RAPID ON BOARD PROTOTYPING OF NEAR-OPTIMAL SPATIAL TRAJECTORIES FOR PILOT'S ASSOCIATE

Oleg A. Yakimenko#
Aviation and Aeronautics Academy of Sciences, Moscow, Russia
Naval Postgraduate School, Monterey, CA (oayakime@nps.navy.mil)

Keywords: *pilot's associate, road-in-the-sky, trajectory optimization and rapid prototyping*

Abstract

The paper deals with a concept, mathematical foundation and results of testing a prototype of onboard system providing an aid for pilot's control actions during more or less long-term manoeuvres, such as takeoff/climbing, waypoint flight through the threat zones, surface-based and air target attack, descent and landing approach. The key issue of this system is to provide an algorithm allowing on-line prototyping of spatial trajectories for these manoeuvres and visualisation of the reference trajectories to a pilot on 3-D flightpath display for further tracking. As an efficient tool for onboard optimisation the, present paper considers different modifications of a direct method of calculus of variations, implemented to the tasks of flight dynamics first time by Prof. Taranenko in the early 60's. The paper contains a brief survey of the methods used so far by the other researches in the field, discloses main ideas of proposed approach, based on independent optimisation of a trajectory and speed profile, shows how to assure a required convergence robustness. The paper is concluded with the reference to the semi-natural and flight test of the proposed system prototype, carried out in Russia in mid 90's in Zhukovskiy Air Force Engineering Academy and Gromov Flight-Test Institute.

1 Introduction

It is well known that a biocybernetic cabin is considered by aviation specialists of leading countries as one of the most progressive ways of modern and perspective aircraft (a/c) efficiency increase (see, for example, Refs. [1,2]). A hardware and software of the last one (also known as a Pilot's Associate (PA) or Electronic Co-pilot) should provide a pilot with the relevant information and intelligent support during flight mission fulfilment – for all tasks from flight planning and preparing up to the landing and post-flight analysis. Pilots expect such support would increase the total "board intellect", i.e. summary possibilities of man-machine (MM) system. Therefore, this system would allow them to solve the different complex flight-mission's tasks creatively, not as a robot. Furthermore, 15-year a/c accidents statistics [3] and pilot's interviewing [4] strongly shows that because of the fact that specifically control errors evoke a majority of the flight accidents, practically all pilots have the urge requirement of their control actions support. (By the way, they consider this last task as the most difficult to be supported [4].) The recent progress in a/c complexity and potential possibilities makes this requirement highly urgent. Thus, the question 'whether we have any means to provide a pilot with a relevant cognitive prompt to assist him in controlling the a/c?' is vital. As a matter of fact, this question can be split into two separate questions – the question of cognitive representation of guidance/control prompt and the question of relevant calculation of the reference controls.

1.1 Perspective flightpath displays

According to pilots [4] the most desirable prompt during manoeuvring would be the reference trajectory (RT) shown on a head-up (HUD) and/or head-down displays in a view of a "road-in-the-sky" (RiS) (also being referred to as a tunnel-in-the-sky, pathway-in-the-sky, highway-in-the-sky, etc.) or in the other words on a 3D flightpath display (FD).

National Research Council Senior Research Associate at the NPGS.

Probably the first time, FD was referred in the technical literature in the early 50's in conjunction with the US Army Navy Instrumentation Program [5]. The goal of this program was to develop an improved MM interface. Of course at that time because of the lack of appropriate computer technologies not many of FD's formats proposed in the context of the mentioned program and latter researches (by Hoover, Farrand, Wilckens, Jensen, and others) was implemented and flown. The major area of interest at that time was to employ perspective FDs for the instrument landing guidance.

Latter research (done by Wattler, Mulley, Grunwald, Adams, Logan, Jauer, Quinn, Mitori, et al. in the 80's) addressed many issues of FD, including RiS representation: its cross-section shape (straight and curved U, brackets, tiles, square, rectangle, etc.), cross-section connections, sizes and so on. They also discussed a RiS as an alternative to the flight director and proposed to employ a synthetic terrain to improve a perception FD's format [6].

One of the first FD prototypes was flown by Hoover in the early 80's. A total of 90 hours of ground simulation and 20 hours of in-flight operation demonstrated that such display provides the pilot with adequate information to perform all types of steady state manoeuvres (including takeoff/climbing, cruise flight, approach and landing) without reference to conventional parametric displays or the real world. Based on the results of the flight it was stated that the concept of continuous command information is one of the most significant innovations that FD can provide to MM interface. This interface serves as external memory, eliminating the necessity for memorising each segment of the flight mission plan and referring to a navigation chart, reducing the load on working memory [6]. Hoover et al. [7] indicated the operational advantages by stating that no coaching by a second pilot with regard to each upcoming event was necessary when the evaluation pilot was flying the RiS. With respect to navigational awareness, they reported that while flying the FD, the majority of the pilots stated that 'there was never any question as to where they were, what they were doing, or what they should be doing' that is not a case with a common symbolic format. Because of mentioned promises FD was included into announced PA program as an output interface in the mid 80's (Fig.1) [1].

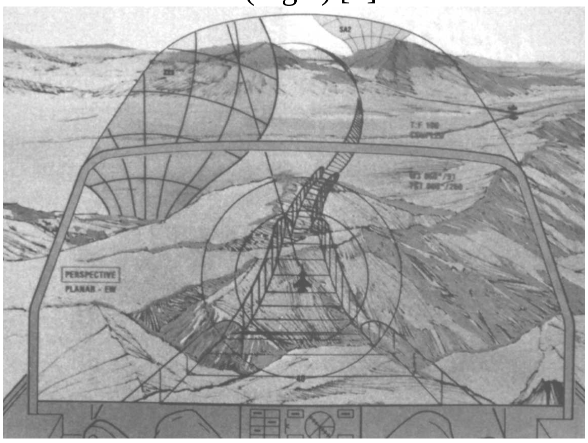


Fig.1 One of the PA's formats

In the 90's, the research teams led by Grunwald, Swenson, Theunissen, and Sachs were able not only to proceed with semi-natural experiments using the modern powerful computers, but also to flight test their FDs on the different-type a/c. They reported a significant improvement in pilot situational awareness and mission effectiveness, as well as a decrease in training and proficiency time required for a near terrain, night-time, adverse weather flight. They also studied an implementation of additional 'dimension' - superimposed predicted vehicle position (4D approach) [6]. Fig.2 shows pilots' desire to have a RiS prompt at the different stages of flight [4].

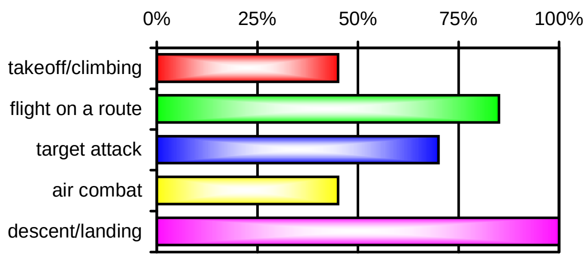


Fig.2 Pilots' desire to use FD at different stages of flight

All preceding research were based on some pre-set trajectory calculated beforehand off-line (usually it was a straight glide-path trajectory or piecewise trajectory consisting of straight lines and pieces of spirals or straight terrain-following trajectory). Today's almost 1GHz-frequency computers with enormous storage capacities and advanced software allow designing any complex adaptive FD and its usage on board. However, one obstacle still prevents the wide employment of these displays - a lack of reliable method allowing on-line prototyping of any spatial trajectory for any stage of flight.

1.2 Pilot's control actions support

As mentioned above, we have to be able to calculate optimal trajectories for a majority of manoeuvres in accelerated scale of time. That is a main goal of the proposed on-board pilot's control actions support system (PSS), considered as an element of supervising PA (Fig.3) [8]. The structure of such PA's subsystem for the majority of different tasks to be supported is shown on Fig.4. In real flight for the real strategic and/or tactic conditions for real state of the a/c, its subsystems and even the pilot, PSS provides computation of RT to be visualised on FD.

