SCATTERING ANALYSIS FOR WRECKAGE OF IN-FLIGHT BREAKUP

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Abstract

Scattering of airframe fragments due to in-flight breakup is studied to estimate flight conditions of an aircraft at the accident. From the distributions of scattering on the ground, breakup location, wind effects, and the initial momentum are estimated by using analytical solutions. Numerical simulations with random variables are also carried out to confirm the analytical results. The results are compared with an actual accident of a small aircraft.

1 Introduction

From time to time, an unfortunate aircraft accident could happen as in-flight breakup. In such an accident, many fragments from the wreckage are scattered around the ground by falling from the sky. The scattering may be affected with breakup sequence, wind, flight velocity, shape of the fragments, and so forth. It seems impossible to trace back the precise trajectory from the ground location of each fragment. If you look at scattering from the stochastic point of view, however, there could be some clues to find out the circumstance of the breakup since each fragment can be regarded as a falling object which is governed by the equation of motion with some random variables. The motivation of the present analysis has come from the real investigation of in-flight breakup of a small airplane. In the investigation, it was necessary to estimate the wind, breakup altitude, and location to clarify the flight path to reach the cause of the accident. Most of the wreckage were recovered from the ground and brought into the laboratory for the investigation. The result of the present analysis is compared with actual data of the accident.

2 Analysis

2.1 Governing Equations

In the analysis, wind is assumed to be horizontal and vertically uniform. The following vectors in the three-dimensional Cartesian coordinates are defined (See Fig.1).

Velocity vector for a piece of wreckage

$$\mathbf{v} = (v_x, v_y, v_z) \tag{1}$$

Wind velocity

$$\mathbf{w} = (w_x, w_y, 0) \tag{2}$$

Velocity vector of drag forces

$$\mathbf{v}_w = \mathbf{v} - \mathbf{w} \tag{3}$$

Direction vector for asymmetric forces

$$\mathbf{u} = (u_x, u_y, u_z) \tag{4}$$

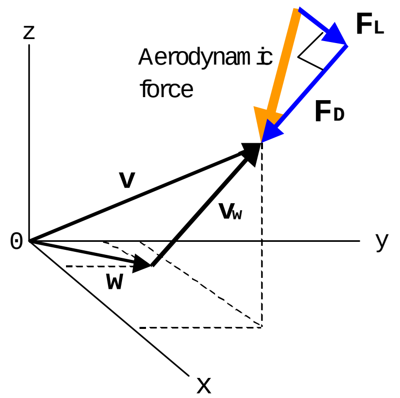

Fig.1 Definition of vectors

The equilibrium equation for a fragment falling in the air can be described as

$$m\,\dot{\mathbf{v}} = \mathbf{F}_D + \mathbf{F}_L + (0,0,\ -mg) \tag{5}$$

where m and g denotes the mass and the acceleration of gravity, respectively. The aerodynamic forces, $\mathbf{F}_D$ and $\mathbf{F}_L$ are the drag forces in the opposite direction against $\mathbf{v}_W$ and the lateral forces due to the asymmetric shape of the fragment, respectively. These aerodynamic forces can be given by

$$\mathbf{F}_D = \frac{1}{2}\rho\tilde{C}_D S(\mathbf{v}_w\cdot\mathbf{v}_w)\left(-\frac{\mathbf{v}_w}{|\mathbf{v}_w|}\right)$$

$$= -\frac{1}{2}\rho\tilde{C}_D S|\mathbf{v}_w|\cdot\mathbf{v}_w \quad \text{and} \tag{6}$$

$$\mathbf{F}_L = \frac{1}{2}\rho\tilde{C}_L S(\mathbf{v}_w\cdot\mathbf{v}_w)\tilde{\mathbf{u}} \tag{7}$$

where the direction vector $\tilde{\mathbf{u}}$ satisfies

$$\tilde{\mathbf{u}}\cdot\mathbf{v}_w = 0 \ \text{ and } \ |\tilde{\mathbf{u}}| = 1 \tag{8}$$

