PROBABILISTIC WAKE VORTEX SAFETY ASSESSMENT TO EVALUATE SEPARATION DISTANCES FOR ATM OPERATIONS

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Keywords: *Wake vortex evolution, Wake encounter, Safety assessment, Separation distances, Risk based policies, Single runway approach, Air Traffic Management*

Abstract

The steady increase in air traffic imposes a need for enhanced airport capacity, and the desire to safely reduce existing separation minima. An important limiting factor in establishing required separation minima is wake vortex induced risk.

A novel probabilistic methodology is under development for the assessment of wake vortex induced accident risk. The methodology is integrated within a stochastic framework. Three probabilistic sub models are being used:

- *Wake vortex evolution model;*
- *Wake encounter model;*
- *Flight path evolution model.*

This probabilistic methodology can be used for an assessment of wake vortex safety of different ATM concepts or procedures. It provides a tool to evaluate separation distances for the current practice, and for promising new ATM concepts which may enable a safe reduction of current separation minima. Numerical results can be fed back to ATM designers, who can use these results to redesign or improve their proposed ATM concept.

The safety management approach to regulate and control wake vortex induced risk can, and should, be based on an assessment of accident risk probabilities, followed by a comparison with risk criteria. Some guidelines for the development of a risk criteria framework, to be agreed upon by involved interest groups, are given.

This paper outlines the probabilistic methodology, and illustrates its initial application for the single runway approach under current flight regulations

1 Introduction

With the steady increase in air traffic, there is an urgent need to use existing and newly proposed technologies in an efficient way. This is reflected in the design of new high capacity aircraft and new advanced ATM concepts and procedures. However, it is also recognized that safety is a key quality that should be guaranteed. In particular the wake vortex problem becomes more important, for example at the busiest airports where incidents (attributed to wake vortex encounters) are reported by pilots, and where there are closely spaced parallel runways and no publicly acceptable extension possibility.

This requires tools and methods to enable a quantitative assessment of wake vortex safety. In view of the uncertainties and the difficulties in understanding of the wake vortex phenomena, this paper proposes a *probabilistic* approach.

To support the design of new aircraft and new advanced ATM concepts, a probabilistic wake vortex induced risk model has been developed. The model is based on a stochastic framework that incorporates the following models:

- Wake vortex evolution model;
- Wake encounter model;
- Flight path evolution model.

The model can be used to evaluate the separation distances for the current practice, and for promising new concepts that may enable a safe reduction of the current separation minima. Identified key safety bottlenecks can be fed back to the ATM designers, who can use these results to redesign or improve their proposed ATM concept.

The current separation minima stem from the early 70's. Although over the last 30 years they have 'proven to be sufficiently safe', the current safety level is unclear and there is a deficiency of tools and methods to determine how to bring into account new developments in operational usage at busy airports.

The proposed modeling approach aims at solving this deficiency. In order to allow for a sufficient level of validation, this approach is applied to conventional single runway situation in this paper.

This paper is organized as follows. Section 2 describes some procedural aspects and requirements relevant for the single runway approach. Section 3 contains guidelines for the development of a risk criteria framework, which is the first step towards risk based policies. In Section 4 the probabilistic wake vortex induced accident risk model is described. Section 5 presents initial numerical results for the single runway approach under current flight regulations. The conclusions and recommendations are given in Section 6.

2 Single runway approach procedure and requirements

2.1 Separation minima

Provisions governing wake turbulence separation minima are published by ICAO [18, 19, 20], and depend on the weight classes of the involved aircraft and the available equipment (e.g. radar or non-radar operations).

The separation minima are based on categories, determined by different aircraft take off weight classes. For aircraft approaching a single runway under radar supported operations, the separation minima as recommended by ICAO are given in table 1.

Table 1 ICAO Separation minima

L / F	Heavy	Medium	Light
Heavy	4	5	6
Medium	-	3	4
Light	-	3	3

Wake turbulence separation minima are not prescribed to VFR approaches, nor to IFR on visual approach. Under these circumstances, it is up to the pilot to guarantee separation with other aircraft.

2.2 Operational requirements

According to available facilities (e.g. ground and onboard equipment), a variety of instrument procedures have been developed to guide the aircraft safely to the runways during Instrument Meteorological Conditions (IMC). In general, an instrument procedure may have five segments: arrival, initial, intermediate, final and missed approach, as sketched in Figure 1.

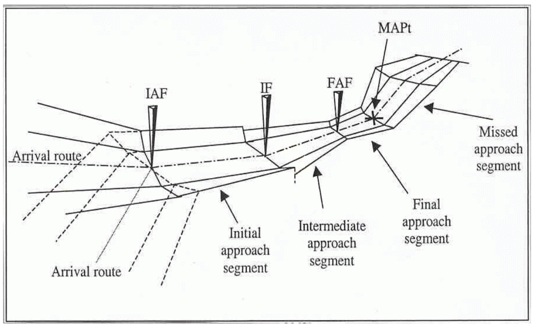

Figure 1 Instrument approach segments

This paper only considers usage of ILS, the presently most common procedure. A detailed description of ILS procedures can be found in the PANS-OPS [18].

It is clear that not only the physical evolution of wake vortices is relevant, human involvement is an important element to be taken into consideration as well. Two important actors are:

- ATC, who is responsible for aircraft separation, including informing pilots to avoid encounters.
- Pilots, who under good visibility conditions may envision the vortices location, and adjust their flight path.

Some operational recommendations to pilots to avoid wake vortices under good visibility conditions are [20, 21]:

- Landing well beyond the preceding larger aircraft touchdown point;
- Passing over flight path of preceding aircraft, or at least 1000 ft under;
- Staying upwind of preceding aircraft flight paths;
- Extra vigilance on calm days when vortices persist longer.

Besides the above preventive measures, a pilot can take counter control actions or initiate a missed approach when he/she experiences a slight roll upset, and to try to minimize the consequences.

3 Risk based policy making

Safety assessments should be expressed in metrics that 'convey the risks clearly to the decision makers, in a way that builds on the safe foundation incorporated into the design of the existing system and also in a form that can be incorporated into cost/benefit assessments' [30]. It is proposed [24] that 'risk characterization should be a decision-driven activity, directed toward informing choices and solving problems. Moreover, it was found [22, 23] that the manner in which risks are expressed has a major impact on people perception of safety and their behavior.

