NEW SOLUTIONS OF THE OPTIMAL INJECTION PROBLEM BASED ON COMPLEX INVESTIGATION OF DYNAMICS AND AERODYNAMICS

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Abstract

The behavior of optimal control laws and injection trajectories of aerospace vehicles is investigated on the basis of the Pontryagin maximum principle at a variation of the aerodynamic shape. The basic types of aerodynamic layouts: conical, cylindrical, and winged, are considered. The geometrical parameters determining the cross section profile and the outboard wing area are varied. The numerical solution of the problem is accomplished using the computer program package ASTER of the thorough optimization of branched trajectories.

The existence of diverse types of optimal control law structures, including those qualitatively different from typical ones for the current space transportation systems is demonstrated. The bifurcation behavior of relations of the optimal solutions to geometrical parameters of the layout is noted. It is shown that sensitivity coefficients for the maximum injected mass to variations of parameters can differ both in order of magnitude and in sign from the traditionally used ones, that may exert an effect on selection of optimal aerospace vehicle layout.

Nomenclature

Symbols:

C_D	drag coefficient
C_{D0}	zero-lift drag coefficient
C_L	lift coefficient
C_L^a	$= \partial C_L / \partial \alpha$
F_0	reference cross section area
F_w	outboard wing area
h	altitude
L/D	lift-to-drag ratio
m	vehicle mass
M	Mach number
q	dynamic pressure
T	thrust value
$\mathbf{T}$	thrust vector
t	time
V	velocity value
$\mathbf{V}$	velocity vector
α	angle of attack

Subscripts:

$( )_f$	at the final point
$( )_i$	at the initial point
$( )_{max}$	maximum value
$( )_{min}$	minimum value
$( )_{opt}$	optimal value

Superscripts:

$( )^*$	at the bifurcation point

1 Introduction

Optimal ascent control laws of advanced aerospace vehicles (ASV), especially with high aerodynamic lift capabilities, can have a new structure as compared with ones for current space transportation systems (STS) of a ballistic type. The use of the new optimal trajectories and control laws, in turn, may call for changes of ASV layout.

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As stated in previous publications [1, 2], changes in aerodynamic characteristics of ASVs can be followed by a qualitative restructuring (including that of bifurcation type) of optimal control laws. A change in the optimal control structure causes, in turn, a jump in sensitivity coefficients for the maximum injected mass to ASV parameter variations. Maximum lift-to-drag ratio, $(L/D)_{\max}$ is a main characteristic governing the structure of optimal injection control [1, 2]. At the same time, the maximum injected mass also depends on other characteristics of the aerodynamic configuration. In view of the fact that a change in the ASV aerodynamic configuration leads, as a rule, to simultaneous changes in all characteristics for all flight regimes, the necessity of ASV modifications can be assessed by analyzing the maximum injected mass as a direct function of geometric parameters of the configuration.

In the present paper, the effect of ASV aerodynamic configurations on the maximum mass injected into low Earth orbit is investigated. Cross section areas of conical and cylindrical ASV are varied, as well as the area of outboard wing of cylindrical ASV. The trajectory optimization is carried out on the basis of the Pontryagin maximum principle [3]. The strict indirect optimization method offers unique capabilities of investigating the ASV weight efficiency not confining the consideration to the framework of traditional control law structure. Parametric investigations of optimal injection trajectories for ASVs of diverse configurations are carried out using the ASTER package [4]. Owing to developed methods of numerical solution of nonlinear boundary-value problems and a convenient interface, the solution of optimization problems based on the Pontryagin maximum principle with the use of this package becomes an almost routine procedure.

2 Aerospace vehicle trajectory optimization

Motion of ASV mass center is considered in the coordinate system fixed to the start point:

$$\frac{d\mathbf{x}}{dt}=\mathbf{f}(\mathbf{x},\mathbf{u},t)\ ,\quad \mathbf{f}=\left\{\begin{matrix}\mathbf{V}\\ \left(\dfrac{\mathbf{T}+\mathbf{A}}{m}\right)+\mathbf{g}+\mathbf{\Omega}\\ -\mu\end{matrix}\right\},\quad (1)$$

where $\mathbf{x}=\{\mathbf{r},\ \mathbf{V},\ m\}^{\mathrm{T}}$ is the state vector, $\mathbf{r}$ is the radius-vector, $\mathbf{A}$ is the vector of aerodynamic forces, $\mathbf{g}$ is the gravitational acceleration vector, $\mathbf{\Omega}$ is the acceleration vector due to coordinate system noninertiality, μ is the mass flow rate.

