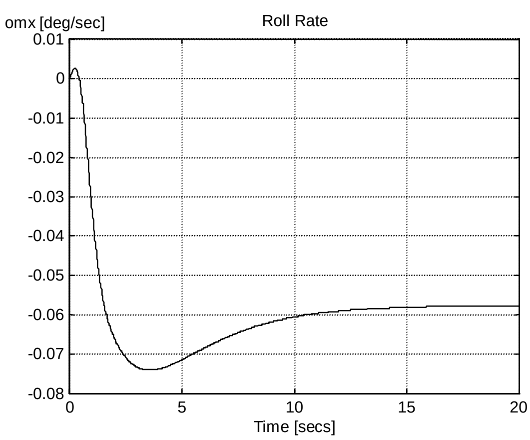

Figure 2 Roll rate time domain behavior

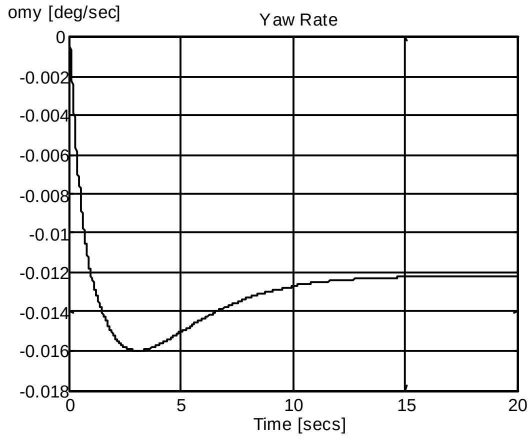

Figure 3 Yaw rate time domain behavior

The closed loop system poles were determined. They are as follows:

$$\lambda_{1,2} = -3{,}92 \pm 3{,}9575\, i;$$
$$\lambda_{3,4} = -2{,}5161 \pm 2{,}6439\, i \qquad (11)$$
$$\lambda_5 = -0{,}6021; \lambda_6 = -0{,}4207$$

3 Closing remarks

The purpose of this paper was to present a new design example for solution of the LQG controller synthesis problem. Main equations and block diagrams for LQG problem had been summarized. For the controller synthesis problem the stabilization of the high performance aircraft during approach has been chosen. The Kalman-filter static gain and the feedback gain matrix had been determined for particular case of heuristically set weighting matrices **Q**, **R**, $\mathbf{Q}_o$ and $\mathbf{R}_o$.

The closed loop system was tested in the time domain. The presented by the authors weighting matrix selection provided for the closed loop system acceptable dynamic performances. For the design and analysis purposes the necessary computer program has been created.

References

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