This stresses the importance of proper risk characterization and consequently using suitable and agreed upon risk metrics. Below some initial guidelines are given for the development of a risk criteria framework. In addition, it is discussed how to proceed towards risk based policies that are agreed upon by the involved interest groups.

3.1 Identification of risk metrics

Up to now several technical metrics have been used in research studies to quantify the hazard imposed by wake vortices: e.g. bank angle, roll angle, roll rate and roll control ratio. Unfortunately, it is not sufficiently clear how these wake encounter type of metrics are related to the safety perception of most involved interest groups (i.e. human operators, regulatory authorities, ATM developers, human society, passengers, controllers). In order to improve this situation, one should develop a probabilistic relation between the occurrence of wake vortex encounter severity and risk metrics that are related to the severity of accidents, incidents and related conditions.

For incident and accident investigation purposes, ICAO consequence definitions are [31, 32]:

- Accident.
- Serious incident, for an incident involving circumstances indicating that an accident nearly occurred.
- Non-serious incident;
- Not determined incidents.

For safety assessment purposes, JAA has defined severity classes for adverse conditions [33]:

- Catastrophic condition;
- Hazardous condition;
- Major condition;
- Minor condition.

The above two classification schemes can be combined into a classification of wake vortex induced consequences as follows:

1. *Catastrophic accident:* the aircraft encountering a wake vortex hits the ground, resulting in loss of life;
2. *Hazardous accident*: the wake vortex encounter results in one or more on-board fatalities or serious injuries (but no crash into the ground);
3. *Major incident*: the wake vortex encounter results in one or more non-serious injuries, but no fatality, on-board the encountering aircraft;
4. *Minor incident*: the wake encounter results in inconvenience to occupants or an increase in crew workload.

The next step is to introduce for each of these four classes suitable risk metrics to regulate and control wake vortex induced risk, such as:

- Risk event probability per movement;
- Risk event probability per year.

3.2 Safety requirements

In Speijker *et al.* [15, 34] initial guidelines are developed for the assessment of safety requirements. Two possible safety management approaches are discussed: the Target Level of Safety (TLS) and the As-Low-As-Reasonably-Practicable (ALARP) approach. The basic idea behind these approaches is to divide the risk continuum into three respectively two risk judgement regions, as sketched in Figure 2.

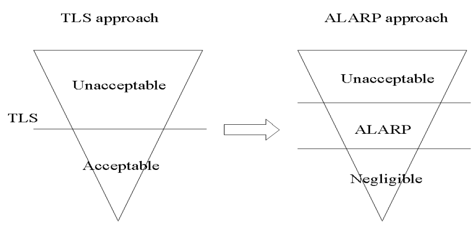

Figure 2 Possible risk criteria frameworks

The ALARP approach contains a tolerable region bounded by maximally negligible and minimally unacceptable levels of risk. Within the tolerable region the risk must be proven to be As Low As Reasonably Practicable (ALARP) in order to be acceptable [8, 9]. Cost-Benefit Analysis (CBA) is a method that can be used to demonstrate that any further risk reduction in the tolerable region is impracticable. Recently the development of the ALARP approach for use in aviation risk management has been investigated within the context of Reduced Vertical Separation Minimum (RVSM) in ECAC countries [8]. It was argued that a combination of these two safety management approaches might be beneficial to aviation risk management under certain conditions.

3.3 Towards risk based policies

The ICAO separation minima shown in Table 1 form a good example of the current prescriptive approach towards safety management in civil aviation. In two recent studies [35, 36] this problem and directions for improvement have been studied. In the RTCA study [35] emphasis was on developing improvements within the existing prescriptive approach towards safety. The resulting recommendations are largely addressing the issues to be addressed by authorities in order to improve the situation. In Blom and Nijhuis [36] the emphasis was on what could be learned by all parties concerned from experiences in other safety critical industries. A key learning example came from the offshore petrochemical industry. Safety policy was there based on a prescriptive approach. However, in reaction to the Piper-Alpha catastrophe in the North Sea in 1988, a major change in safety policy has been developed and introduced in the UK. The key change is the introduction of a goal-setting safety management approach (see Figure 3).

The basic idea is that safety monitoring and feedback to all management levels in an organization becomes standard practice, and that top management has the responsibility to agree with the authorities with respect to the safety goals and the safety monitoring and feedback mechanisms. Most remarkably, top management in offshore industry has become an active promoter of such goal-setting safety management approach.

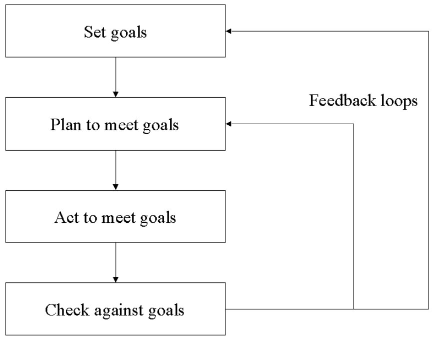

Figure 3 Goal-setting safety management

For wake vortices the adoption of a goal-setting safety management approach would give service providers the possibility to develop and optimize operations that account for technological developments and the local conditions of a particular airport (airport layout, meteorological conditions, equipment level, etc.) without the need to await authority initiatives. One of the important feedback tools is to start the building of modern safety cases for a new operation under development in collaboration with other stakeholders [37]. The key stakeholders to collaborate with are the major airport users, the airport service provider and the manufacturers of ground/airborne equipment and aircraft.

4 Safety assessment model

4.1 Risk assessment methodology

As the basis for the development of the WAke Vortex Induced Risk assessment (WAVIR) methodology use is made of the TOPAZ (Traffic Organization and Perturbation AnalyZer) methodology to assess accident risks for advanced ATM operations [7]. TOPAZ supports a spiral development cycle that is of the form:

A. Design of an ATM operational concept.
B. Assessment of the ATM concept, resulting in a cost-benefit overview.
C. Detailed analysis of the assessment results, resulting in recommendations to improve the ATM concept.
D. Review ATM concept development strategy and plan.
E. Back to A: adapted and/or more detailed ATM concept design using the results from C resulting in a new or optimised ATM concept.