The vector of aerodynamic forces can be written in the form [2]:

$$\begin{aligned}\mathbf{A}=\frac{1}{2}\rho V^{2}F_{0}\Big(C_{L}^{\alpha}\mathbf{e}_{\tau}-\big(D_{0}+\\ +\left(C_{L}^{\alpha}+D_{\alpha}\right)(\mathbf{e}_{\tau},\mathbf{e}_{\mathrm{v}})\big)\,\mathbf{e}_{\mathrm{v}}\Big),\end{aligned}\quad (2)$$

where $\mathbf{e}_{\tau}$ is the unit vector directed along the vehicle's longitudinal axis, $\mathbf{e}_{\mathrm{v}}$ is the unit vector in the direction of the velocity, ρ is the atmospheric density.

The following form for aerodynamic coefficients is used [2]:

$$C_{L}=C_{L}^{\alpha}\sin\alpha,\quad C_{D}=D_{0}+D_{\alpha}\cos\alpha\,,\quad (3)$$

that is in accordance with the square aerodynamic polar at a small angle of attack:

$$\begin{aligned}&C_{L}\cong C_{L}^{\alpha}\alpha,\quad C_{D}\cong C_{D0}+k\alpha^{2},\\ &D_{0}=C_{D0}+2k,\quad D_{\alpha}=-2k.\end{aligned}\quad (4)$$

The coefficients C_{L}^{a}, C_{D0} and k depend on flight regimes.

The engine thrust is assumed to be directed along the longitudinal axis: $\mathbf{T}=T\mathbf{e}_{\tau}$, $T_{\min}\le T \pounds T_{\max}$.

The ASV initial position at the moment t_i and velocity vector absolute value are fixed, the velocity vector orientation can be free:

$$\mathbf{r}(t_i)=\mathbf{r}_i,\ \mathrm{v}(t_i)=\mathrm{v}_i,\ m(t_i)=m_i.\quad (5)$$

The task is to find control

$$\mathbf{u}=\{\mathbf{e}_{\tau},\eta\},\quad (6)$$

where $\eta = T/T_{\max}$, to provide transition of ASV from the initial point to a specified Earth orbit with a minimum propellant consumption that corresponds to maximization of the final vehicle mass:

$$F \equiv m_f \Rightarrow \max_{\{e_\tau,\eta\}} . \qquad (7)$$

In solving problems with the use of the Pontryagin maximum principle, optimal control at every point is found from the condition [4]:

$$\{\mathbf{e}_\tau,\eta\}_{opt} = \operatorname*{argmax}_{\{\mathbf{e}_\tau,\eta\}} \mathcal{H} , \qquad (8)$$

where

$$\mathcal{H} = \mathbf{\Psi}^T \mathbf{f} \qquad (9)$$

is the Hamiltonian of the system (1), $\mathbf{\Psi}$ is the adjoint vector that satisfies the equation [4]:

$$\dot{\mathbf{\Psi}} = -\left(\frac{\partial \mathcal{H}}{\partial \mathbf{x}}\right)^T \qquad (10)$$

and the transversality conditions. Thus, the reference optimization problem reduces to a multipoint boundary-value problem for sets of equations (1), (10).

Numerical solution is found using the ASTER package [4], in which the practically routine procedure of solving respective multipoint boundary-value problems of this class is realized.

Owing to the application of the Pontryagin maximum principle, simultaneously with calculation of an optimal trajectory the sensitivity coefficients $\frac{\partial \Phi}{\partial p}$ of a functional Φ to variation of a parameter p of the problem can be obtained [5] with a high accuracy without noticeable additional computing costs simply by integrating:

$$\frac{\partial \Phi}{\partial p} = \int_{t_i}^{t_f} \frac{\partial \mathcal{H}(\mathbf{x}_{opt}, \mathbf{u}_{opt}, p)}{\partial p} dt , \qquad (11)$$

where $\mathbf{x}_{opt}$ and $\mathbf{u}_{opt}$ are the state vector and the control vector, respectively, on the nominal optimal trajectory.

3 Aerodynamic shapes of an aerospace vehicle

Optimal injection trajectories are investigated for three types of aerodynamic configurations covering most-used vehicle configurations:

– conical and cylindrical with an elliptic cross section,
– cylindrical with a delta wing.

For all types of ASVs, the influence of their geometric configuration modifications, ensuring enhanced lifting capabilities, on the injected mass and optimal injection trajectories is analyzed. Given below is a detailed description of ASV geometric parameters.

Aerodynamic configurations of the first type are blunt cones with elliptic cross sections. The base of this family is a circular cone with an apex angle of 11.42° and bluntness radius/base radius ratio of 0.05.

In order to enhance the lifting properties, the cone contours are modified by passing over the whole vehicle length from circular cross sections to elliptic ones with a specified ratio of width a to height b, the section area remaining the same. The parameter a/b, hereinafter referred to as the "contraction parameter", is varied from 1 to 3 with an interval of 0.25.