The symbol ~ implies probabilistic characteristics of coefficients. The parameters ρ and S denote constant density of the air and representative area of a fragment, respectively. In addition to the symbols above, the following definitions will be used.

$$V_w = \sqrt{\mathbf{v}_W\cdot\mathbf{v}_W} \tag{9}$$

$$\tilde{\alpha} = \frac{\rho}{2(m/S)}\tilde{C}_D \tag{10}$$

$$\tilde{\beta}_x = \frac{\rho}{2(m/S)}\tilde{C}_L\tilde{u}_x \tag{11}$$

$$\tilde{\beta}_y = \frac{\rho}{2(m/S)}\tilde{C}_L\tilde{u}_y \tag{12}$$

Then, Eq.(5) can be reduced to

$$\dot{v}_x = \tilde{\beta}_x V_w^{\ 2} - \tilde{\alpha}V_w\ (v_x - w_x) \tag{13}$$

$$\dot{v}_y = \tilde{\beta}_y V_w^{\ 2} - \tilde{\alpha}V_w\ (v_y - w_y) \tag{14}$$

$$\dot{v}_z = -g - \tilde{\alpha}V_w\cdot v_z \tag{15}$$

In deriving Eq.(15), the lateral forces in the z-direction have been neglected by considering the order of magnitude. Furthermore, the relative air speed defined by Eq.(3) can be replaced by the velocity of the z-direction if we neglect the contribution from the wind and the horizontal component of initial velocity. Since the most of difference due to this replacement appears at only the initial stage of falling, it does not affect much on scattering. It should be mentioned that the assumption does not mean the negligence of initial speed of the fragments. Thus, if we assume

$$V_w = |v_z| \tag{16}$$

then, we obtain, in lieu of Eqs.(13)~(15),

$$\dot{v}_x + \tilde{\alpha}|v_z|v_x = \tilde{\beta}_x v_z^{\ 2} + \tilde{\alpha}|v_z|w_x \tag{17}$$

$$\dot{v}_y + \tilde{\alpha}|v_z|v_y = \tilde{\beta}_y v_z^{\ 2} + \tilde{\alpha}|v_z|w_y \tag{18}$$

$$\dot{v}_z = -g + \tilde{\alpha}v_z^{\ 2} \tag{19}$$

These three equations are the governing equations to be solved for falling objects. Generally speaking, the lateral coefficients $\tilde{\beta}_x$ and $\tilde{\beta}_y$, which can have stochastically arbitrary direction perpendicular to the relative airflow, come from the asymmetric shape of fragments and much smaller if you compare with their drag coefficient $\tilde{\alpha}$.

2.2 Exact solution

Although the simultaneous differential equations, Eqs.(17)~(19), are nonlinear, the exact solution can be obtained analytically for this case. First, we solve an independent nonlinear equation, Eq.(19) for the z-direction. The method of separation of variables gives us the following form.

$$\int\frac{dv_z}{\tilde{\alpha}\ v_z^{\ 2} - g} = \int dt \tag{20}$$

Since the vertical acceleration should be always negative, the right hand side of Eq.(19) renders a condition,

$$|v_z| < a/\tilde{\alpha} \quad \text{where} \quad a = \sqrt{\tilde{\alpha}g} \tag{21}$$

The Eq.(20) can be integrated with the condition of Eq.(21) as

$$a^{-1}\,\mathrm{arctanh}(-ag^{-1}v_z) = t + C_0 \tag{22}$$

We tentatively assign the initial condition of v_z =0 at t =0 for the sake of simplicity.
Then, solving Eq.(22) for v_z yields

$$v_z = -a\tilde{\alpha}^{-1}\tanh at \tag{23}$$

Substitution of Eq.(23) into Eq.(17) gives a differential equation in the form of

$$\dot{v}_x + p(t)\cdot v_x = q(t) \tag{24}$$

where

$$p(t) = a\tanh at \tag{25}$$

$$q(t)=\tilde{\beta}_x a^2\tilde{\alpha}^{-2}\tanh^2 at+aw_x\tanh at \quad (26)$$