The TOPAZ methodology is based on a stochastic modelling approach towards risk assessment and has been developed to provide designers of advanced ATM with safety feedback following on a (re)design cycle, see Figure 4.

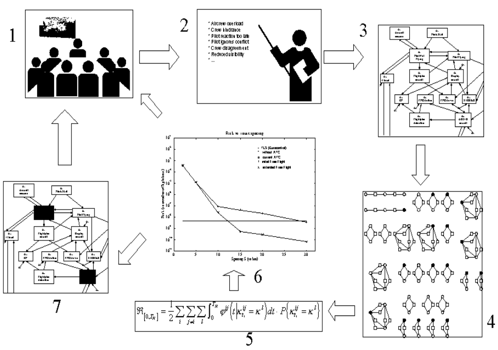

Figure 4 TOPAZ risk assessment cycle

During the assessment cycle four stages are sequentially conducted:

1. Identification of operation and hazards (upper box in Figure 4)
2. Mathematical modelling (lower right box in Figure 4)
3. Accident risk assessment (middle box in Figure 4)
4. Feedback to operational experts (lower left box in Figure 4)

For the second and third stages use can be made of different TOPAZ tool sets. For the assessment of wake vortex induced risk, the WAVIR and SIMULATOR tool sets are used.

4.2 Overview of the risk model

To determine the metrics for the possible wake vortex induced risk events, an appropriate safety assessment model is required. In view of the uncertainties and the difficulties in understanding of the wake vortex phenomena, it is proposed to follow a *probabilistic* approach.

This probabilistic model should enable evaluation of wake vortex safety under various operational and weather conditions. It should also be possible to evaluate the current practice as well as promising new concepts, such as new operational improvements, aerodynamic aircraft designs, or weather related separation minima. The approach should be able to cover the situation of a sequence of aircraft that fly towards different kinds of runway configurations.

Considering these requirements, three probabilistic sub models are integrated within a stochastic framework:

- Wake vortex evolution model
- Wake encounter model
- Flight path evolution model

An extensive literature survey [10] led to the selection of the deterministic wake vortex related sub models described in Section 4.4, 4.5, and 4.6. These models are probabilitised in three steps [15], and integrated in a stochastic framework [6].

Figure 5 below gives an overview of the main elements of the probabilistic safety assessment.

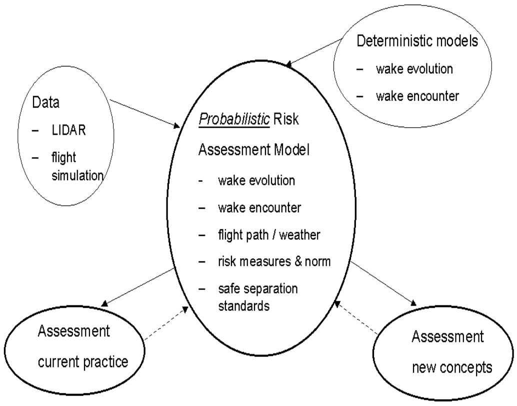

Figure 5 Overview probabilistic approach

4.3 Wake vortex risk assessment

To numerically assess wake vortex induced accident risk the models mentioned in Section 4.1 are integrated. The safety assessment is carried out in a seven-step procedure.

Step 1: The parameters in the wake vortex evolution model are identified and the parameter distributions are based on empirical data and/or state-of-the-art literature. In addition a set of relevant longitudinal positions *x* is determined.

Step 2: Run Monte Carlo simulations with the wake vortex evolution model for the case that the wake vortex is generated when the leading aircraft has longitudinal position *x*. The position, strength, and core radius of the wake vortex are obtained at the time instant that it has the same longitudinal co-ordinate as the trailer aircraft. The latter time instant follows from Monte Carlo simulations with the SIMULATOR tool.

Step 3: The simulation results from Step 2 are analysed. Based on this analysis a dedicated probability density fitting procedure is identified that accounts for dependencies between the position co-ordinates, the strength, and the core radius of the wake vortex. The probability density fitting procedure is carried out and the joint distribution of the wake vortex position, strength, and core radius is obtained.

Step 4: Monte Carlo simulations are carried out to simulate the wake vortex encounter. In this step the joint distribution from Step 3 is used and distributions of the position of the trailer aircraft obtained with the SIMULATOR tool set are used.

Step 5 concerns the numerical evaluation of the wake induced accident risk due to a wake vortex that is generated when the leading aircraft was at position *x*.

Step 6: The wake induced accident risk is obtained by maximising over *x* the risk obtained in Step 5.

Step 7: Perform a qualitative uncertainty analysis of the influence of modelling assumptions on estimated accident risk.

4.4 Flight path evolution model

The flight path evolution model yields the following stochastic variables:

- The lateral and vertical co-ordinates of the leader if its longitudinal co-ordinate *x* is given,
- The period of time elapsed between the generation of the wake and the time instant that the trailer has longitudinal position *x*,
- The lateral and vertical co-ordinates of the trailer when it has longitudinal co-ordinate *x*.

The flight path evolution model is a stochastic dynamical model, which incorporates the established ICAO-CRM [5] as baseline, and which has been further developed to handle the dependent usage of closely spaced runways [11]. This model is represented in a form [14] that allows a straightforward extension of the SIMULATOR tool set for new air and/or ground procedures and advanced vortex detection and decision-support systems.

4.5 Wake vortex evolution model

This section provides a mathematical description of the wake vortex evolution model, which accounts for stratification, atmospheric turbulence, ground effects (rebound, divergence) and crosswind (advection, shear) [1, 2,12]. It is extended with probabilistic wind field models to include the impact of wind in the vertical and lateral direction [6, 15, 29].

The model enables determination of the wake vortices motion, decay and strength in time at certain positions relative to the leader. This aircraft generates two counter rotating vortices of which the positions and strengths are to be determined. The positions are given relative to a rectangular xyz-coordinate system, with x-axis in longitudinal direction, y-axis in lateral direction and z-axis in the vertical direction.