The aerodynamic configurations of the second type are blunt cylinders with elliptic cross sections. The base of this family is a circular cylinder with a blunt conical nose. The geometric dimensions of the cylinders are derived from the condition that its volume is equal to the volume of the above-considered vehicle of conical type. The ratio of the length of the circular cylinder to its diameter is taken to be 12, conical nose apex angle is 20°, and bluntness radius/cylinder radius ratio of 0.1.

As in the first case, the modification of this configuration version implies transition from circular cross sections to elliptic ones over the whole vehicle length. The contraction parameter is varied from 1 (circular cylinder) to 2 with an interval of 0.25, the cross section area remaining constant.

For the third configuration type, the lifting capabilities are enhanced by installing a mid-mounted delta wing on the base model of the second type. The wing leading edge sweep angle is 60°. The relative outboard wing area $\bar{F}_w = F_w/F_0$ is 0, 10%, 20%, 50%, 100%, 150%, and 200% of the mid-section area F_0.

The aerodynamic characteristics are determined relying on the available experimental data and numerical calculations by the technique described in [6]. Characteristics at intermediate points are obtained by linear approximation in parameters *a/b* and `F_w.

In optimizing trajectories by the Pontryagin maximum principle in accordance with the model of aerodynamic forces (2), (3), aerodynamic coefficients D_0 , D_α , C_L^α with respect to the Mach number: $\frac{\partial D_0}{\partial M}, \frac{\partial D_\alpha}{\partial M}, \frac{\partial C_L^\alpha}{\partial M}$, must be found at every trajectory point. To simplify the procedure of numerical solution of the reference problem, these derivatives must be continuous.

The coefficients D_0 and D are calculated by (4). The coefficients C_L^a, C_{D0} and k versus Mach number are specified in the form of third-order splines. The splines are created by processing the reference dependencies of aerodynamic characteristics C_D and C_L on angle of attack and Mach number. At every Mach number corresponding to a spline node, the coefficients of polar (3) approximating the dependencies $C_D(\alpha)$ and $C_L(\alpha)$ are calculated. Coincidence both of the zero lift drag coefficient C_{D0} and the maximum lift-to-drag ratio $(L/D)_{\max}$, defined for the polar by

$$(L/D)_{\max} = \frac{C_L^\alpha}{\sqrt{C_{D0}\left(C_{D0}+4k\right)}} ,$$

is required for the approximating polar and reference characteristics. The induced drag factor k is obtained from the condition of minimization of root-mean-square C_D deviations from the reference data.

Fig. 1 exemplifies comparison of approximations of $C_D(\alpha)$ (solid lines) with the reference data (markers) for two values of Mach Number and *a/b*.

After calculation of the coefficients C_L^a, C_{D0} and k for node points in terms of the Mach number, the third-order spline coefficients are calculated from the condition of minimization of root-mean-square $C_L^a(M)$, $C_{D0}(M)$ and $k(M)$ deviati-

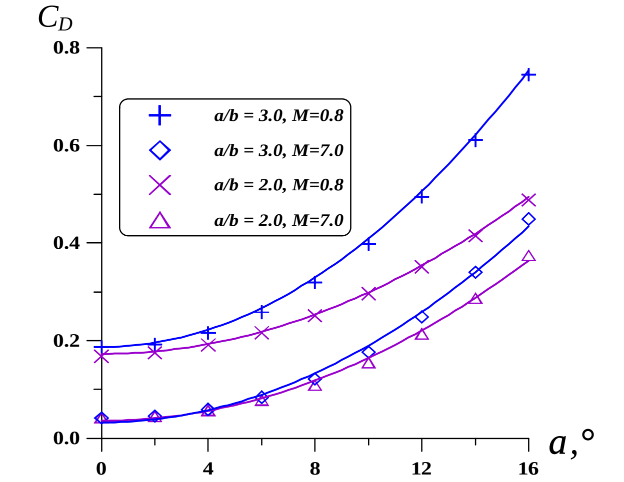

Fig. 1 Approximation of reference values of C_D as a function of angle of attack α in the form of (3).