Equation (24) can be solved by the separation of variables and the variation of constant as

$$v_x=C_1 e^{-\int p(t)dt}+e^{-\int p(t)dt}\int e^{\int p(t)dt}q(t)dt \quad (27)$$

Since the function *p(t)* in Eq.(27) is given by Eq.(25), the integrations can be carried out as

$$\int p(t)dt=\log(\cosh at) \quad (28)$$

$$e^{-\int p(t)dt}=\operatorname{sech} at \quad (29)$$

$$e^{\int p(t)dt}=\cosh at \quad (30)$$

Substitution of Eq.(26) and the functions above into Eq.(27) gives

$$v_x=C_1\operatorname{sech} at+\tilde{\beta}_x a\tilde{\alpha}^{-2}\{\tanh at-\operatorname{sech} at\arctan(\sinh at)\}+w_x \quad (31)$$

An arbitrary constant C_1 in Eq.(31) can be determined to satisfy the initially given value for v_x.

$$\lim_{t\to 0} v_x=C_1+w_x=v_{x0} \quad (32)$$

Thus, the final form of the exact solution for Eq.(17) with Eq.(19) becomes

$$v_x=(v_{x0}-w_x)\operatorname{sech} at+\tilde{\beta}_x a\tilde{\alpha}^{-2}\{\tanh at-\operatorname{sech} at\arctan(\sinh at)\}+w_x \quad (33)$$

Since the governing equation for the velocity component in the y-direction has a similar form, the solution of Eq.(18) can be obtained in a same manner as Eq.(33).

$$v_y=(v_{y0}-w_y)\operatorname{sech} at+\tilde{\beta}_y a\tilde{\alpha}^{-2}\{\tanh at-\operatorname{sech} at\arctan(\sinh at)\}+w_y \quad (34)$$

Therefore, the position of the fragments in the x-y plane can be provided as the function of time by integrating Eqs.(33) and (34) with the initial location (x_0,y_0).

$$x=(v_{x0}-w_x)a^{-1}\arctan(\sinh at)+w_x t+\tilde{\beta}_x\tilde{\alpha}^{-2}\left\{\log(\cosh at)-\frac{1}{2}\arctan^2(\sinh at)\right\}+x_0 \quad (35)$$

$$y=(v_{y0}-w_y)a^{-1}\arctan(\sinh at)+w_y t+\tilde{\beta}_y\tilde{\alpha}^{-2}\left\{\log(\cosh at)-\frac{1}{2}\arctan^2(\sinh at)\right\}+y_0 \quad (36)$$

Since the position at the z-coordinate can also obtained by integrating Eq.(23) as

$$z=z_0-\tilde{\alpha}^{-1}\log(\cosh at) \quad (37)$$

Thus, the loss of altitude $H=-(z-z_0)$ can be related to a floating time T in the air by

$$aT=\operatorname{arccosh} e^{\tilde{\alpha}H} \quad (38)$$

Substitution of Eq.(38) into Eqs.(35) and (36) yields the fragment location on the ground as functions of H.

$$x=(v_{x0}-w_x)a^{-1}\arccos e^{-\tilde{\alpha}H}+w_x a^{-1}\operatorname{arccosh} e^{\tilde{\alpha}H}+\tilde{\beta}_x\tilde{\alpha}^{-1}\left\{H-(2\tilde{\alpha})^{-1}\arccos^2 e^{-\tilde{\alpha}H}\right\}+x_0 \quad (39)$$

$$y=(v_{y0}-w_y)a^{-1}\arccos e^{-\tilde{\alpha}H}+w_y a^{-1}\operatorname{arccosh} e^{\tilde{\alpha}H}+\tilde{\beta}_y\tilde{\alpha}^{-1}\left\{H-(2\tilde{\alpha})^{-1}\arccos^2 e^{-\tilde{\alpha}H}\right\}+y_0 \quad (40)$$