Since the requirement of optimisation rapidity and analytical representation of the extremal solution are the main, an employment of classical methods of calculus of variation becomes strongly limited. This has led to a search for variational methods of a different kind, known as direct methods (DMs) (which do not entail the reduction of variational problems to problems involving differential equations). The fundamental idea of DMs is to consider a variational problem as a limit problem of the extreme of a function of a finite number of optimisation parameters (OPs) to be solved by usual methods. Because of their convergence robustness, DMs give the safest approach for rapid prototyping of spatial trajectories for an a/c [9].

Although DMs ensure much better robustness in comparison with indirect methods, it is still impossible to use them onboard for on-line optimisation. That is why in expectation of more powerful computers, it was proposed to employ the databases of initial guesses of OPs or the "banks of trajectories" (TBs) essentially improving a convergence robustness.

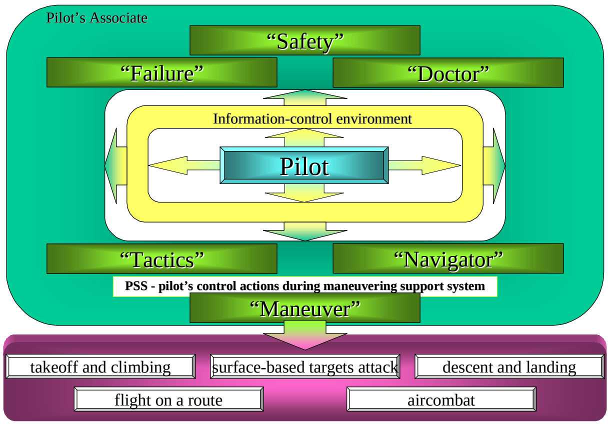


Fig.3 General structure of PA expanded with PSS

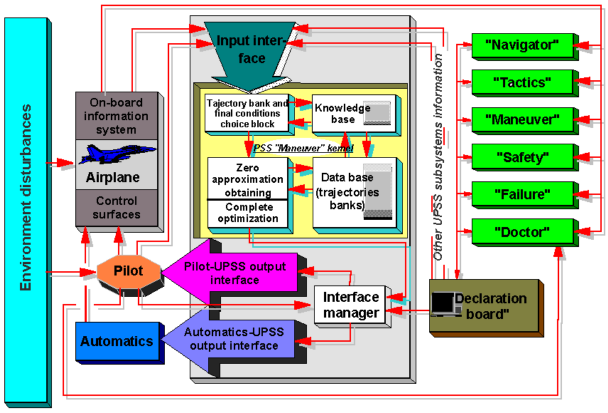


Fig.4 Block-scheme of PSS

The left and right "boundaries" of the trajectory are in general full defined. If any of states is not fixed, it can be chosen with the help of corresponding knowledge-bases, created on the basis of modelling and pilots' interviewing (see example on Fig.5). Controls' constraints are supposed to be known from the other PA's subsystems.

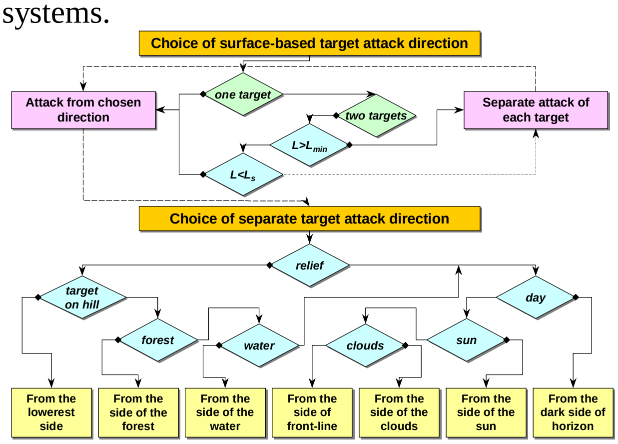


Fig.5 Knowledge base's fragment

The present paper discloses some basic algorithms of the proposed PSS and is organised as follows. Section 2 provides a mathematical foundation of one of modifications of the mentioned DM for rapid prototyping (DMRP) and shows some simulation results. Section 3 briefly discusses semi-natural experiments and flight test performed at that point.

2 Real-time trajectory optimisation

2.1 General statement of optimisational problem

The most general statement of the optimal control problem, determining a/c trajectory from the current position to a given point, may be specified as follows.

There is a set of admissible trajectories satisfying: *i)* the system of ordinary differential equations, *ii)* initial conditions and final (terminal) conditions, *iii)* constraints on the state space, on controls, and on the controls' derivatives. The problem is to find an optimal trajectory that minimises some cost function (CF) and an optimal control corresponding to this trajectory. Mentioned CF in general can be represented not only as integral function (the simplest examples are the manoeuvre time or fuel consumption), but also as the function of current co-ordinates and controls in the terminal point or at some event-conditioned instant of time (e.g., terminal load factor or bank angle at aiming point, etc.).

Although the terminal point in the preceding definition is considered as completely defined, in general, some state variables and/or controls might not be pre-set. In this case, they are also needed to be optimised.

As a system of ordinary differential equations, the 3D point-mass equations over a flat Earth with zero sideslip angle are considered as usual (totally seven equations). In this case the common controls are a throttle, load factor and bank angle with appropriate constraints. Nonlinear aerodynamic characteristics and propulsion performance are presented by corresponding coefficients defined by multidimensional tabulated data.

2.2 Direct methods in flight dynamics

It was already mentioned that the direct methods are the best base for the rapid trajectory prototyping. The main idea of the direct methods is to consider a function as a finite set of variables. This is fairly evident, if it is assumed that the admissible function can be represented by an infinitive power series, or by a Fourier series, or by any series of the form $y(x)=\sum_{k=1}^{\infty}a_k\varphi_k(x)$, where $\varphi_k(x)$ are given functions. Then CF will be the function of a set of unknown coefficients allowing to reduce the task by considering a finite series instead of infinite.

The first direct approaches to solve the problem of integral minimisation were introduced by Euler and later by Ritz. Another more simple but more universal procedure was introduced by Galerkin in 1915 as a means of obtaining approximate solutions to the boundary value problems. When combined with interpolation equations of the method of finite elements, which is a variation of the Rayleigh-Ritz procedure, Galerkin's method becomes a very useful procedure for solving both initial and boundary value problems.

It was Taranenko, who developed and applied the method like Ritz-Galerkin to the problems of flight dynamics with constraints on state variables and controls [10]. Following the main idea of the direct methods, Taranenko suggested defining the reference functions (RFs) for both a/c's Cartesian co-ordinates and its airspeed as $x_i=x_{i0}+\dfrac{x_{if}-x_{i0}}{\tau_f-\tau_0}(\tau-\tau_0)+\Phi_i(\tau)$, $i=\overline{1,4}$. Here τ is an argument, and $\Phi_i(\tau)$ are continuous, unequivocal and differentiable functions, satisfying to obvious boundary conditions $\Phi_i(\tau_0)=\Phi_i(\tau_f)\equiv 0$. Taranenko proposed to use one of the following functions $\sum_{k=1}^{n}A_k\sin k\pi\dfrac{\tau-\tau_0}{\tau_f-\tau_0}$, or $\sum_{k=1}^{n}A_k(\tau-\tau_0)^k(\tau-\tau_f)^k$, or $(\tau-\tau_0)^{m_1}(\tau-\tau_f)^{m_2}$, or their linear combination. However, in principle, there are no limitations, so one can use any convenient functions for the particular task at hand.

An explicit mean to increase flexibility is to increase a number of elements n in a mentioned series or to increase powers m_1 and m_2. However, Taranenko proceeded with another approach – he subdivided an interval $[\tau_0;\tau_f]$ on several pieces and employed low order polynomials to describe behaviour of state variables x_i, $i=\overline{1,4}$ along each of them (the parameters of pieces' collocation can then be considered as an additional OP). The higher number n (or m_1 and m_2), and the higher the number of pieces in piecewise case, the closer a near-optimal (NO) solution is to the optimal one.