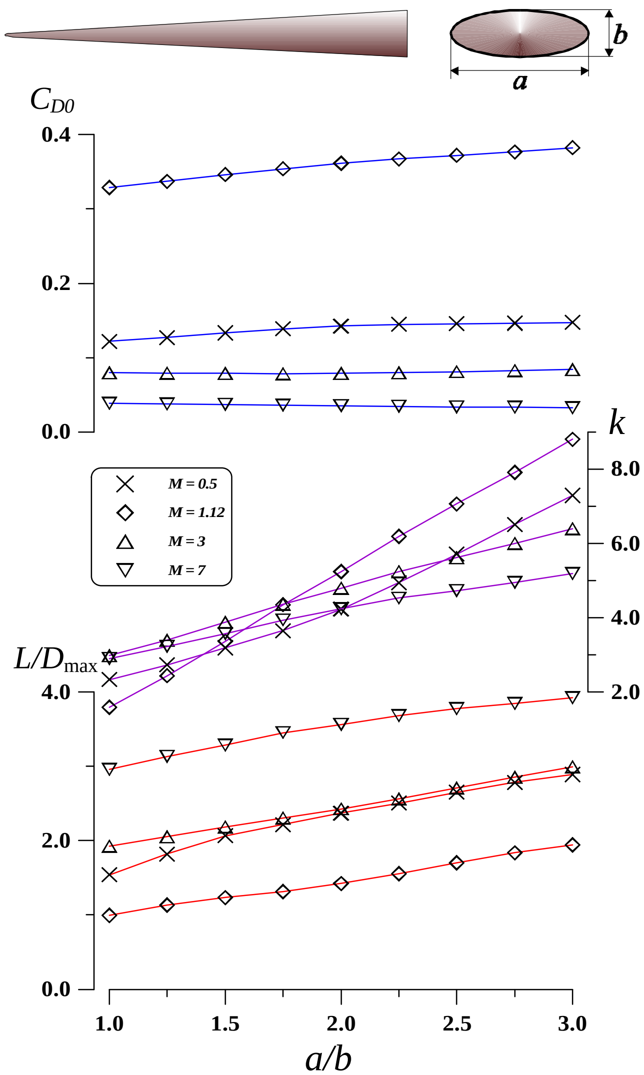

Fig. 2 The aerodynamic characteristics of the conical ASV.

ons from piece-linear functions passing through reference points.

The above-considered procedure of forming the aerodynamic force coefficients as a function of angles of attack and Mach numbers is performed automatically by the ASTER package.

Figs. 2-4 show the aerodynamic characteristics versus the Mach number for different parameter values.

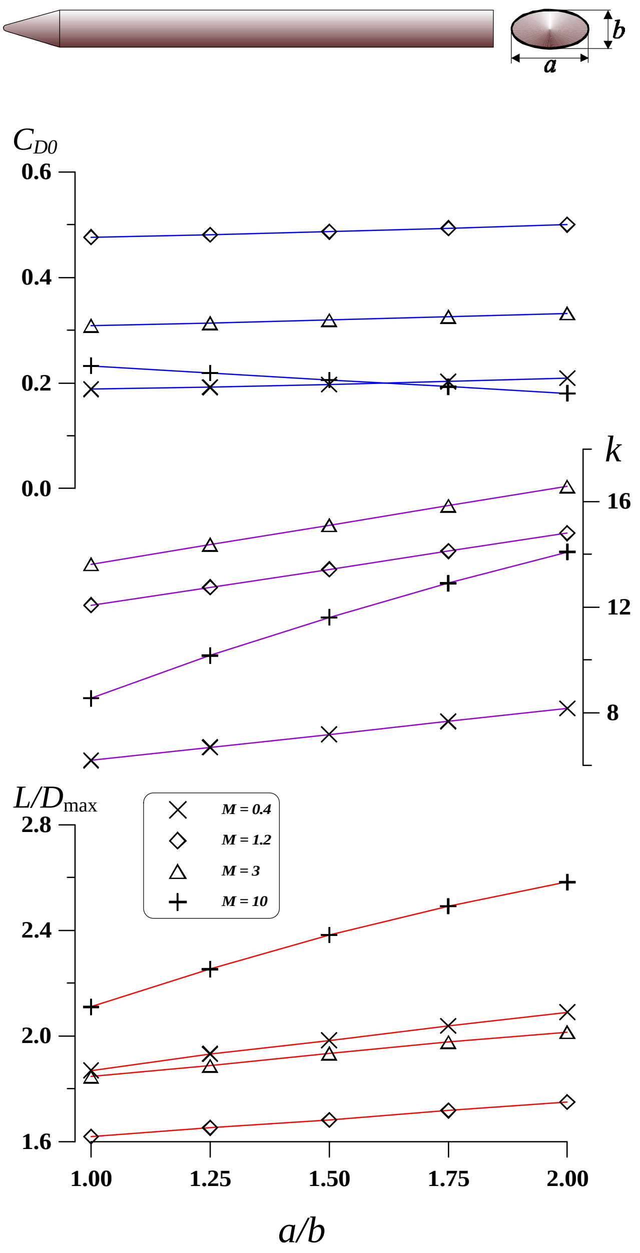

Fig. 3 The aerodynamic characteristics of the cylindrical ASV.