2.3 Case of $v_{z0}\neq 0$

In obtaining the solution of Eq.(23), we assumed the initial condition in the z-direction with no vertical speed. If a fragment has a certain vertical speed at an instance of breakup as $v_z=v_{z0}$ at t=0, Eq.(23) should be replaced by

$$v_z=-a\tilde{\alpha}^{-1}\times\tanh\left\{at+\operatorname{arctanh}\left(-ag^{-1}v_{z0}\right)\right\} \quad (41)$$

In accordance with Eq.(41), Eqs.(25) and (26) should become

$$p(t)=a\tanh\left\{at+\operatorname{arctanh}\left(-ag^{-1}v_{z0}\right)\right\} \quad (42)$$

$$q(t)=\tilde{\beta}_x\times\left[-a\tilde{\alpha}^{-1}\tanh\left\{at+\operatorname{arctanh}\left(-ag^{-1}v_{z0}\right)\right\}\right]+aw_x\tanh\left\{at+\operatorname{arctanh}\left(-ag^{-1}v_{z0}\right)\right\} \quad (43)$$

Although a similar procedure to the previous solution with the condition v_{z0} =0 may be traced, it would be too complicated. Therefore, we shall formulate in different way by utilizing the solutions with a vertically zero initial condition. Let us denote t = t* when the vertical velocity becomes $v_z = v_{z0}$ by the solution with the zero initial condition.

$$v_{z0} = -a\tilde{\alpha}^{-1} \tanh at^* \quad (44)$$

The solution to satisfy the condition, $v_x = v_{x0}$ and $v_y = v_{y0}$ at $t = t^*$ must have the virtually initial conditions v_{x0}^* and v_{y0}^* which satisfy

$$\begin{aligned} v_{x0} &= (v_{x0}^* - w_x)\,\mathrm{sech}\, at^* \\ &+ \tilde{\beta}_x a\tilde{\alpha}^{-2}\left\{\tanh at^* - \mathrm{sech}\, at^* \arctan(\sinh at^*)\right\} \\ &+ w_x \end{aligned} \quad (45)$$

$$\begin{aligned} v_{y0} &= (v_{y0}^* - w_y)\,\mathrm{sech}\, at^* \\ &+ \tilde{\beta}_y a\tilde{\alpha}^{-2}\left\{\tanh at^* - \mathrm{sech}\, at^* \arctan(\sinh at^*)\right\} \\ &+ w_y \end{aligned} \quad (46)$$

Equation (44) gives

$$\mathrm{sech}\, at^* = \sqrt{1 - V_z^{\,2}} \quad (47)$$

$$\sinh at^* = V_z / \sqrt{1 - V_z^{\,2}} \quad (48)$$

where

$$V_z = \tilde{\alpha} a^{-1} v_{z0} \quad (49)$$

Thus, Eq.(45) can be written as

$$\begin{aligned} v_{x0} &= (v_{x0}^* - w_x)\sqrt{1 - V_z^2} + w_x \\ &- \tilde{\beta}_x a\tilde{\alpha}^{-2}\left(V_z + \sqrt{1 - V_z^2} \arcsin V_z\right) \end{aligned} \quad (50)$$

This equation yields

$$\begin{aligned} v_{x0}^* &= (v_{x0} - w_x)/\sqrt{1 - V_z^2} + w_x \\ &+ \tilde{\beta}_x a\tilde{\alpha}^{-2}\left(V_z / \sqrt{1 - V_z^2} + \arcsin V_z\right) \end{aligned} \quad (51)$$

$$\begin{aligned} v_{y0}^* &= (v_{y0} - w_y)/\sqrt{1 - V_z^2} + w_y \\ &+ \tilde{\beta}_y a\tilde{\alpha}^{-2}\left(V_z / \sqrt{1 - V_z^2} + \arcsin V_z\right) \end{aligned} \quad (52)$$

The virtual loss of altitude H^* at the time of t^* can be given by solving Eq.(38).