The choice of an argument τ also depends on a particular task. Generally speaking, one can use any continuous monotonic parameter: time, path, etc. In case of defining both a/c's co-ordinates and airspeed, as Taranenko did, we may use only an abstract parameter, otherwise we would be unable to vary trajectory and speed history independently. Taranenko called τ a virtual arc [10].

Finally, to be able to satisfy boundary conditions imposed on state variables, their first and second derivatives, Taranenko and his followers preferred the sum of three cubic polynomials as RFs, where polynomials' coefficients are automatically defined by pre-set boundary conditions at the origin and the end of the trajectory [11]. In the case of control's constraint violation, they shift to a direct integration of state equations with the marginal value of this (violated) control. Since the 60's, with the use of this method, a lot of optimisation problems have been solved for different a/c, including helicopters, strike and civil a/c, airspace vehicle.

Neljubov uses fifth-seventh order time-polynomials for Cartesian co-ordinates only and implemented them as "basic trajectories" during the following tracking with a real automation [12]. Hargraves [13], Convay [14], and others use the so-called collocation-based or direct transcription methods, which is similar in many respects to Galerkin's procedure. They reduce the initial problem by segmenting the time interval into the 5-20 pieces and representing the solution both for state variables and controls by piecewise polynomials (constants). The tens of unknown coefficients are then determined by enforcing continuity at the nodes and by satisfying the differential equations at some specified points in each segment. Seywald [15] and others use the so-called differential inclusion approach. They eliminate controls from the state equations by employing a description of the dynamical system in terms of its attainable set. Lu [16] uses, for each piece, an approach similar to Taranenko's method. He calls it inverse dynamics approach. For optimisation of planar trajectory for aerospace vehicle at each of the twenty pieces he pre-determines one of the state variables' time history and one of the controls' time history by cubic splines, and then he solves the inverse task of flight dynamics.

Unfortunately neither of preceding methods has properties required by PSS concept. Ones approaches do not provide analytical representation of the entire trajectory, another imply relatively difficult numerical calculations with numerous OPs. The accuracy of piecewise-based approaches directly depends on the number of segments used in the approximation. Finally, although all cited approaches have been successfully used for off-line trajectory optimisation, nobody tried to employ DM for on-line optimisation.

2.3 Introduction to the rapid-prototyping approach

Without loss of generality let us consider a computational procedure of the basic modification of DM, suitable for short-term spatial manoeuvres like takeoff/climbing, curved approach, etc. [17]. This modification combines a number of advantages over both previously mentioned methods by Taranenko and Neljubov, and consequently a close position to Lu's approach. Though having less possibilities of varying the trajectory itself, this algorithm assures the following: *i)* the boundary conditions are satisfied 'a priori'; *ii)* an a/c control is physical and realisable (smooth), meaning a pilot can easily perform it; *iii)* the iterative process converges well, making it possible to proceed with on-line optimisation; *iv)* RT has an analytical representation.

We take the RFs for an a/c's co-ordinates x_i, $i=\overline{1,3}$ as algebraic polynomials of degree n with the virtual arc τ as an argument:

$$x_i(\tau)=\sum_{k=0}^{n} a_{ik}\frac{(\max(1,k-2))!\tau^k}{k!} \tag{1}$$

The degree n of these polynomials is determined by the number of boundary conditions to be met, so that all coefficients a_{ik} were determined algebraically, rather than varied. The higher the maximum degree of time derivative of an a/c co-ordinate at initial and terminal points, whose values (the derivatives) are known, the higher the degree of the polynomial. The minimum degree of the polynomial is $n=d_0+d_f+1$, which is greater by one than the sum of the maximum orders of the time derivative of the a/c co-ordinates at the initial and terminal points (d_0 and d_f, respectively) [12].

For example, if we consider the task without pre-setting the initial and terminal values of second time derivatives of a/c co-ordinates' (proportional to controls), that means $d_0=d_f=1$, the minimum polynomials' order is $n=3$. Substituting the corresponding values of x_{i0}, x'_{i0}, $i=\overline{1,3}$ for $\tau=0$, and x_{if}, x'_{if}, $i=\overline{1,3}$ for $\tau=\tau_f$ into Eq.(1) (where τ_f, the length of a virtual arc, is considered as the first OP), we obtain a set of 12 linear algebraic equations for 12 unknown coefficients a_{ik}, $i=\overline{1,3}$, $k=\overline{0,3}$ being resolved as:

$$\begin{aligned} a_{i0}&=x_{i0}, \quad a_{i2}=-\frac{2x'_{if}+4x'_{i0}}{\tau_f}+6\frac{x_{if}-x_{i0}}{\tau_f^2}\\ a_{i1}&=x'_{i0}, \quad a_{i3}=6\frac{x'_{if}+x'_{i0}}{\tau_f^2}-12\frac{x_{if}-x_{i0}}{\tau_f^3}\end{aligned} \tag{2}$$

Of course, we can compound the RFs as superposition of several cubic polynomials [10,11], and then satisfy the boundary conditions for controls. Otherwise, we should employ the higher-order polynomials. For instance, fifth-order polynomials allow to satisfy the boundary values for the a/c co-ordinates, their first and second time derivatives at both ends of the trajectory ($d_0=d_f=2$). To have a final part of the trajectory more smooth, it is better to employ a case when $d_f=3$ with $x'''_{if}\equiv 0$, $i=\overline{1,3}$.

The only OP so far was a length of a virtual arc. However sometimes to make RT more flexible, it is worth to add some more OPs. For instance, we can add one fictive boundary condition x'''_{i0}, $i=\overline{1,3}$ to the case $d_0=2$, $d_f=3$, and for $n=7$ obtain relations for 24 coefficients a_{ik}, $i=\overline{1,3}$, $k=\overline{0,7}$ in the same manner, as in (2). Now we can use these fictive boundary values as additional OPs.

Since RT is defined not in the time frame, it does not explicitly determine a history of speed. This gives a great advantage - an a/c can fly along the same trajectory with different speed histories. In general, the dependence $V(\tau)$ may be determined either by pre-setting a separate RF $V(\tau)$, as in Taranenko's method, or by integrating the corresponding state equation for a speed with predetermined thrust history

$$V'(\tau)=g(n_x-\sin\gamma)\frac{dt}{d\tau}=\frac{g(n_x-\sin\gamma)}{\lambda(\tau)} \tag{3}$$

where $\lambda=\dot{\tau}$ is a virtual speed, γ is a flightpath angle, and n_x is a tangential projection of a load factor. It means that we can explicitly employ the results of controls' synthesis obtained with the help of indirect methods. We will further deal specifically with the last approach, assuming that throttle versus time (arc) history is known qualitatively beforehand.

For example, for the time-optimum problem, the optimum thrust control is the on/off control. Hence, when solving such optimisation problem, we can set several (for the majority of cases at hand - two) switching points: from maximum thrust $\delta_{T\max}$ to minimum $\delta_{T\min}$ at the moment τ_T^* (t_T^*), and back from $\delta_{T\min}$ to $\delta_{T\max}$ at the moment τ_T^{**} (t_T^{**}) ($0\le\tau_T^*<\tau_T^{**}\le\tau_f$).

Therefore, the search of the NO thrust's control will be made among three admissible thrust histories: $\delta_{T\,\max}$-$\delta_{T\,\min}$, $\delta_{T\,\min}$-$\delta_{T\,\max}$ and $\delta_{T\,\max}$-$\delta_{T\,\min}$-$\delta_{T\,\max}$. Of course, for other type of CF we can try other reasonable thrust time-histories.

