MATHEMATICAL MODEL-BASED METHODS TO INVESTIGATE MANUFACTURING ANOMALIES

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Abstract

During the design of new aircraft systems, their manufacturing tolerances should be determined basically by working requirements of the system and technological possibilities of the manufacturers. Therefore the determination of manufacturing tolerances is a very important and difficult task.

It is possible that system parameter values are inadequate, but every unit and element of the system meets its own requirements. Because of the manufacturing tolerances of the units and elements have been determined incorrectly.

It is possible too, that the working requirements of the designed system have to determine strictly the system parameter tolerances. For example, these system requirements can be velocity or acceleration of the piston of aircraft hydraulic servo actuator, from the flight mechanical point of view. These external parameters should require very strictly the tolerances of internal parameters of the given system.

Using linearized mathematical diagnostic model of the given system, the problems mentioned above can be investigated and solved.

Their densities, expected values and variances can characterise manufacturing anomalies of internal parameters. These random characteristics determine the densities, expected values and variances of external parameters that is their manufacturing anomalies.

This paper will show the methodology of the linearized mathematical-diagnostic model's usage to investigate effects of manufacturing anomalies. Investigation methods and their inverse methods will be shown in case of the Gauss-distributions and in case of unknown (general) distributions of the internal parameters.

1. Methodology of the Mathematical Diagnostics

The setting up mathematical model should start from splitting up of the investigated system into its functional units. These units should be examined and the interdependencies between their input and output parameters should be established mathematically [8].

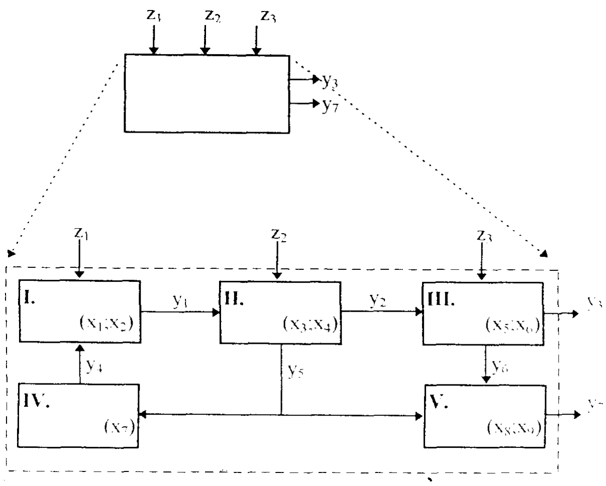

Figure 1.
Block-Diagram of a system

In the technical practice, the mathematical model can be written basically in two ways:

- WHITE BOX METHOD;

The model is written by analytical equation on the basis of scientific knowledge.

✈ BLACK BOX METHOD.

The model is written by analyzing of output parameters responded to known input ones.

The equations mentioned above form the model of the system consisting k units. This model can be written in general case:

$$
\begin{aligned}
f_1(y_1; y_2; \dots y_k) &= g_1(x_1; x_2; \dots x_p) \\
f_2(y_1; y_2; \dots y_k) &= g_2(x_1; x_2; \dots x_p) \\
&\vdots \\
f_k(y_1; y_2; \dots y_k) &= g_k(x_1; x_2; \dots x_p)
\end{aligned}
$$

or in simpler way: (1)

$$\mathbf{f(y)} = \mathbf{g(x)}$$

For setting up of the linear diagnostic model, the mathematical model should be linearized. For linearization following methods can be used:

✈ LOGARITHMIC LINEARIZATION;

Firstly, the natural logarithm (to e base) of both sides of the general non-linear equation(s) should be formed. Then the total differential of the logarithmic equation should be determined. Its usage is admissible basically in case of exponential (thermodynamic) equations.

✈ DIRECT DIFFERENTIATION;

As a first step, the total differential of both sides of the general equation(s) should be formed. As the next step, same sides of the general equation should multiply both sides of the last equation. This method is suggested if the general equation cannot be decomposed to multipliers.