$$\begin{aligned} H^* &= \tilde{\alpha}^{-1} \log(\cosh at^*) \\ &= -(2\tilde{\alpha})^{-1} \log(1 - V_z^2) \end{aligned} \quad (53)$$

Therefore, using the virtually initial location (x_0^*, y_0^*) to adjust these initial conditions, the location (x,y) on the ground can be calculated by

$$\begin{aligned} x &= \left(v_{x0}^* - w_x\right) a^{-1} \arccos e^{-\tilde{\alpha}(H+H^*)} \\ &+ w_x a^{-1} \,\mathrm{arccosh}\, e^{\tilde{\alpha}(H+H^*)} \\ &+ \tilde{\beta}_x \tilde{\alpha}^{-1}\left\{H + H^* - (2\tilde{\alpha})^{-1} \arccos^2 e^{-\tilde{\alpha}(H+H^*)}\right\} \\ &+ x_0 - x_0^* \end{aligned} \quad (54)$$

and

$$\begin{aligned} y &= \left(v_{y0}^* - w_y\right) a^{-1} \arccos e^{-\tilde{\alpha}(H+H^*)} \\ &+ w_y a^{-1} \,\mathrm{arccosh}\, e^{\tilde{\alpha}(H+H^*)} \\ &+ \tilde{\beta}_y \tilde{\alpha}^{-1}\left\{H + H^* - (2\tilde{\alpha})^{-1} \arccos^2 e^{-\tilde{\alpha}(H+H^*)}\right\} \\ &+ y_0 - y_0^* \end{aligned} \quad (55)$$

where

$$\begin{aligned} x_0^* &= \left(v_{x0}^* - w_x\right) a^{-1} \arccos e^{-\tilde{\alpha}H^*} \\ &+ w_x a^{-1} \,\mathrm{arccosh}\, e^{\tilde{\alpha}H^*} \\ &+ \tilde{\beta}_x \tilde{\alpha}^{-1}\left\{H^* - (2\tilde{\alpha})^{-1} \arccos^2 e^{-\tilde{\alpha}H^*}\right\} \end{aligned} \quad (56)$$

and

$$\begin{aligned} y_0^* &= \left(v_{y0}^* - w_y\right) a^{-1} \arccos e^{-\tilde{\alpha}H^*} \\ &+ w_y a^{-1} \,\mathrm{arccosh}\, e^{\tilde{\alpha}H^*} \\ &+ \tilde{\beta}_y \tilde{\alpha}^{-1}\left\{H^* - (2\tilde{\alpha})^{-1} \arccos^2 e^{-\tilde{\alpha}H^*}\right\} \end{aligned} \quad (57)$$

2.4 Approximate solution for high altitude

If the start of falling is relatively high in the altitude, the solutions of Eqs.(39) and (40) may have an approximate form by assuming large values of $\tilde{\alpha}H$ as

$$\begin{aligned} x &\approx \pi\left(v_{xo} - w_x\right)/(2a) \\ &\quad + w_x H \sqrt{\tilde{\alpha}/g} + \tilde{\beta}_x H / \tilde{\alpha} \end{aligned} \quad (58)$$

$$\begin{aligned} y &\approx \pi\left(v_{yo} - w_y\right)/(2a) \\ &\quad + w_y H \sqrt{\tilde{\alpha}/g} + \tilde{\beta}_y H / \tilde{\alpha} \end{aligned} \quad (59)$$

These formulae can give the scattering distributions for light objects as a function of height with the assumption of vertically uniform wind.

2.5 Wind effects

Further, if we roughly estimate the wind effects in scattering, the floating distance d by the wind w is calculated by

$$d \approx \mathrm{w} H \sqrt{\tilde{\alpha}/g} \quad (60)$$

3 Accident analysis

3.1 Wreckage of an accident

Wreckage brought in the laboratory is shown in Fig.2. It was a small airplane with a single engine and had a pressurized cabin.