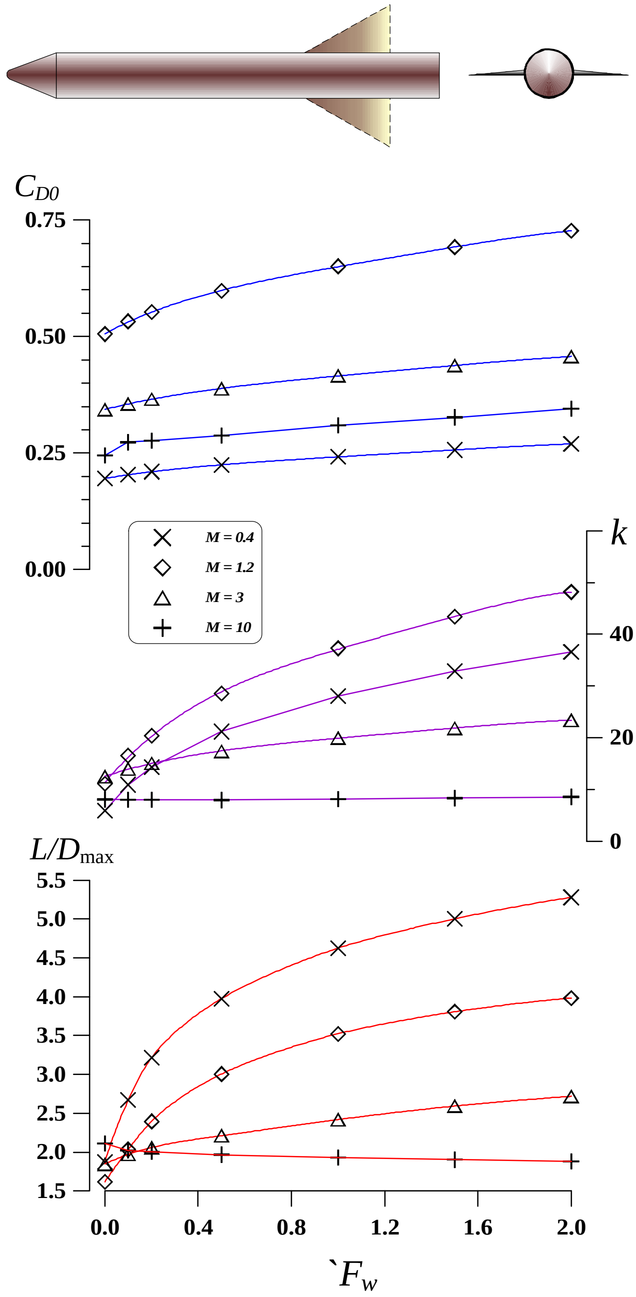

Fig. 4 The aerodynamic characteristics of the cylindrical ASV with delta wing.

4 Influence of geometric configuration parameters on the injected mass

It is established in [2] that the maximum lift-to-drag ratio, $(L/D)_{max}$, is a main characteristic governing the optimal injection control structure. If the value of ASV $(L/D)_{max}$ does not exceed some critical value $(L/D)^*_{max}$ (at subsonic speed) the lifting capabilities of the vehicle are almost not in use on optimal injection trajectories. According to classification given in [2], such trajectories and relevant optimal control laws are assigned to type *B* (Ballistic). The structure of *B*-type optimal control laws are qualitatively consistent with control laws traditionally used at the present time for STS (thereinafter they will be referred to as traditional ones). The traditional control laws are characterized by almost zero angles of attack on the atmospheric trajectory segment (gravitational turn) and quasilinear laws in pitch on subsequent flight segments. For ASVs, which require the *B*-type control laws for optimal injection, increase in the injected mass with variations of the geometric configuration is basically achieved only when the aerodynamic drag reduces.

If the value of $(L/D)_{max}$ exceeds $(L/D)^*_{max}$, qualitatively other trajectories and optimal control laws become optimal, namely those of type *A* (Aerodynamic) [2]. On extremals of this type, the lifting capabilities of the vehicle are used to a greater extent, and the optimal control laws on atmospheric trajectory segment have a pronounced oscillatory nature. For these vehicles, increase in the injected mass can be achieved by enhanced $(L/D)_{max}$ even though the aerodynamic drag does not decrease.

Although $(L/D)_{max}$ is one of the main parameters of the aerodynamic configuration that govern the structure of optimal control laws and trajectories, it follows from [2] that variations of other parameters, for example, C_{D0}, can also make an effect on the functional and the optimal control law. In view of the fact that a change in the aerodynamic configuration influences, as a rule, the whole set of aerodynamic characteristics, the analysis of the dependency of optimal solutions on geometric parameters of the ASV configuration is of practical interest. In the present paper, the effect of the ASV cross section configuration and the outboard wing area F_w is investigated.