✈ TAYLOR SERIES;

I this case Taylor-series of the general equation(s) should be developed and its more the first-order terms have to be neglected. This linearization method can be used if general equation can be derivable any times.

✈ LIE-MAGNUS SERIES.

Using a differential matrix, the Lie - Magnus series both sides of the general system of equations should be formed and more than first-order terms should be neglected.

System of equation got in this way describes interdependencies between relative changes of independent ($\delta \underline{x}$) and dependent ($\delta \underline{y}$) parameters in point of view of given investigation. This model can be written in the following matrix formula:

$$\mathbf{A}\delta\mathbf{y} = \mathbf{B}\delta\mathbf{x} \quad . \tag{2}$$

Using the

$$\mathbf{D} = \mathbf{A}^{-1}\mathbf{B} \tag{3}$$

diagnostic matrix, the equation

$$\delta\mathbf{y} = \mathbf{D}\delta\mathbf{x} \tag{4}$$

can be used for diagnostic investigations [3].

2. Investigation of Effects of Manufacturing Anomalies

During design of new aircraft system the determination of manufacturing tolerances is very important task.

It is possible that system parameter values are inadequate, but every units and elements of the system meet own requirements. Because of the manufacturing tolerances of units or elements have been determined incorrectly [7].

It is possible too, that working requirement of the system determine strictly the system parameter tolerances. For example, such system requirements can be velocity or acceleration of the piston of hydraulic servo actuator, in flight-mechanical point of view. These external parameters should require such strictly the tolerances of internal parameters of the system.

Using mathematical diagnostic model of the given system, the problems mentioned above can be investigated and solved.

Their densities, expected values and variances can characterise manufacturing anomalies of internal parameters. These random characteristics determine the densities, expected values and variances of external parameters so their manufacturing anomalies.

For investigation, the vectors of minimal and maximal values of the internal parameters should be introduced and filled up [3].

3. Case of the Gaussian Distributions

Firstly we supposed that manufacturing anomalies of internal parameters are interdependent random variables with normal distribution.

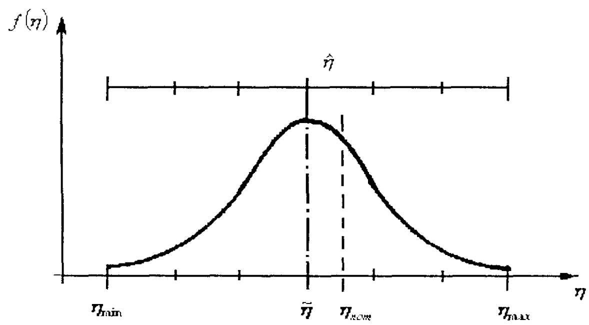

Figure 2.
Usage of the “3σ-rule”

In this case the expected values of internal parameters are the means of their tolerance zones (see Figure 2.):

$$\tilde{\eta} = \frac{\eta_{\max} + \eta_{\min}}{2}$$

that is: (5)

$$\tilde{\mathbf{x}} = \frac{\mathbf{x}_{\mathbf{max}} + \mathbf{x}_{\mathbf{min}}}{\mathbf{2}} \quad .$$

It is important to mention, if the tolerance zones are asymmetric, the expected value will not be equal the nominal value of the given parameters.

The variance of this parameters should be determined as a sixth parts of tolerance zones due to so called "3σ-rule". Because the random variables of normal distribution with expected value *m* and variance σ will fall "practically certainly" in the (*m-3σ*, *m+3σ*) interval - its probability in fact is 0,9973 [6].

$$\hat{\eta} = \frac{\eta_{\max} - \eta_{\min}}{6}$$

that is: (6)

$$\hat{\mathbf{x}} = \frac{\mathbf{x}_{\mathbf{max}} - \mathbf{x}_{\mathbf{min}}}{6} \quad .$$