In practice we only have the boundary values of state variables and controls, so we need to calculate boundary conditions for the derivatives of the state variables with respect to the argument τ down to the order of d_0 and d_f respectively (2). We can make it with the help of known kinematic and dynamic equations of an a/c, using obvious relations $x_i' = \lambda^{-1}\dot{x}_i$ and $x_i'' = \lambda^{-2}[\ddot{x}_i - \dot{x}_i\lambda']$, $i = \overline{1,3}$ [11]. Corresponding values of λ and λ' at the boundary points are determined as $\lambda_{0;f} = V_{0;f}$, $\lambda'_{0;f} = \dot{V}_{0;f}V_{0;f}^{-1}$.

During numerical solution, the parameters of RT are calculated in N points equidistantly placed over the virtual arc, so that $\Delta\tau = \tau_f(N-1)^{-1}$. This sampling period for $j = \overline{1, N-1}$ corresponds to the time intervals $\Delta t_j = 2\sqrt{\sum_{i=1}^{3}\left(x_{i\,j} - x_{i\,j-1}\right)^2}\left(V_j + V_{j-1}\right)^{-1}$. With these values of $\Delta\tau$ and Δt_j, the virtual speed λ is calculated at each step according to $\lambda_j = \Delta\tau\Delta t_j^{-1}$.

The explicit laws for a/c's co-ordinates (1), with account of calculated speed history (3), uniquely determine the a/c attitude – flightpath and azimuth angles, - and remaining controls – bank angle ϕ and normal projection of load factor n_z. It can be done through the solution of the inverse problem of flight dynamics (have to define the controls' time-histories corresponding to a desired RT).

Therefore, for each set of OPs we calculate the value of a CF along with the value of a penalty function, summarising possible violations of constraints and terminal speed error. As a result, we reduce the original problem to a nonlinear programming problem, meaning we obtain a problem of minimisation for the scalar function of several (but not tens as in Refs.13-16) OPs.

Because of erroneous gradient information (because of tabulated character of aerodynamic and thrust data, because of event-conditioned step-changing mass or aerodynamic configuration, etc.), zero-order algorithms are preferred to quadratic programming. Another and possibly the most important reason for the use of these simple algorithms is that since we are going to implement them on board of an a/c, we need a probability of solution equalling to one, meaning an absolute reliability. In practice, a number of major-loop iterations required by any non-gradient algorithm to converge the task did not exceed 25-30 iterations (with an arbitrary initial guess). With account of searching iterations, the total number of CF evaluations was an average of 100-120. The run time t_{CPU} for IBM486-type processor was not more than 3% of trajectory duration itself.

2.4 Examples of near-optimal manoeuvring

By now within the PSS paradigm with the help of proposed DMRP, tens of thousand trajectories have been calculated and tested for different types of a/c for such stages of flight as take-off/climb, surface-based target attack (SBTA) and curved landing approach (CLA). Figs.6-8 demonstrate some examples of such trajectories, calculated for the multi-regime fighter with the use of seventh-order polynomials.

Fig.6 illustrates the possibility of applying DMRP to calculate SBTA (a) and CLA (b) trajectories. Fig.7 shows an example when the final states' manifold is determined by the terminal speed, terminal range to the origin of the inertial frame and by final flightpath angle, but the final azimuth angle is not pre-set. In this case, the final azimuth angle was also optimised.

Since DMRP takes into account the prescribed values of high order derivatives at the boundary points there is no problem to calculate (optimise) a series of manoeuvres ensuring a smooth change of controls. Fig.8 gives an example of such SBTA trajectory composed of two manoeuvres optimised one after another (the terminal state of the first manoeuvre serves as an initial state for the second).

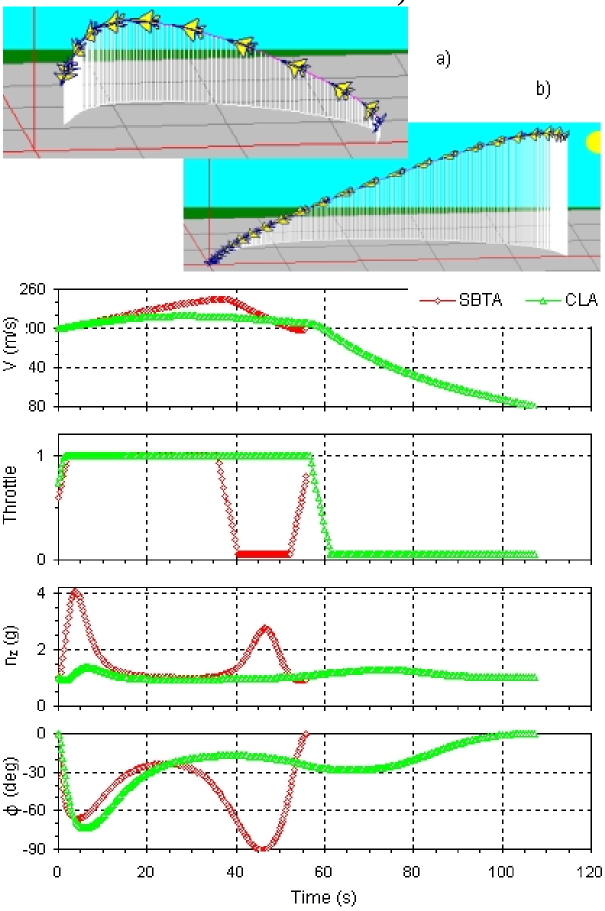


Fig.6 Examples of NO trajectories

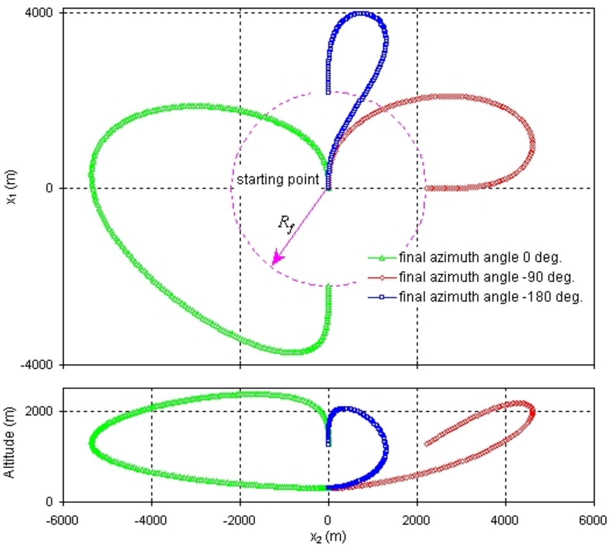


Fig.7 Illustration of RFs' flexibility

Note, this feature allows implementation of the same approach for "dynamic" optimisation, when the constraints on the state variables or desired final condition change in time. The latter means that a trajectory should be recalculated from the current (prognosis) condition every several seconds, as it can take place during collision avoidance in free flight or evasion-pursuing in a dog-fight (Fig.9), for example.