The effect of geometric parameters of the ASV configuration on optimal control laws and injection trajectories is analyzed for the following nominal conditions:

– initial speed V_i = 50m/s, altitude h_i = 200m,
– final orbit is circular with an altitude of h_{orb} = 275 km,
– initial thrust-to-weight ratio $(T/mg)_i$ = 1.1,
– maximum relative mass flow rate $\mu / m_i = 3.83{*}10^{-3}$ s^{-1},
– initial specific mid-section load $m_i /F_0 = 6{\cdot}10^3$ kg/m^2 (for the cylindrical shapes) and $1.6{\cdot}10^3$ kg/m^2 (for conical shapes)

It is obvious from Figs 2-4 that changes in the cross section geometry and outboard wing area give rise, as a rule, to $(L/D)_{max}$ and C_{D0}. Thus, a change in the injected mass with variations of the aerodynamic configuration depends on trade-off between two opposite trends: increase in aerodynamic drag and relative growth of ASV lifting capabilities. The investigations show that the effect of variations of the ASV aerodynamic configuration is significantly different for optimal trajectories of types *A* and *B* type.

Fig. 5 presents the relative injected mass $\grave{}m_f = m_f(a/b)/m_f(1)$ for different values of the contraction parameter of the conical ASV obtained using optimal (upper curve) and traditional (lower curve) control laws. It is seen that the parametric analysis based on approximate (traditional) control laws qualitatively distort the objective parameter dependency of the ASV weight efficiency.

To the left of the bifurcation point, increased contraction parameter results in reduced injected mass. The optimal trajectories for these configurations correspond to conventional STS injection schemes [7, 8] (type *B* according to the classification [2]) and almost do not use the lifting capabilities. Therefore, increase in the contraction parameter followed by increased aerodynamic drag results in reduced injected mass.

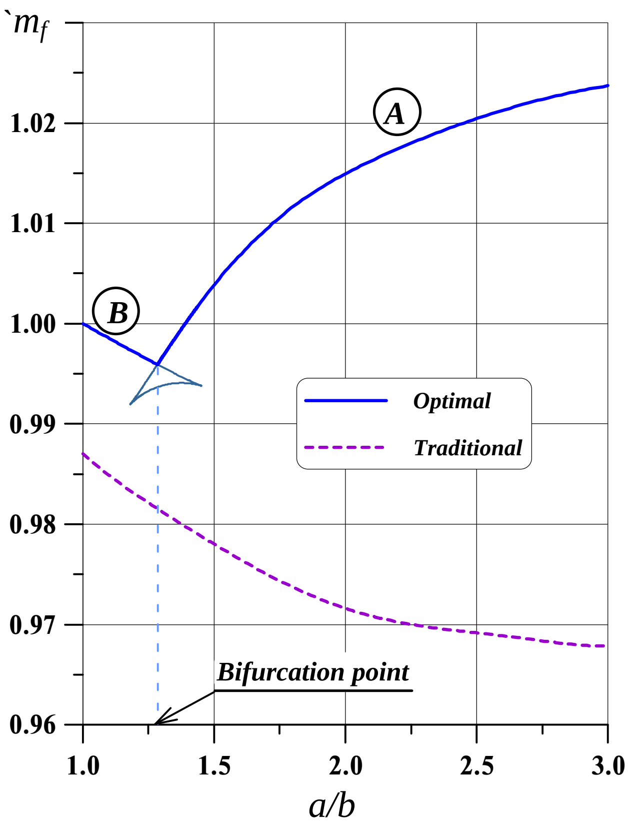

Fig. 5 Relative injected mass versus contraction parameter for a conical ASV. The values marked out by a bold line are relevant to the global optimum.

Increase of the contraction parameter after excess of the bifurcation value $a/b \approx 1.3$ leads to growth of the injected mass. As noted above, a qualitative change in the maximum injected mass versus the contraction parameter is caused by a change in the optimal control law structure. To the right of the bifurcation point, the injected mass is the maximum when the *À*-type control law is used.

The Pontryagin maximum principle is based on necessary conditions of optimality, therefore the ASTER complex involves special procedures of selecting local extremals. In Fig. 5, the dependencies of the functional on the contraction parameter are shown by thin lines in the vicinity of the bifurcation point that correspond to local extremals existing simultaneously with global ones.

Comparison of the functional values on different-type trajectories shows that when optimal control laws are used the gain in the injected mass on *B*-type trajectories is 1.3% as compared with traditional control laws. When the lifting capabilities are used (on the optimal trajectories of type *A*), this gain already at $a/b \approx 2.5$ is four times greater.