The positions of the left and right centers of two vortices, are represented by $X_t^- = \{x_t^-, y_t^-, z_t^-\}$ and $X_t^+ = \{x_t^+, y_t^+, z_t^+\}$. The strengths of the vortices are denoted by $\Gamma_t^- \in \Re$ and $\Gamma_t^+ \in \Re$. The initial positions at time t=0 are denoted by X_0^- and X_0^+, and are determined by the three dimensional position of the center of the leader aircraft at time t=0. The initial strengths are denoted by Γ_0^- and Γ_0^+.

4.5.1 *Wake vortex strength and decay*

The basic equation of wake vortex decay is that the rate of change of circulation strength equals the sum of the rates of change of circulation due to viscosity, buoyancy, turbulence, and crosswind:

$$\frac{d\Gamma^\pm}{dt} = \left.\frac{d\Gamma^\pm}{dt}\right|_{visc} + \left.\frac{d\Gamma^\pm}{dt}\right|_{buoy} + \left.\frac{d\Gamma^\pm}{dt}\right|_{turb} + \left.\frac{d\Gamma^\pm}{dt}\right|_{crossw}$$

The rate of change for viscosity depends on vortex descent speed, viscous force coefficient (C_D), wake oval width (L_{wv}), angle between force and drift velocity of a vortex (θ_t), and the initial spacing between the vortices (b_0), and is equal to

$$\left.\frac{d\Gamma^\pm}{dt}\right|_{viscosity} = \frac{\left|\dot{X}_t^\pm\right|^2 C_D L_{wv} \cos\theta_t^\pm}{2b_0}$$

The rate of change for buoyancy force depends on the area of the wake oval (A_{wv}), the Brunt–Väisälä frequency (N), the descent distance of a vortex, the angle between the force and drift velocity of a vortex (θ_t), and the initial spacing between the two vortices (b_0), and is given by:

$$\left.\frac{d\Gamma^\pm}{dt}\right|_{buoyancy} = \frac{A_{wv} N^2 \left[z_t^\pm - z_0^\pm\right] \cos\theta_t^\pm}{b_0}$$

The rate of change for atmospheric turbulence depends on the rms turbulence velocity (q), the vortex circulation (Γ_t), and the initial spacing between the vortices (b_0), and is given by:

$$\left.\frac{d\Gamma^\pm}{dt}\right|_{turbulence} = -0.82\frac{q\Gamma_t^\pm}{b_0}$$

An effect of crosswind is the acceleration of the decay of the vortex with the opposite sign vorticity from the crosswind. The decay rate of the other vortex is not influenced significantly. This effect can be modeled by adding a term in the basic equation of wake vortex decay:

$$\left.\frac{d\Gamma^+}{dt}\right|_{crossw} = -\frac{2}{3} C_{DV}\sigma w_0 b_0 \quad \text{and} \quad \left.\frac{d\Gamma^-}{dt}\right|_{crossw} = 0$$

with C_{DV} the viscous coefficient caused by crosswind, the wind shear coefficient, and w_0 the crosswind magnitude at initial height z_0^+.

The initial value of the circulation at t=0 depends on the weight of the leader (W^i), the initial aircraft true airspeed, the initial spacing between the vortices (b_0) and the density (ρ) [13]:

$$\Gamma_0^{\pm} = \frac{W^i}{\rho b_0 [\dot{x}_0^i - \omega_{x,0}]}$$

The vortex residence time depends on two influencing phenomena: Crow instability and vortex bursting. An analytical model has been proposed that assumes bursting and linking to happen in time as a function of some meteorological parameters [1].

To better account for observed data, in WAVIR the probabilistic bursting and linking period is modelled independently of the vortex evolution and decay as a stochastic variable with a Rayleigh density, the mean of which is assumed equal to 50s. This density is depicted in Figure 6 together with empirical data for vortex residence period. It is assumed that the curve is independent of height, and is also valid at higher altitudes. This Rayleigh density modelling differs significantly from the theoretical probability density model of Kuzmin [3].

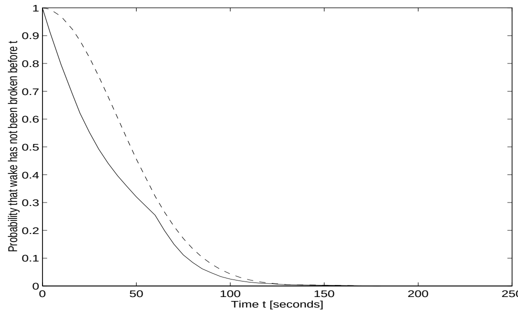

Figure 6 Solid line: observed residence time distribution for B-747 vortices with initial height 30 metres, initial strength of the wake of 600 m^2/s and wingspan 60 metres [17]. Dashed line: Rayleigh density adopted in WAVIR for vortex bursting or linking time

4.5.2 Wake vortex position

In order to determine the wake vortex induced rolling moment on an encountering aircraft j, the trajectories of the counter-rotating wake vortices are also required. Basic equations for these trajectories are provided by Corjon and Poinsot [1], thereby accounting for divergence and rebound effects. The model gives equations for the total induced velocity of primary and secondary vortices. These equations are modified to include the wind speed ω in all three directions. The equations from which the trajectories of the two counter-rotating vortices can be evaluated are given by:

$$\frac{dx_i}{dt} = \omega_x$$

$$\frac{dy_i}{dt} = \omega_y + \sum_{j\neq i}\frac{\Gamma_j}{2\pi}\frac{z_j - z_i}{r_{ij}^2} + \frac{\Gamma_i'' \cos\theta_i}{2\pi r_i''}$$

$$\frac{dz_i}{dt} = \omega_z + \sum_{j\neq i}\frac{\Gamma_j}{2\pi}\frac{y_j - y_i}{r_{ij}^2} + \frac{\Gamma_i'' \sin\theta_i}{2\pi r_i''}$$

$$\frac{d\theta_i}{dt} = \frac{\Gamma_i - \Gamma_i''}{2\pi [r_i'']^2}$$

$$r_{ij}^2 = (y_j - y_i)^2 + (z_j - z_i)^2$$

where i=1,2 and j=1,..,4. An explanation of the terms in these equations can be found in references [1], [2].