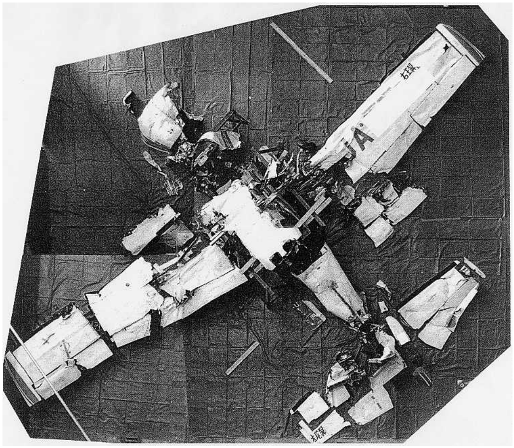

Fig.2 Wreckage by in-flight breakup

The airframe has been broken into 26 fragments including the fuselage. Two cuts in front of the windshield and at the rear of the cabin were artificially made after the accident for recovery.

3.2 Actual scattering of fragments

The fragment distributions on the ground of an accident are shown in Fig.3.

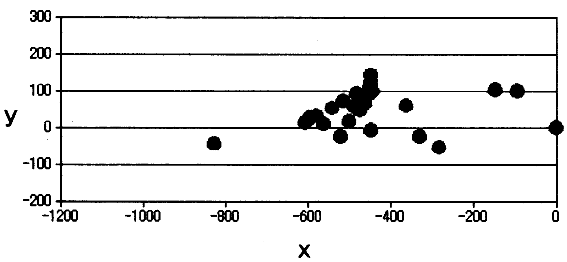

Fig.3 Distribution of fragments

It comprises twenty-six points including the main wreckage of fuselage at the origin. The unit of the coordinates is one meter. In the laboratory, each fragment was measured one by one for its mass and representative area to calculate the value of mass per unit area, *m/S*. The area is defined as a projection on the ground. Uncertainties for the definition of area on such random shape of fragments can be considered as the effects of probabilistic aerodynamic coefficients.

3.3 Wind direction

In order to find out the wind direction at the time of accident, distance from a certain point was calculated and plotted versus the square root of inverse of *m/S* since the wing effects are involved with that parameter as can be found in Eq.(60). A typical result for the point (250,300) is shown in Fig. 4. The regression line is also

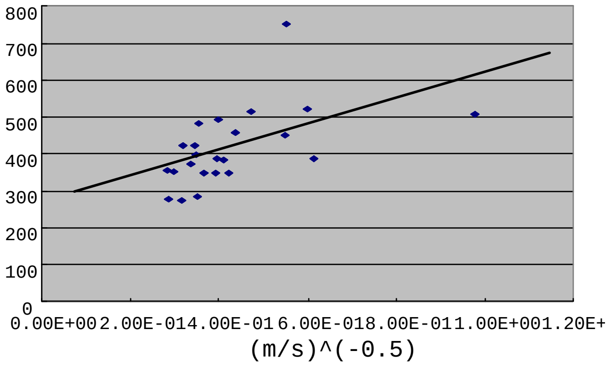

Fig.4 Distance versus $1/\sqrt{m/S}$

illustrated in the figure. From these kinds of calculations, a contour map for the R2 values (square of Peason's correlation coefficient) can be formed as shown in Fig. 5. The figure clearly indicates the direction that is most correlated with the wind parameter. The wind direction was determined from the information revealed as an angel of 23 degrees from x-axis.