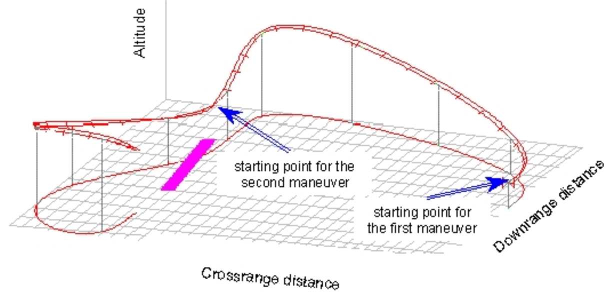


Fig.8 Illustration of consistent manoeuvres calculation

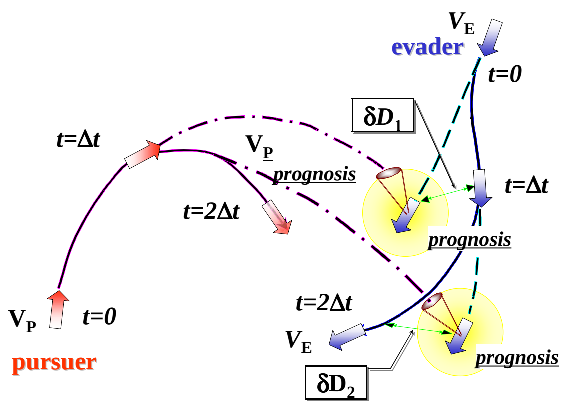


Fig.9 Step-by-step optimisation

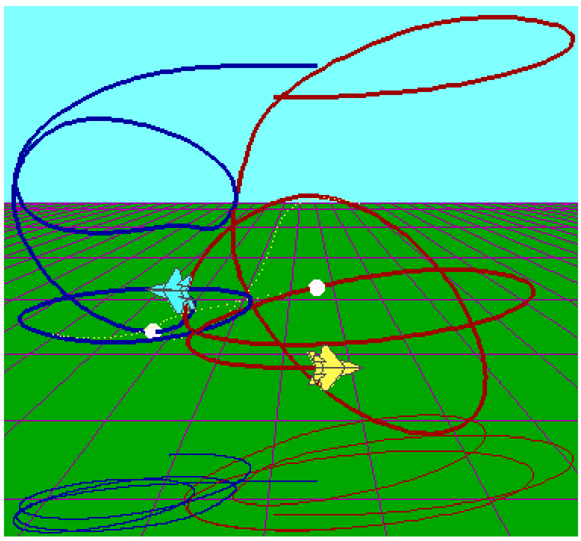


Fig.10 Example of dogfight modelling

Fig.10 illustrates the example of NO pursuing-evading in an aggressive dogfight with the use of DMRP by both sides. As a CF, the kill probability computed with the use of original approach [18] was used in this case. Another feature of a dogfight is that a final position of an a/c and its velocity is not known beforehand. Therefore they were considered as additional OPs (in total nine OPs were used for each side).

As mentioned earlier, DMRP has a very important advantage from the standpoint of employing it in PSS - the rapidity of two-point boundary-value problem solution even by means of non-gradient method ($t_{CPU} = 0.03 t_f$). To correctly account a run time, the predicted on t_{CPU} a/c's state should be used as an initial point for RT calculation. For example, for PentiumII-class processors it means no more than one-second prognosis for 60-s manoeuvre. Unfortunately today's onboard computers are much slower than Pentium-type ones. Should we wait until the last one will be available on board, or it is possible to employ the proposed DMRP right away? The answer is 'yes', we can use it on today's a/c, but we have to employ a known idea of initial-guess database.

2.5. Trajectories Databases Employment

The idea of convergence robustness improvement through the employment of databases of initial guesses is fairly clear. If some kind of TB, containing the values of OPs for a certain number of node trajectories, would be available on board computer, then we could use it to obtain an initial guess of OPs for the current two-point boundary-value problem (see Fig.11a). The developed DMRP allows creating these TBs due to two main reasons. First, the entire trajectory is defined only by a few parameters. Second, these parameters have a clear physical sense, and they change smoothly with a change of boundary conditions and constraints' rigidity.

Fig.11b demonstrates an example of effectiveness of TB employment. Suppose we have two node trajectories T_1 and T_2, defined by sets of OPs $\vec{\Xi}_1$ and $\vec{\Xi}_2$, and differ from each other only by state x_{20}. Then as initial guess for an arbitrary trajectory with $x_{20}^* \in \left[x_{20}^{T_1}; x_{20}^{T_2}\right]$, we can use the vector $\vec{\Xi}^* = (1-\rho)\vec{\Xi}_1^{opt} + \rho\vec{\Xi}_2^{opt}$, where $\rho = \dfrac{x_{20}^* - x_{20}^{T_1}}{x_{20}^{T_2} - x_{20}^{T_1}}$. As it turns out, a trajectory calculated with the use of $\vec{\Xi}^*$ and a trajectory finally optimised with the use of this initial guess practically coincide, meaning that the interpolated vector $\vec{\Xi}^*$ is fairly close to the optimised one $\vec{\Xi}^{opt}$ (this means that $\vec{\Xi}^*$ is a good guess for $\vec{\Xi}^{opt}$).

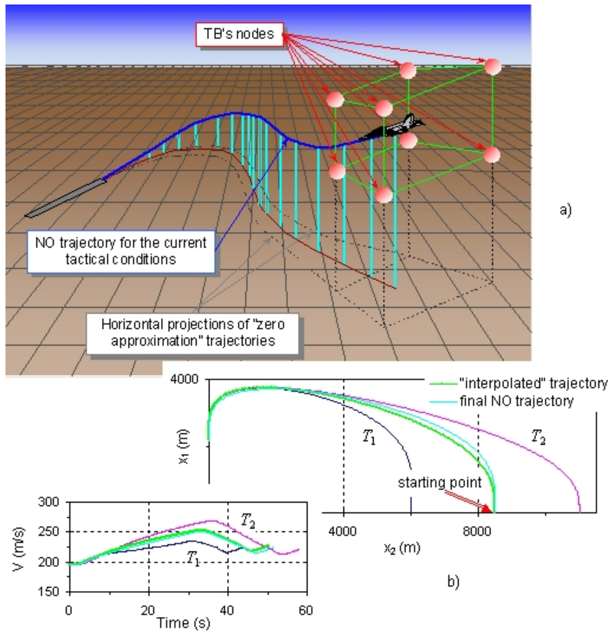


Fig.11 Illustration of TB usage

In general case, there is more then one input parameter; therefore a multi-parametrical (multi-entry) interpolation is used. The only question is what set of variables should be used as an entry parameters, and how many nodes for each of them should be established. Obviously, the more nodes for each entry parameter TB has, the better the convergence robustness is. However in general, there are too many entry parameters: initial and final values of state variables, controls constraints, a/c and its engine characteristics, atmospheric conditions. As performed research shows, the DMRP is more sensitive to changes of only 13 of those parameters and it is enough to have from two to six nodes to improve the convergence of algorithm by the factor of three (e.g., for an IBM486-type processor it means $\bar{t}_{CPU} \le 1\%$). It was found that the values of the initial and terminal azimuth angles are the most influential. For this reason, TBs should have the largest mesh point frequency for these parameters, whereas for the majority of other parameters it was sufficient to have only two nodes: for minimum and maximum possible (expected) values. Consequently, the required RAM volume to keep the OPs for all varieties of those entry parameters is reasonable. For instance, the TB for onboard computation of SBTA-type trajectories using an IBM486-type processor contains 47,040 trajectories and covers (with respect to the target) any initial and final azimuth angles, initial and final velocity in the range of [170;250]*m/s*, initial range up to 15,000*m*, final range within [1,800;3,600]*m*, initial altitude within [200;1,000]*m*, final diving angle within [10;30]º, any operative constraints on controls. To store the values of three OPs and the value of CF, it requires less then 1Mb RAM. For other stages of flight like CLA or take-off/climb because of a/c restriction to a runway, the number of TB's node trajectories is approximately eight times less.

Another good thing about TB - it ensures a unit probability of the NO trajectory computation for a certain time due to its construction. Moreover, before implementation on board, it can and has to be cross-checked in a series of intermediate points.

2.6 Waypoint flight

A little bit different modification but with the use of the same basic principles was designed for the solution of the tasks with "soft" (penetrative) restrictions on state co-ordinates like a waypoint flight through the field of prohibited zones of plight (PZF) (air threat zones, thunderstorm fronts, etc.) [19].

In this modification instead of pre-setting a thrust history, the separate RF for speed was employed. Following the basic idea of DM, the search of NO trajectories was made in the class of global cubic splines passing through the set of waypoints, location of which have to be optimised. Fig.12 explains the essence of this modification.