Of course, the wind field model has to be tuned for the airport situation. This can be done on the basis of statistical measurement based data.

4.6 Wake encounter model

This section provides a description of the used wake encounter model, consisting of two parts:

- A wake vortex interaction model;
- An aircraft control capability model.

The wake encounter model yields the probability that the wake vortex induced rolling moment is larger than the maximum control capability – in terms of rolling control moment – of the encountering aircraft.

4.6.1 Wake vortex interaction model

The wake vortex interaction model is based on Kuzmin [3]. The description of the deterministic version that has been probabilitised is given below.

The aircraft encountering the vortex alters, to some extent, the wake vortex flow field as generated by the leader. In general, one effect is to reduce the rolling moment as calculated with the wake vortex evolution model.

The vortex-induced rolling moment on the encountering aircraft j is modeled as a function of vortex strength and the distance between aircraft axis and vortex axis. The non-dimensionalised rolling moment is estimated for the situation with vortex axis parallel to the aircraft axis with the assumption of a rectangular wing, and is given by:

$$M^{j}_{induced}(t) = \frac{\Gamma^{\pm}_{t} C^{j}}{2\pi V_{t}^{\,j} b^{j}} F(\tilde{dy}, \tilde{dz})$$

The vortex-induced rolling moment depends on the flight speed of the encountering aircraft (V_t^j), its wing span (b^j), the vortex strength (Γ_t), the aircraft specific coefficient C^j , and a function F.

This function F, describing the influence of the distance between vortex axis and aircraft axis, is:

$$F(\tilde{dy}, \tilde{dz}) = 1 + \frac{\tilde{dz}}{2} \ln\left[\frac{\tilde{dy}^{2} + (1/2 - \tilde{dz})^{2}}{\tilde{dy}^{2} + (1/2 + \tilde{dz})^{2}}\right] - \tilde{dy}\left[\arctan\left[\frac{1/2 - \tilde{dz}}{\tilde{dy}}\right] + \arctan\left[\frac{1/2 + \tilde{dz}}{\tilde{dy}}\right]\right]$$

where the required input values of F depend on the distance between vortex axis and aircraft axis in lateral and vertical direction (dy and dz), the vortex core radius (r_{core}) and the wing span (b^j) of the encountering aircraft j, according to:

$$\tilde{dy} = \frac{\sqrt{r_{core}^{\;2} + dy^{2}}}{b^{j}}$$

$$\tilde{dz} = \frac{dz}{b^{j}}$$

For vortex core radius growth in time of vortices that did not have changed state by bursting or linking the following equation is used:

$$r_{core}(t) = \max(r_{core,t=0}, 0.0125\sqrt{\Gamma_{o} t})$$

Note that the vortex-induced rolling moment attains its maximum at distance equal to the vortex core radius from the vortex axis. This indicates that the majority of angular momentum is in the regions farthest from the core. Outside the core radius, the rolling moment is negligible.

4.6.2 Control capability model

The aircraft control capability model is based on Kuzmin [3] and Woodfield [4]. The deterministic version that has been probabilitised is described below.

The basic equation for the maximum roll control moment of an aircraft j depends on the wing span (b^j), the wing area (S^j), the air density (ρ), the aircraft true airspeed ($\dot{x}_t^j - \omega_{x,t}$), the maximum steady roll rate ($\hat{p}$), and the roll damping coefficient (C_{rd}^j), and is given by:

$$M_{control}^j(t) = -\rho \frac{S^j [b^j]^2}{4} \left[\dot{x}_t^j - \omega_{x,t}\right] C_{rd}^j \hat{p}$$

The equation for the maximum steady roll rate depends on encounter time (t_{enc}), bank angle ($\phi(t_{enc})$) and roll mode time constant (τ), and is:

$$\hat{p} = \frac{\phi(t_{enc})}{t_{enc} - \tau(1 - e^{-t_{enc}/\tau})}$$

A method for estimating $\hat{p}$ can be based on the minimum control capability requirements of aircraft rolling a certain bank angle within a certain period of time [3, 4]. Assuming that an aircraft performs two times better than such a requirement, and using the fact that the roll mode time constant is usually around 1 sec., $\hat{p}$ can easily be estimated.

The equation for the roll damping coefficient depends on the local lift curve slope of the wing (a^j) and the ratio between local wing chord (c^j) and standard mean chord ($\bar{c}^j$), and is given by:

$$C_{rd}^j = -4 \int_0^{1/2} a^j \frac{c^j}{\bar{c}^j} \left[\frac{y}{b^j}\right]^2 d\left[\frac{y}{b^j}\right] = -4 \int_0^{1/2} a^j \frac{c^j}{\bar{c}^j} y'^2 dy'$$

The roll damping coefficient clearly depends on the shape of an aircraft wing, thus reflecting the aircraft design in the developed risk model. To estimate this coefficient, some assumptions must be made regarding the shape of the aircraft wing.

5 Numerical evaluations

To illustrate the wake vortex induced accident risk assessment methodology a (single) runway is considered, on which a B737-400 aircraft, which is in the ICAO medium weight class, is landing behind a B747-400 aircraft, which is in the ICAO heavy weight class.

Three different scenarios, with controller expected separation distance of 4 Nm, 5 Nm, and 6 Nm when the heavy is at the threshold, are being considered. For both involved aircraft it is assumed that the approach is ILS Cat I.

5.1 Modeling assumptions

The landing phase starts at about 20 km before the threshold, and ends at touchdown, which is 300 metres beyond threshold. Figure 6 shows the side view of the runway and glide slope, where the *x*-axis is along the runway centerline and positive in runway direction.