Note especially the fundamental difference in sensitivity coefficients ¶`m_f/¶ *(a/b)* characterizing the effect of the contraction parameter on the injected mass for optimal and traditional control laws. It is widely believed that approximate estimates of the injected mass are sufficient at the initial ASV design stages when optimal variations of the configuration parameters are often based on the analysis of only partial derivatives, i.e., sensitivity coefficients. In this case, the hypothesis of their independence from the parameter to be varied is implicitly accepted. However, it is seen in Fig. 6 that the real dependence of sensitivity coefficients on the parameter, which takes account of the optimal use of lifting capabilities of ASV, is not only inquasi-constant, as opposed to traditional control laws, but also experiences a bifurcation change. During the jump, not only the derivative magnitude changes (sometimes in orders) but also its sign (Fig. 6).

It is important to stress that in this case the bifurcation behavior is characteristic for the injected mass sensitivity coefficients not only to variations of the contraction parameter but also to almost all other ASV configuration parameters (for example, initial thrust-to-weight ratio, specific load on mid-section etc.). It follows from Fig. 6 that if traditional approximate control laws were used in determining the influence of the parameters on the functional, the derivative ¶`m_f/¶ *(a/b)* would be essentially constant.

Compare now the optimal contraction parameter values for optimal and traditional control laws. Fig. 5 shows that the approximate (traditional) approach gives the only "optimal" configuration solution: $(a/b)_{opt} = 1$, i.e., the circle is the best cross section shape of a conical ASV.

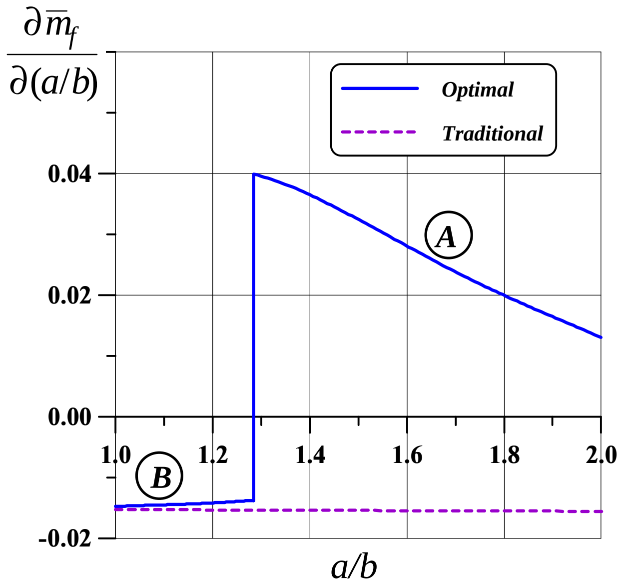

Fig. 6 Jump in sensitivity coefficient for the maximum relative injected mass to variations of contraction parameter of the conical ASV when the global extremal type is changed.

But when the strict optimization procedure taking account of structural changes in optimal control laws is used, it is obtained that the ellipse with a great contraction parameter is sufficiently better in the functional than the circle. Thus, the approximate approach to constructing the trajectory control laws for such ASVs can violate the optimal concept of the vehicle under design. Investigations of diverse aerodynamic configurations reveal that complex analysis of the influence of vehicle parameters on effectiveness of the vehicle use with due regard for significantly nonlinear dependency of optimal solutions on ASV parameters is of great importance in designing ASV. One of main conditions of the investigation result reliability is use of the regular procedure of constructing optimal trajectories with a specified accuracy. In the present paper, the trajectory optimization is carried out with the use of the ASTER package enabling the solution to be obtained by applying the Pontryagin maximum principle in automatic mode without manual selection of an initial approximation for varied parameters in the boundary-value problem.