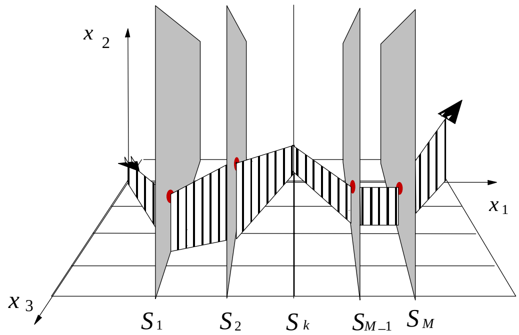


Fig.12 Illustration of spline-variations method essence

Generalised function of losses, compound of accumulated damage and a total fuel consumption during entire flight (with coefficient *k*), was considered as a CF for this kind of manoeuvres. The real values of coefficient $k \in [10^{-5};10^{-3}]$ factually lead to neglecting of the functional losses, but in case of lack of PZF we automatically receive a NO solution for the fuel-minimum flight. Because of polymodal nature of the CF in this task, Strongin's information-statistical method [20] was employed in order to find an area of global extremum attraction. To assure the required robustness a zero-order method was employed then for a final local optimisation.

Fig.13 shows an example of a route optimisation in presence of PZF field, formed by five stationary air threat zones, at variation of different number of states (free and fixed speed history). Terrain features are also accounted here. Fixing one of the states (some concrete tasks might require certain altitude or speed profile), we can receive different sub-optimal solutions. It demonstrates a flexibility of the designed DM.

Fig.14 demonstrates one more example of spatial trajectory optimisation in condition of flight through the complex PZF field, which besides three fixed circle-type zones and one triangle-type zone, contains also a moveable PZF, crossing the line of pre-set a/c path with a constant speed.

Optimisation algorithm turned to be fairly sensitive to a/c performance (it smartly uses the features of a concrete a/c dynamics). Where a condition of non-growth of damage permits, a/c flies on fuel-optimum regime.

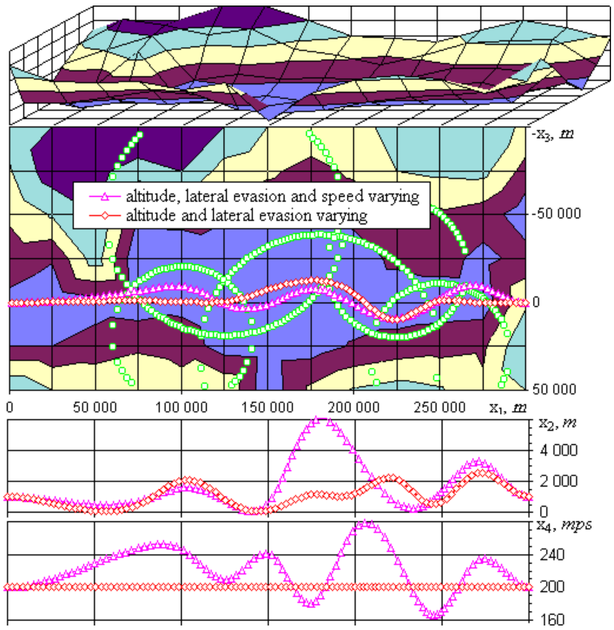


Fig.13 Examples of NO waypoint flight

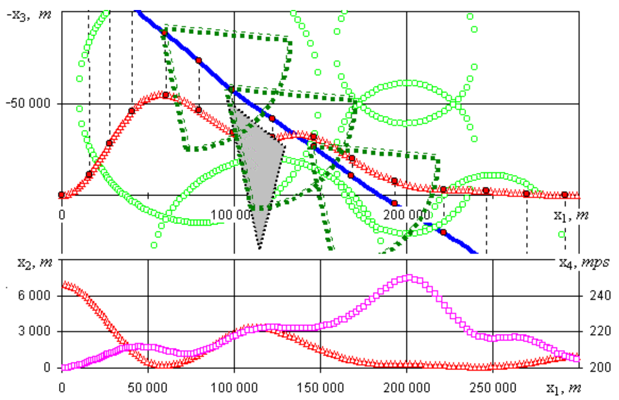


Fig.14 Route optimisation in presence of moveable PZF

Run time for this modification depends on task dimensionality, making in average of $\bar{t}_{CPU} = 5-10\%$ for IBM466-type computer.

3 Testing and onboard implementation experience

Proposed PSS's algorithms were tested not only in PC simulation, but also in real simulation with pilots.

3.1 Research simulator

A semi-natural experiment was mainly done with the use of adoptive cockpit-transformed flight simulator (SpMATS) on the modern multi-regime fighter [21].

SpMATS represents a complex consisting from the real a/c, local network of two field PC linked to onboard computer, and FD (Fig.15). First PC carries out a modelling of an a/c's 6-DOF dynamics (an object-oriented approach allows to quickly change an a/c type), synthesis of cockpit's instrumental panel, HUD images and synthetic outcabin world, as well as PSS prototype functioning. SpMATS functioning requires an a/c to be under current with switched-on objective control system (OCS) and released control column (without start of engines). The pilot, sitting inside the a/c cabin, with the help of FD observes synthetic world, targets, basic indicators, and HUD images corresponding to a current flight mode and simulation/training task. He operates with the a/c and armament control system models by usual controls. Modelling PC takes these controls inputs from onboard PC (OCS).

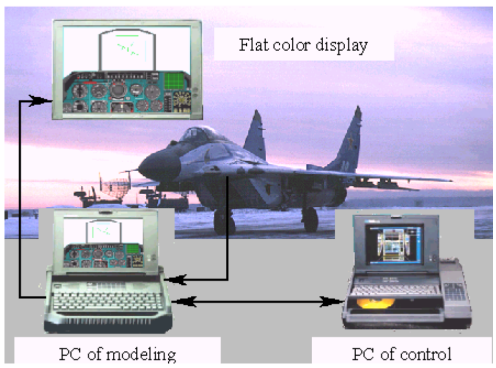


Fig.15 SpMATS block-scheme

An instructor/researcher uses the second PC for the management of simulation/training process. He can not only supervise the training process, but also participate in it as instructor (co-pilot) or opponent. He also can introduce any disturbances, a/c' systems failures and fictive damages, new targets, etc. Both PC are linked through the standard bi-directional port RS-232 and use a specially developed protocol.

LCD-based FD is installed with the help of rubber bandage in front of HUD (Fig.16).

3.2 Semi-natural experiments

During testing with the help of SpMATS pilots were asked to observe different NO RT, which were generated on-line for real tactical conditions (Fig.17 shows examples of FD formats). RiS was equipped with appropriate wayside signs, like “Throttle-up”, “Throttle-down”, “Fire”, “Drag flaps”, “Gears”, etc.

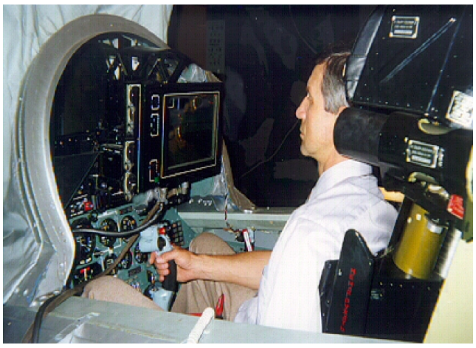
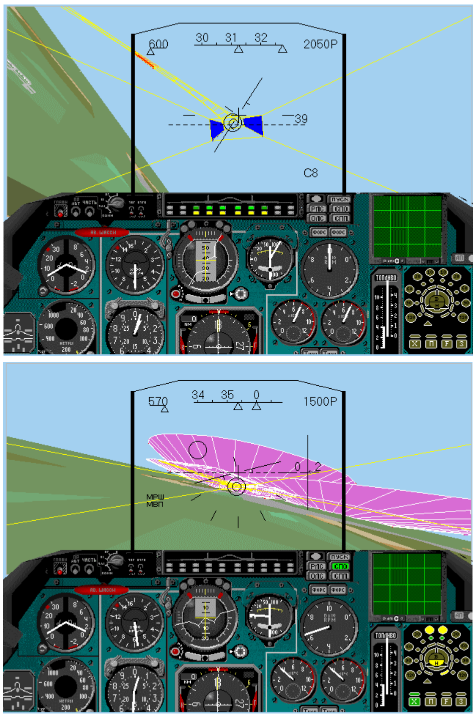


Fig.16 FD accommodation



a) b)

Fig.17 FD formats: a) SBTA, b) waypoint flight

In total 14 pilots participated in the semi-natural experiment on PSS’ algorithms testing [22]. Some estimates and recommendations were also given by test-pilots Pugatchev, Bichkov, Kutuzov, Votintzev, Solovjov, and Tzoj. Totally there were analysed over a hundred and fifty realisations of different manoeuvres. For each type of manoeuvres an accuracy of trajectory observation, an accuracy of terminal conditions satisfaction, a character of pilot’s control actions, and dependence on flight regime were analysed. The PSS’s and RiS’s parameters, e.g. RiS’s cross-section’s size, RiS’ type (gutter, tunnel); image type (real, compressed), etc. were analysed and optimised also. Of course all experiments were carried our with the model of real atmosphere.