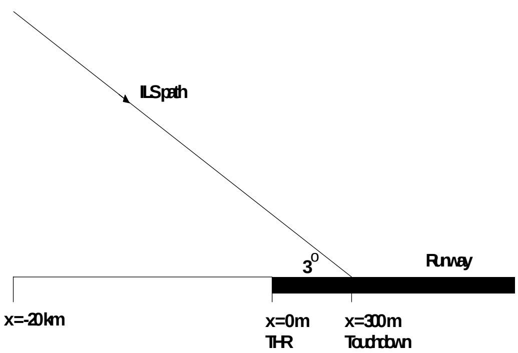

Figure7 Side view of runway and glide slope

The novel wake induced risk assessment methodology clearly allows to bring the assumptions made to the foreground. For this example, the following main assumptions have been adopted:

A.1. Long landings (landings far beyond threshold) do not happen.
A.2. A wake vortex induced accident is characterised by the wake induced rolling moment being larger than the aircraft control capability. The latter is assumed to be equal to two times the aircraft certification requirement.
A.3. A pilot does not initiate a missed approach when experiencing a slight roll upset.
A.4. Bursting and linking probabilities are modelled by a Rayleigh density with mean 50 seconds.
A.5. There is no head wind, no tail wind and no vertical wind. The wind speed in lateral direction is normally distributed with expectation 0 and standard deviation 1.5 m/s.
A.6. There are no wind shear layers.
A.7. Turbulence of the air is 10% of the wind speed.

In addition to these main assumptions, several other assumptions have been made. It would go beyond the scope of this paper to list all these assumptions.

5.2 Numerical results

With support of the toolsets WAVIR and SIMULATOR, the wake vortex induced risk is evaluated for the single runway approach. Monte Carlo simulations are performed for six cases, where the wake vortices are generated when the leader has distance 0, 400, 2000, 4000, 7000, and 17147 m from threshold. The latter is associated with a height of 3000 ft.

Below some plots are presented for the case associated with the controller expected separation distance of 5 Nm at the threshold. Similar data plots associated with the cases 4 Nm and 6 Nm are not included in this paper. Figures 8e, 8d, 8c, 8a show histograms with the lateral and vertical position, strength, and radius of the left vortex for wake vortices generated at 17147, 7000, 2000 and 0 m from the threshold.

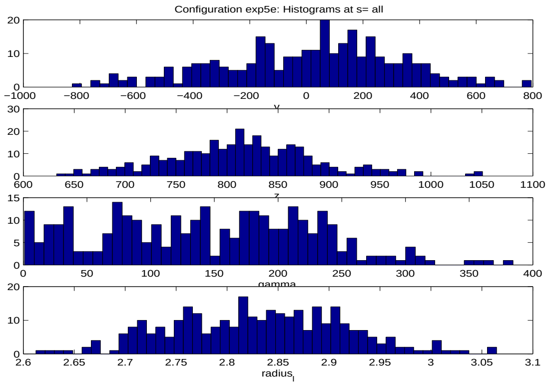

Figure 8e – 5 Nm Histograms, at x=17147 m

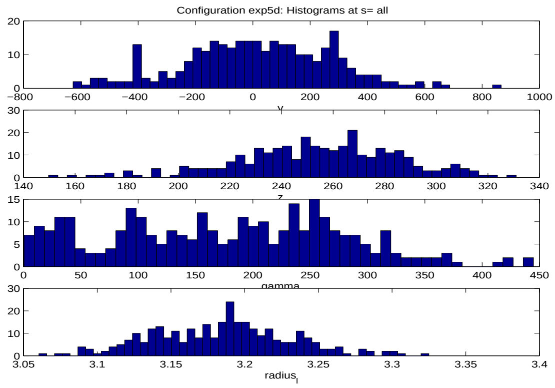

Figure 8d – 5 Nm Histograms, at x=7000 m

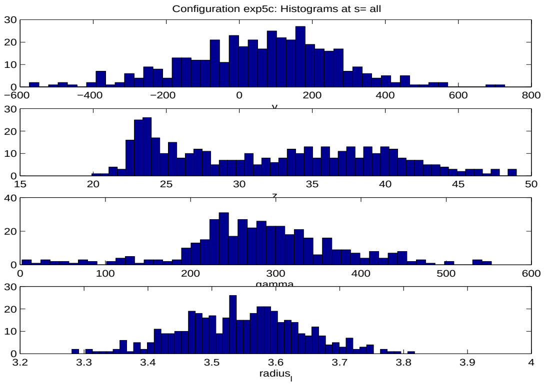

Figure 8c – 5 Nm Histograms, at x=2000 m

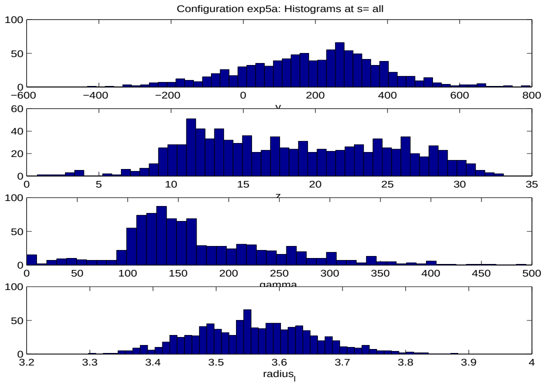

Figure 8a – 5 Nm Histograms, at threshold

The decrease in uncertainty about the position is for some part determined by the increase in navigation performance along the glide path. The navigation performance of the leader is based on the ICAO-CRM [5], and is incorporated in these figures. With increased navigation performance, the uncertainty about the wake vortex position will decrease. Note the impact of ground effect in Figure 8a.