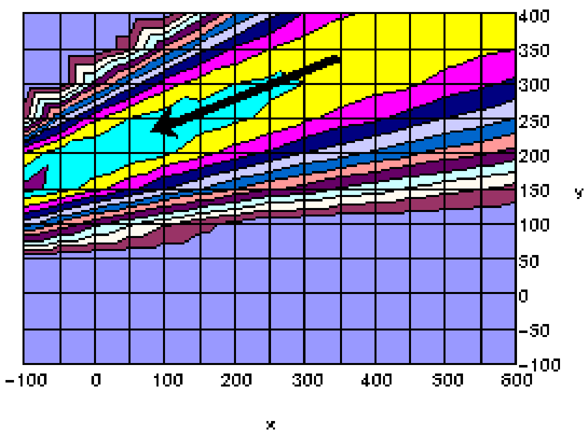

Fig. 5 Contour map of R2 values

3.4 Parameter estimation by using analytical solutions

In order to find out wind conditions and other circumstances of the breakup, a parametric study has been performed with various combinations of parameters in Eqs.(39) and (40). It was performed in a way of the try-and-error method by examining the pattern of distributions. The altitude of breakup was confirmed through this procedure as around 4,100ft that was the last flight data by active radar information. The most typical result is shown in Fig.6 where you can see a favorably resembling pattern of scattering to the real case

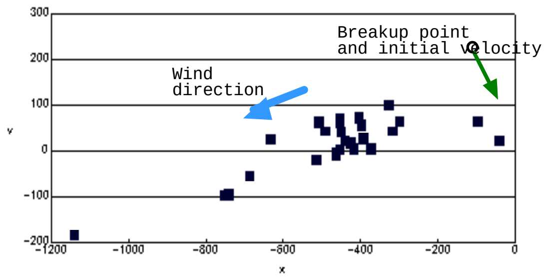

Fig.6 Results of estimation

shown in Fig.3.In this calculation, the following aerodynamic values were used.

$$\tilde{C}_D = 0.2 \pm 0.005\ ,\ \tilde{C}_L \tilde{u} = \pm 0.01$$

The results are thought to sustain the estimation of the breakup velocity and the direction of flight. The accuracy of the approximate solutions, Eqs.(58) and (59) was also examined for this case. The results from these equations were illustrated in Fig. 7. Most of the locations are the same as Fig.6 except two fragments

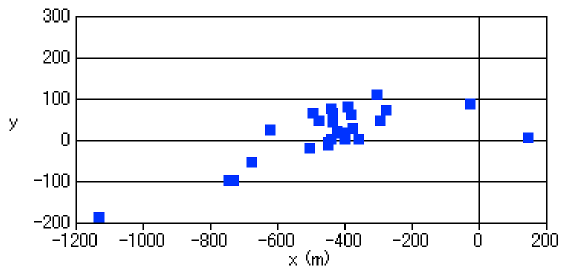

Fig.7 Approximate solution

in the right most area. The difference can be attributed to their higher *m/S* that deteriorates the assumption as a large $\tilde{\alpha}H$ in the approximation.

3.5 Monte Carlo numerical simulation with Eqs.(13)-(15)

In order to confirm the assumption for the analytical solution, especially for the effects of difference at initial stage, Eqs.(13)-(15) have been integrated numerically with the same random coefficients used in the previous results of Fig.6. The result is shown in Fig.8. It can be seen in comparison of Fig.6 and Fig.8 that the analytical solution has no appreciable difference

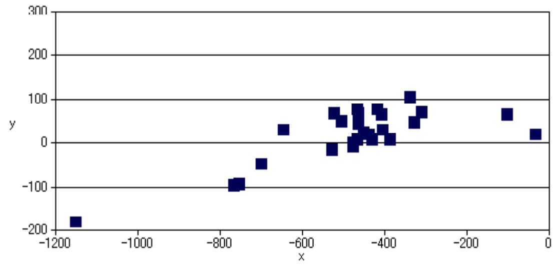

Fig.8 Numerical simulation

with the numerical simulation. It can be said that the method described herein provides an effective tool for the accident investigation.

4 Conclusions

Nonlinear simultaneous equations to analyze distributions of scattering for falling objects were derived and exactly solved to investigate an aircraft accident. The flight conditions at the time of in-flight breakup were estimated by using those solutions with random aerodynamic coefficients. The location, momentum and a wind speed at the instance of breakup could be estimated appropriately. It is believed that the results have made significant contribution to the accident investigation. Approximate formulae for light objects to find out the location and the floating distance by the wind are also derived.