The vigorous (high-G) trajectories, specifically SBTA and pursuing-evading trajectories, as expected, were the most difficult ones for tracking. However even for them, the accuracy of RT tracking was good enough: $I_D \sim 60m$, $I_{\Delta V} \sim -10m/s$, $I_{|\Delta n_y|} = 0.6$, $I_{|\Delta\gamma|} \sim 10^{\circ}$, $\Delta T \sim 3s$ (here $I_D = \frac{1}{T}\int_T \sqrt{\sum_{i=2}^{3}\left(x_i(t) - x_i^{ref}(t)\right)^2}\,dt$, T is a duration of manoeuvre, $I_{\Delta a} = \frac{1}{T}\int_T \left(a(t) - a^{ref}(t)\right)dt$).

Terminal conditions were also satisfied with a high accuracy (final attitude errors in vertical and horizontal planes were of order of two degrees).

Experiments proved that RTs can be effectively calculated on-line, and that RiS-image keeps all necessary information for their effective tracking. It was also confirmed that the pilots master a new control mode with the RiS assistance fairly quickly. Fig.18 shows one example when after the proper training the pilot could perform a manoeuvre, which is closed to the NO one, himself, without prompt (it means PSS paradigm may be used also for pilots’ training). The spectral analysis of control actions proves pilots’ subjective opinion that there is no additional workload because of this new kind of indication.

Obviously, the accuracy of tracking depends upon RiS’s cross-section dimensions (Fig.19) and RT curvature. That is why for the CLA trajectories the averaged tracking accuracy was much better - $I_D \sim 10m$.

The final errors at glide-path capture made the order of 5*m/s* in terminal speed, ~ 4*m* in lateral deviation, ~2*m* in altitude, ~2° and ~1° in terminal pathangle and azimuth. Obviously they did not affect on the subsequent gliding and landing.

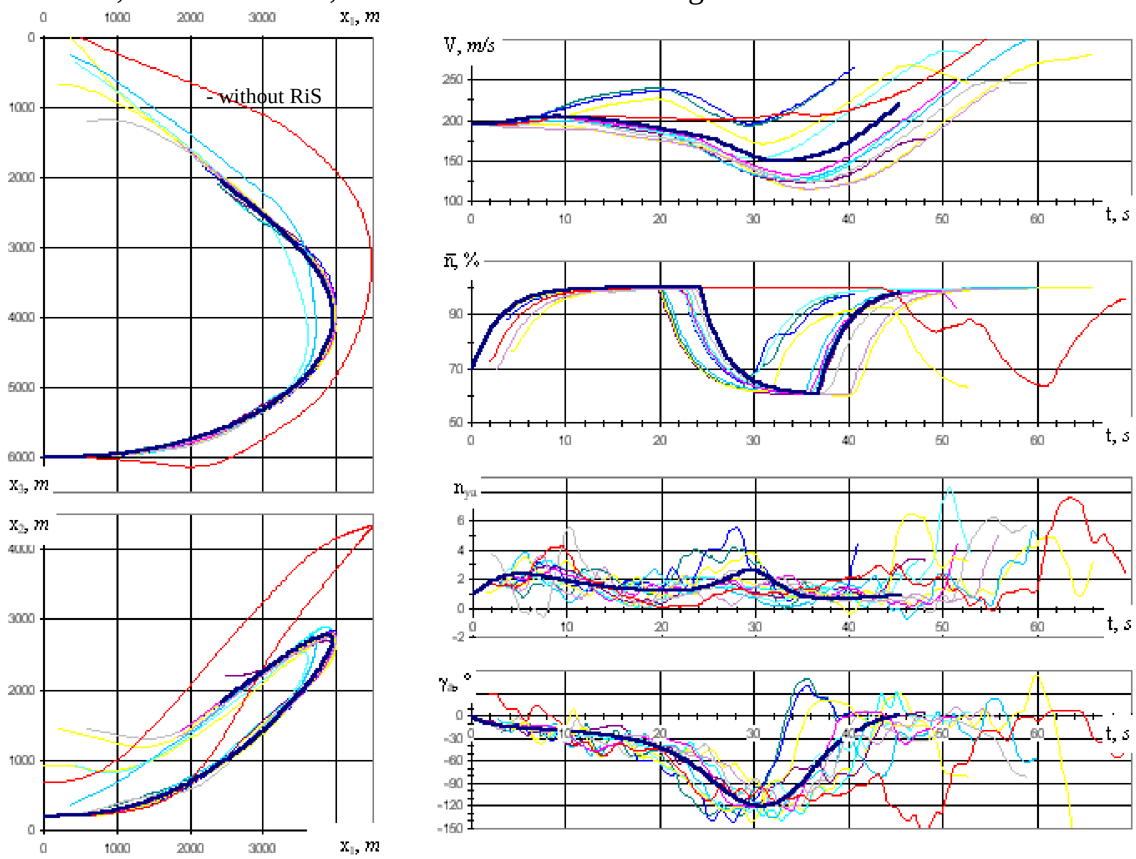


Fig.18 SBTA trajectory tracking

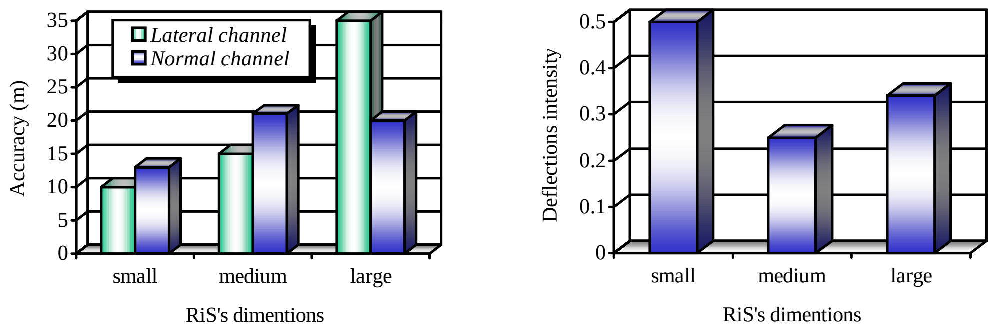


Fig.19 Accuracy and intensity of pilot's control actions

3.3 Flight test

Finally, designed software was flight-tested for the CLA trajectories by the test pilots Birjukov and Kazin onboard the flying laboratory An (Antonov)-72 equipped with Glonass and Navstar/GPS data receivers in the Gromov Flight-Test Institute [23]. FD was placed in front of pilots over the instrument panel. On-line RT computation was provided onboard by the separate IBM486-type computer.

It was shown that proposed FD with PSS’ algorithms allowed performing CLA with the high accuracy and pilot’s confidence. Pointing a prognosis marker along the RiS provided the movement inside the corridor by light adjusting control actions. Operating by the throttle, the pilot provided the required decrease of airspeed.

Several CLA were performed in twilight condition and with strong wind turbulence. Fig.20 shows an example of final part of CLA in condition of lateral wind transition of 10*m/s*.