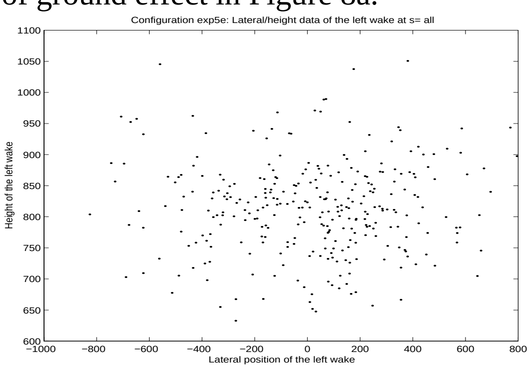

Figure 9e – Position data plot, at x=17147 m

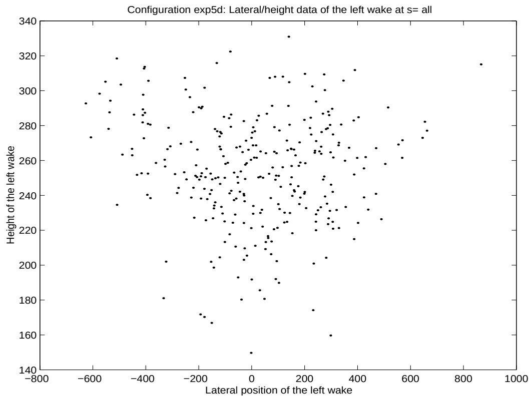

Figure 9d – Position data plot, at x=7000 m

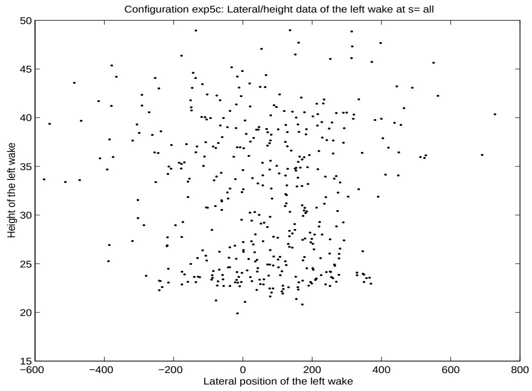

Figure 9c – Position data plot, at x=2000 m

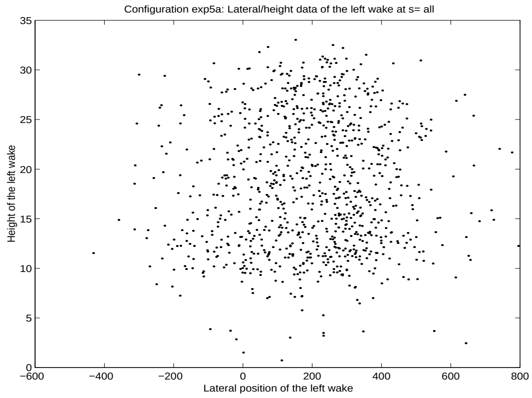

Figure 9a – Position data plot, at threshold

Figures 9e, 9d, 9c, 9a show the lateral position versus the height of the left vortex for the case associated with 5 Nm separation.

Figure 10 shows the instantaneous risk resulting from a wake that is generated at –*x* km before the threshold.

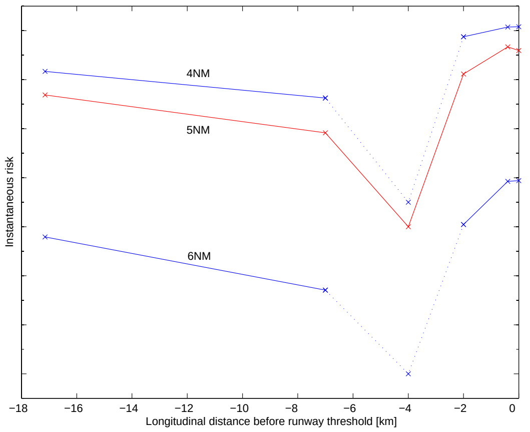

Figure 10 Instantaneous wake vortex induced risk along the glide slope. The vertical axis has a logarithmic scale.

Figure 10 shows that the instantaneous risk decreases from 20 km till about 4 km before the threshold. The decrease is due to the higher navigation precision of the trailer and leader by which the chances of flying such low to encounter a wake is reduced. At shorter distance from the threshold the instantaneous risk increases due to the rebound of wakes near the ground.

Figure 11 shows the wake vortex induced accident risk versus controller expected separation distance, when the heavy is at the threshold.

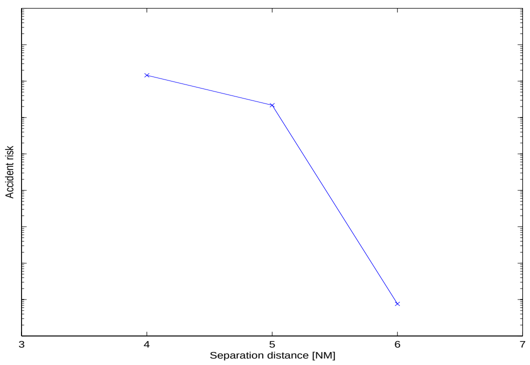

Figure 11 Accident risk versus separation. The vertical axis has a logarithmic scale.

Note that the risk decreases far more rapidly when the separation distance increases from 5 Nm to 6 Nm than from 4 Nm to 5 Nm.

5.3 Uncertainty analysis

A straightforward maximisation over x for the risk curves in Figure 9 leads to an overall maximum risk at the threshold. However, one should bring into account that the calculated wake vortex induced accident risk curve may bear significant bias and/or uncertainty both in positive and negative directions. Usage of such a curve without taking into consideration existing bias and/or uncertainty can inspire undue conclusions.

In order to understand the impact of the assumptions on the wake vortex induced risk, A.1-A.7 have been analysed in a qualitative way. The results are given in Table 2, where an optimistic expected direction means that the modelled risk reduces due to the assumption.

The effect of other assumptions on the wake induced accident risk has been estimated as either minor or negligible.

Table 2 Effect of the main assumptions on the assessed risk

A.#	Expected direction of effect on wake vortex induced accident risk	Expected magnitude
A.1	Optimistic	Significant
A.2	Neutral	Significant
A.3	Pessimistic	Major
A.4	Neutral	Significant
A.5	Optimistic	Significant
A.6	Optimistic	Major
A.7	Pessimistic	Significant

A similar reasoning as given in Kos *et al.*[29], shows that the very right and left part of the curve in Figure 9 have a major level of uncertainty with a clear bias in the pessimistic direction and optimistic direction respectively.

5.4 Discussion of the results

It has been shown that there are two areas along the approach where the risk can be considerable:

- Near the threshold, due to the ground effect on the wake evolution;
- At distances larger than 10 km from threshold, due to larger navigation errors further away from the runway.

This result is consistent with statistical analysis of data from the ETWIRL wake encounter database, see Figure 12 [28].