3.1. Determination of Variances

To determine the variances of external parameters, the vector of relative variances of interval parameters should be determined by equation

$$\delta\hat{\eta} = \frac{\hat{\eta}}{\eta_{nom}} \quad ,$$

that is: (7)

$$\delta\hat{\mathbf{x}} = \left(\mathbf{Ex}_{\mathbf{nom}}\right)^{-1}\hat{\mathbf{x}} \quad .$$

Using the diagnostic matrix of investigated system, the vector of relative variances of external parameters is:

$$\delta\hat{\mathbf{y}} = \mathbf{D}\,\delta\hat{\mathbf{x}} = \left(\mathbf{Ex}_{\mathbf{nom}}\right)^{-1}\mathbf{D}\hat{\mathbf{x}} \tag{8}$$

Knowing the nominal values of the external parameters, the vector of their measured variances should be determined by following equation:

$$\hat{\mathbf{y}} = \left(\mathbf{Ey}_{\mathbf{nom}}\right)\delta\hat{\mathbf{y}} = \left(\mathbf{Ex}_{\mathbf{nom}}\right)^{-1}\mathbf{D}\left(\mathbf{Ey}_{\mathbf{nom}}\right)\hat{\mathbf{x}} \tag{9}$$

We introduced the "MEASURED DIAGNOSTIC COEFFICIENT MATRIX":

$$\mathbf{S} = \left(\mathbf{Ex}_{\mathbf{nom}}\right)^{-1}\mathbf{D}\left(\mathbf{Ey}_{\mathbf{nom}}\right) \tag{10}$$

the equation (9) can be simplified:

$$\hat{\mathbf{y}} = \mathbf{S}\hat{\mathbf{x}} \tag{11}$$

3.2. Determination of Expected Values

To determine expected values of external parameters, the vector of relative expected values of integral parameters should be determined. Because of the diagnostic matrix describe interdependencies between relative changes of internal and external parameters, this vector has to show the relative values of difference between measured expected and nominal values to nominal ones:

$$\delta\tilde{\eta} = \frac{\tilde{\eta} - \eta_{nom}}{\eta_{nom}} \quad ,$$

that is (12)

$$\delta\tilde{\mathbf{x}} = \left(\mathbf{E}\mathbf{x}_{\mathbf{nom}}\right)^{-1}\left(\tilde{\mathbf{x}} - \mathbf{x}_{\mathbf{nom}}\right) \quad .$$

Knowing the diagnostic matrix, the vector of relative expected values of external parameters should be determined by equation

$$\delta\tilde{\mathbf{y}} = \mathbf{D}\,\delta\tilde{\mathbf{x}} = \left(\mathbf{E}\mathbf{x}_{\mathbf{nom}}\right)^{-1}\mathbf{D}\left(\tilde{\mathbf{x}} - \mathbf{x}_{\mathbf{nom}}\right) \quad . \tag{13}$$

Then, the vector of measured expected values of external parameters should be determined:

$$\begin{aligned}\tilde{\mathbf{y}} &= \mathbf{y}_{\mathbf{nom}} + \left(\mathbf{E}\mathbf{y}_{\mathbf{nom}}\right)\delta\tilde{\mathbf{y}} = \\ &= \mathbf{y}_{\mathbf{nom}} + \left(\mathbf{E}\mathbf{x}_{\mathbf{nom}}\right)^{-1}\mathbf{D}\left(\mathbf{E}\mathbf{y}_{\mathbf{nom}}\right)\left(\tilde{\mathbf{x}} - \mathbf{x}_{\mathbf{nom}}\right)\end{aligned} \quad . \tag{14}$$

Applying the measured diagnostic coefficient matrix **S** — see equation (10) — the equation (14) can be simplified:

$$\tilde{\mathbf{y}} = \mathbf{y}_{\mathbf{nom}} + \mathbf{S}\left(\tilde{\mathbf{x}} - \mathbf{x}_{\mathbf{nom}}\right) \quad . \tag{15}$$