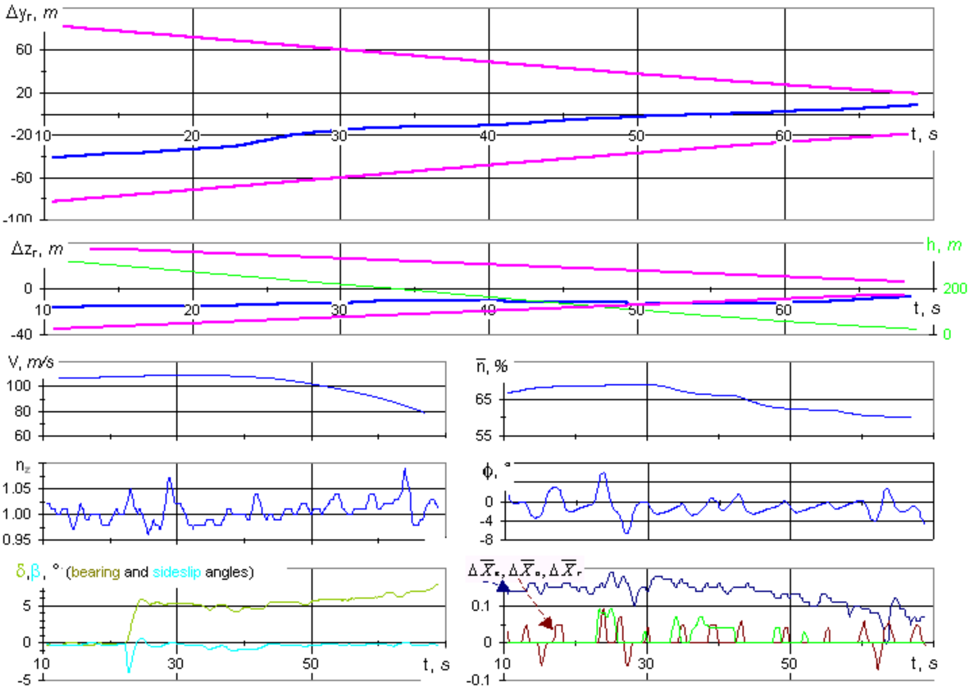


Fig.20 Example of final straight-in approach with lateral wind

Concluding Remarks

In conclusion it can be stated that the developed DM solve optimisational boundary-value problems very effectively. Resulting algorithms that have an excellent convergence robustness can be easily integrated with existing navigation/control algorithms, making possible the on-line prototyping of spatial manoeuvres for their following tracking by a pilot with the use of FD. The high efficiency and necessity for pilots of such kind of support was convincingly proved in all carried out semi-natural experiments and flight test. Therefore, performed research gives the grounds for further proceeding with implementation of PSS as a part of PA onboard an a/c in order to support the lowest levels of pilot's decision making.

References

[1] Stein K. "DARPA Stressing Development of Pilot's Associate System; Wright Laboratories Broadens Advanced Technologies Initiatives," *Aviation Week and Space Technologies*, Vol.122, No.16, pp 69-74, 77-84, 1985.

[2] Winter H et al. "Intelligent Decision Aids for Human Operators," *AGARD Symposium on "future Aerospace Technologies in the Secure of the Alliance*, Palaisean, France, 1997.

[3] Khatwa R and Helmreich R. "Flight Safety Foundation Approach-and-Landing Accident Reduction Task Force Analysis of Critical Factors During Approach and Landing in Accidents and Normal Flight: Data Acquisition and Analysis Working Group Final Report," *SAE&AIAA World Aviation Congress Proceedings*, San Francisco, CA, 1999.

[4] Yakimenko O. "Pilot Requirements to Universal Airborne Intelligent Pilot's Decisions Support System," *Proceedings of IEEE National Aerospace and Electronics Conference*, Dayton, OH, pp 446-452, 1995.

[5] Quinn T. "Pictorial Formats Literature Review," *AFWAL-TR-81-3156*, Flight Dynamics Laboratory, Wright-Patterson AFB, OH, 1982.

[6] Theunissen E. "Integrated Design of a Man-Machine Interface for 4-D Navigation". *PhD Thesis*, Delft University Press, 1997.

[7] Filarsky S and Hoover G. "The command flight path display," *Naval Air Development Centre*, Warminster, PA, 1983.

[8] Yakimenko O. "Realization of Airplane Piloting along the Spatial Near-Optimal Trajectories in Director Control Mode with Sight within the frame of Pilot Intellectual Support System," *Airspace Techniques and Engineering*, No.4, 1998 (in Russian).

[9] Betts J. "Survey of Numerical Methods for Trajectory Optimisation," *Journal of Guidance, Control, and Dynamics*, Vol.21, No.2, pp 193-207, 1998.

[10] Taranenko V. "Experience of Employment the Ritz's, Poincare's, and Lyapunov's Methods for Solving the Problems of Flight Dynamics," Air Force Engineering Academy n.a. Prof. N.Zhukovskiy Press, Moscow, 1968 (in Russian).

[11] Taranenko V and Momdzhi V. "Direct Variational Method in Boundary Value Problems of Flight Dynamics," Mashinostroenie Press, Moscow, 1986 (in Russian).

[12] Neljubov A. "Mathematical Methods of Calculation of Combat, Takeoff/Climb, and Landing Approach Manoeuvres for the Aircraft with 2-D Thrust Vectoring. Flight Characteristics and Combat Manoeuvring of Manned Vehicles," *AFEA Press*, Moscow, 1986 (in Russian).

[13] Hargraves C and Paris S. "Direct Trajectory Optimisation Using Nonlinear Programming and Collocation," *Journal of Guidance, Control, and Dynamics*, Vol.10, No.4, pp 338-342, 1987.

[14] Enright P and Convay B. "Discrete Approximations to Optimal Trajectories Using Direct Transcription and Nonlinear Programming," *Journal of Guidance, Control, and Dynamics*, Vol.15, No.5, pp 994-1002, 1992.

[15] Seywald H. "Trajectory Optimisation Based on Differential Inclusions," *Journal of Guidance, Control, and Dynamics*, Vol.17, No.3, pp 480-487, 1994.

[16] Lu P. "Inverse Dynamics App roach to Trajectory Optimisation for an Aerospace Plane," *Journal of Guidance, Control, and Dynamics*, Vol.16, No.4, pp 726-732, 1993.

[17] Yakimenko O. "Direct Method for Rapid Prototyping of Near-Optimal Aircraft Trajectories," To be published in *Journal of Guidance, Control, and Dynamics*, 2000.

[18] Dobrokhodov V and Yakimenko O. "Synthesis of near-optimal evading of the aircraft inducing the quickest loss of pursuer's velocity and maneuverability," *A Collection of Technical Papers of AIAA Atmospheric Flight Mechanics Conference*, Portland, OR, pp 568-575, 1999.

[19] Alekhin D and Yakimenko O. "Synthesis of Optimization Algorithm for Planned-Route Flight Trajectory by Direct Variational Method," *Journal of Computer and Systems Sciences International*, Vol.38, No.4, pp 650-666, 1999.

[20] Strongin R and Markin D. "Minimization of Multiextremal Functions Under Nonconvex Constraints," *Cybernetics*, Vol.22, No.4, pp 486-493, 1987.

[21] Firsov A and Yakimenko O. "Mobile Adaptive Training System for Pilot's On-Board Pre-Flight Preparation and Tactical Thinking Training," *Proceedings of 10th ITEC*, The Hague, The Netherlands, 1999.

[22] Yakimenko O and Demidov V. "Use of a Cockpit-Transformed Simulator for Testing of Proposed Algorithms of Pilot's Control Actions During Aircraft Maneuvering Support System," *A Collection of Technical Papers of AIAA Modeling & Simulation Technologies Conference*, Portland, OR, pp.160-170, 1999.

[23] Yakimenko O, Krjuchkov L, Demidov V, et al. "Development and Testing of an Integrated Display with a Cognitive Guidance Prompt for Improving the Safety of Landing Operations," *Proceedings of Advances in Aviation Safety Conference*, Daytona Beach, FL, 2000.