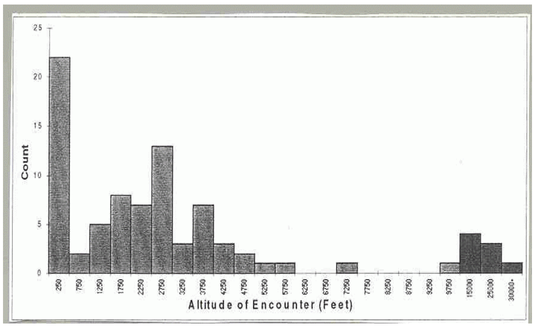

Figure 12 Histogram of encounter altitude

However, the absolute value of the wake vortex induced risk depends largely on separation distance. The risk decreases far more rapidly when the separation distance increases from 5 Nm to 6 Nm than from 4 Nm to 5 Nm.

The Rayleigh density for bursting and linking probability is to some significant extent responsible for decrease of the accident risk as separation distance increases. This implies that modeling the wake vortex residence time with appropriate and validated probability distributions is of major importance for wake vortex safety assessment. In particular, predictions of wake vortex residence times under different weather conditions should enable a more detailed analysis of the impact of weather on risk.

5.5 Influence of wind conditions

In the above, numerical evaluations have been carried out for the situation with no head wind, no tail wind and no vertical wind. Of course, although the results illustrate the methodology under development, in reality for most airports the wind conditions are more diverse. In the following the effect of wind on the wake vortex induced risk is discussed.

The horizontal wind model accounts for height dependency. It appeared that horizontal wind can have a major impact on the wake vortex induced risks. Head wind enlarges the effective vertical distance between the trailer and the wake vortex that has been generated by the leader, whereas tail wind effectively reduces this distance. Therefore, the risks that are evaluated with the WAVIR model are often higher for the case of tail wind than for the case of head wind. This corresponds to pilot experience. Detailed numbers depend on the ATM procedures at the airport (e.g. procedures for tail wind landings) and on the horizontal wind field. Strong crosswind may transport the wake vortex so that it is laterally far from the trailer, which reduces the wake vortex induced risks.

The vertical wind field model accounts for varying weather conditions. The strongest vertical wind speeds occur in case of a convective atmosphere. In this case, wakes can travel significant distances. In addition, the left wake can be in an upwind, whereas the right wake is in a downwind. Hence the distance between the left and right wake can become so large that they may be considered as isolated wakes. In a convective atmosphere there may be isolated wakes that stay at the height at which they have been generated (or they may rise). Since the wake vortex induced risks are mainly due to wakes generated at low altitude (near the threshold), the vertical wind is considered to be of minor importance for wake vortex induced risk.

5.6 Influence of weather

It is important to realize the major influence of specific weather conditions, in particular wind fields, turbulence, stable stratification and wind shear [16].

Generally vortex decay is enhanced in an ambient turbulence environment. Under stable stratification conditions, vortices will decay but may stall or rise. Wind shear, with weak turbulence and weak stable stratification may enhance stalling or rising vortices without significant decay. It was shown that vortices may stall or rebound to the glide path in the convective, stable stratified and sheared boundary layer [28]. Important from a safety point of view is that rising vortices have been observed at higher altitudes that cannot be explained by rebound and ground effect [25]. Interesting is also that vortices seem unaffected by uniform fog or rain [27].

Recent research focuses on the development of ATM concepts that may enable reduction of separation minima under certain weather conditions, so as to increase capacity. Examples are the High Approach Landing System (HALS) procedure, the Aircraft Vortex Spacing System (AVOSS) [26], and the Wake Vortex Warning System (WVWS).

An important first step towards implementing an operational weather based ATM concept to increase capacity, is the definition of, and agreement on, weather classes that allow evaluation of the wake vortex induced risk under different meteorological conditions.

6 Conclusions

6.1 Safety assessment

This paper describes a novel probabilistic WAke Vortex Induced Risk (WAVIR) assessment methodology. It can be used as a tool to evaluate separation minima for the current practice and for promising new ATM concepts that may enable a safe reduction of the current separation minima. The methodology incorporates:

- Wake vortex evolution model;
- Wake encounter model;
- Flight path evolution model;
- Risk criteria framework.

6.2 Single runway example

The methodology has been illustrated for a B737 landing behind a B747-400 aircraft, with expected separation distance equal to 4 Nm, 5Nm, and 6 Nm at the threshold. The results clearly show the high potential of the methodology towards risk based policies for safe and appropriate separation minima of existing and new ATM concepts or procedures. However, key modelling areas appeared that ask for increased research *before* adequate understanding of wake vortex safety can be reached. Summarized, these areas are [29]:

General modelling areas:

- Navigation performance and long landing models;
- Pilot reactions when experiencing a slight roll upset;
- Bursting and linking phenomena.

Airport specific modelling areas:

- Weather, including impact of stable stratification, wind shear, turbulence;
- Runway dependencies involving combinations of wake vortex

induced risk and collision risk between aircraft or with the ground.

Numerical results showed two areas along the single runway approach path where the risk can considerable:

- Near the threshold, due to rebound and ground effect of the vortices;
- At distances further than about 10 km from the threshold, due to larger navigation errors further away.

These results are in line with statistical analysis of data from e.g. the ETWIRL wake encounter data base [28].

6.3 Some recommendations

In support of the development of new inventive ATM concepts, a thorough wake vortex safety assessment that identifies the key safety bottlenecks should be carried out.

It is of major importance to incorporate the view of pilots and controllers at an early stage of the design and development of such an ATM concept. It is equally important that involved interest groups (a.o. regulatory authorities, pilots, controllers, safety analysts) agree upon a risk criteria framework to be used within risk based policy making.

6.4 Ongoing research

Apart from this single runway approach illustration, which has been carried out within NLRs basic research programme, also wake vortex induced risk related to the newly proposed High Approach Landing System (HALS) procedure at Frankfurt airport has been evaluated under contract to DFS. Since January 2000, NLR is leading the major three-year S-Wake project and is also involved in the related C-Wake project, both for the European Commission. Under co-ordination of the Thematic Network Wake Vortices (WakeNet), NLR is collaborating with key European wake vortex experts to further develop validated tools for assessment of wake vortex safety, and to define inventive solutions to cope with the risks induced by wake vortices.

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