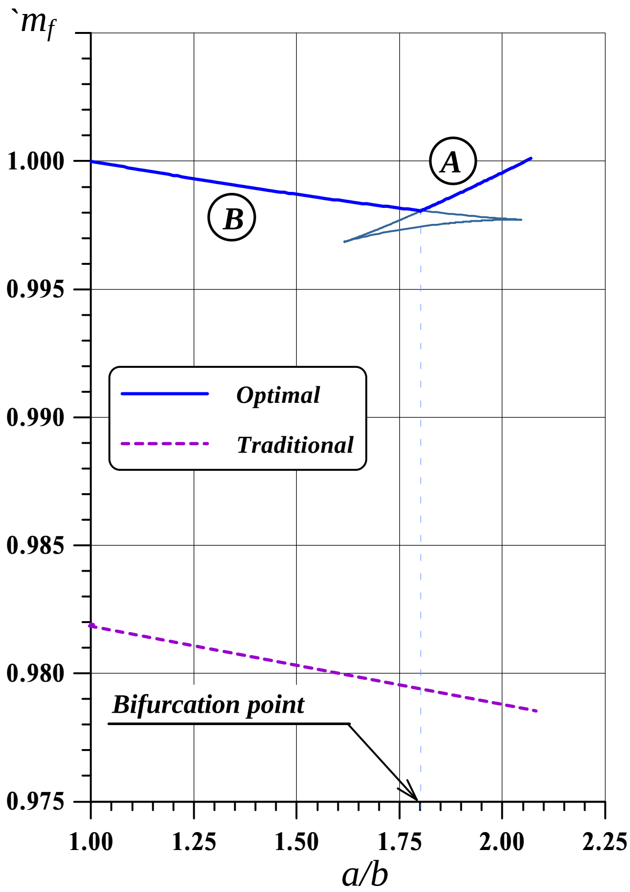

Fig. 7 Relative injected mass versus contraction parameter for a cylindrical ASV. The values relevant to the global optimum are bolded.

The dependency of the relative injected mass on the contraction parameter a/b of the cylindrical ASV cross section shown in Fig. 7 is similar to that for the conical vehicle (Fig. 5). When compared to conical vehicles, the range of changing `m_f is much less because the aerodynamic characteristics of the cylindrical ASV with variation of a/b vary to smaller extent (Fig. 3).

Changes in the relative injected mass `$m_f = m_f($`$F_w)/m_f(0)$ with variations of the relative outboard wing area $\overline{F}_w = F_w/F_0$ (Fig. 8) have the same peculiarities as with variations of the contraction parameter. There exists a bifurcation value of the parameter $\overline{F}_w^*$, which separates the optimality regions for extremals of types *A* and *B*. At $\overline{F}_w < \overline{F}_w^*$ the *B*-type extremals is globally optimal, while at $\overline{F}_w > \overline{F}_w^*$ the

A-type extremals are of this sort because they better use the lifting capabilities of space vehicles. It must be emphasized that in this case qualitative changes in the dependency of the maximum injected mass on the outboard wing area are exhibited already at very small outboard wing panels which area are only several per cent on the mid-section one.

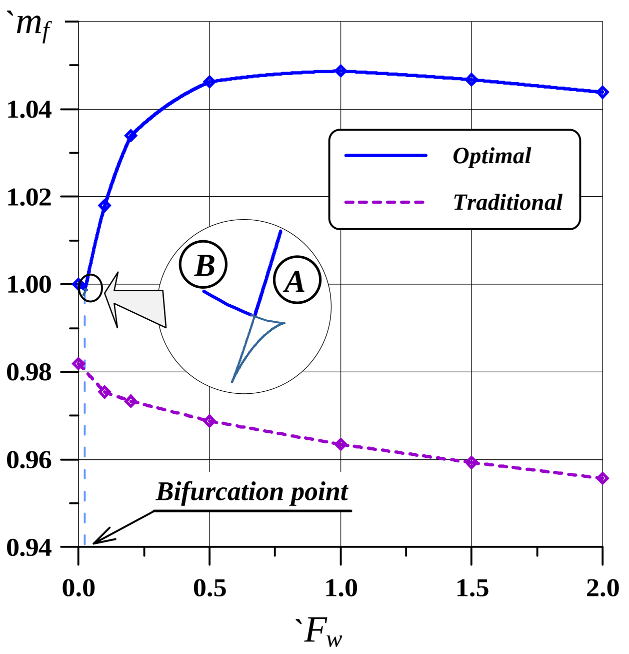

Fig. 8 Relative injected mass versus relative outboard wing area for a winged cylindrical ASV. The bold line is relevant to the global optimum.

Conclusions

The investigation of the influence of the ASV aerodynamic configuration on the maximum mass injected into low Earth orbit has shown that this dependency can be significantly nonlinear, and bifurcation of sensitivity coefficients is possible. At jump points, the sensitivity coefficients can change both in order of magnitude and in sign. The nature of such behavior of optimal solutions consists in a bifurcation change in the optimal control law structure with variations of configuration parameters. To cause the qualitative change the relatively small variations can be sufficient. For example, in the situation considered in the paper, a variation of the relative ASV outboard wing area results in a qualitative restructuring of the optimal control law and sensitivity coefficients already in installing the outboard wings with the area of only several per cent of the mid-section area. Revealing such features of fundamental importance in designing advanced ASVs becomes possible in using the strict approach to optimization of the ASV trajectory control. In the present paper, the advantages of the Pontryagin maximum principle realized in the automated package ASTER are demonstrated.

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