Knowing variances and expected values of external parameters, their "manufacturing tolerance zones" that are result of manufacturing anomalies of units and elements of the system can be determined. These interval should be determined using the above mentioned "3σ–rule", that is the vector of their minimum and maximum values:

$$\begin{aligned}\mathbf{y}_{\mathbf{min}} &= \tilde{\mathbf{y}} - 3\hat{\mathbf{y}} \\ \mathbf{y}_{\mathbf{max}} &= \tilde{\mathbf{y}} + 3\hat{\mathbf{y}}\end{aligned} \tag{16}$$

3.3. The Inverse Method

It is possible, that task and work of given system limit strictly output parameter values and their tolerances determined by equations (16) cannot meet these requirements. Then the manufacturing tolerances of internal parameters have to be determined on the basis the required tolerances of external system parameters. This task can be solved by inverse method of the problem [4].

The required variances of internal parameters should be determined by equation (11). The vector $\hat{\mathbf{x}}$ that satisfies the equation

$$\left(\hat{\mathbf{y}} - \mathbf{S}\hat{\mathbf{x}}\right)^2 = 0 \tag{17}$$

should be estimated by using any search of optimum method. This one can be Gauss-Seidel, Random or Gradient method.

For determination of required expected values of internal parameters the equations (15) should be rearranged

$$\tilde{\mathbf{y}} - \mathbf{y}_{\mathbf{nom}} = \mathbf{S}\left(\tilde{\mathbf{x}} - \mathbf{x}_{\mathbf{nom}}\right) \tag{18}$$

and auxiliary vectors

$$\mathbf{u} = \tilde{\mathbf{y}} - \mathbf{y}_{\mathbf{nom}} \tag{19}$$

and

$$\mathbf{v} = \tilde{\mathbf{x}} - \mathbf{x}_{\mathbf{nom}} \tag{20}$$

should be introduced. Then the equation (18) will modify to following form:

$$\mathbf{u} = \mathbf{S}\mathbf{v} \tag{21}$$

On the basis of the auxiliary vector **u** — see equation (19) — and tolerance coefficient matrix **S** — see equation (10) —, the auxiliary vector **v** — see equation (20) — should be estimated by using scalar-vector function

$$(\mathbf{u}-\mathbf{S}\mathbf{v})^2 = 0 \quad . \tag{22}$$

Then the vector of required expected values of internal parameters should be determined by equation

$$\tilde{\mathbf{x}} = \mathbf{x}_{\mathbf{nom}} + \mathbf{v} \quad . \tag{23}$$

The vectors of minimum and maximum values of the required manufacturing tolerance zones of internal parameters:

$$\begin{aligned} \mathbf{x}_{\mathbf{min}} &= \tilde{\mathbf{x}} - 3\hat{\mathbf{x}} \\ \mathbf{x}_{\mathbf{max}} &= \tilde{\mathbf{x}} + 3\hat{\mathbf{x}} \end{aligned} \quad . \tag{24}$$

This data have to be investigated in technological and manufacturing point of view. If the technological possibilities do not meet required quality, on the basis of the practicable tolerance zones of internal parameters should be determined and the base investigation should be performed once more while the external system parameters will meet the requirements.

4. Case of unknown (General) Distributions

To determine maximum and minimum values of external system parameters, as a first step, the vectors of relative maximum and minimum internal parameter values should be determined (see Figure 3):

$$\delta\eta_{\max} = \frac{\eta_{\max} - \eta_{nom}}{\eta_{nom}} \tag{25}$$

and

$$\delta\eta_{\min} = \frac{\eta_{\min} - \eta_{nom}}{\eta_{nom}} \quad , \tag{26}$$

that is

$$\delta\mathbf{x}_{\max} = (\mathbf{E}\mathbf{x}_{nom})^{-1}(\mathbf{x}_{\max} - \mathbf{x}_{nom}) \tag{27}$$

and

$$\delta\mathbf{x}_{\min} = (\mathbf{E}\mathbf{x}_{nom})^{-1}(\mathbf{x}_{\min} - \mathbf{x}_{nom}) \quad . \tag{28}$$

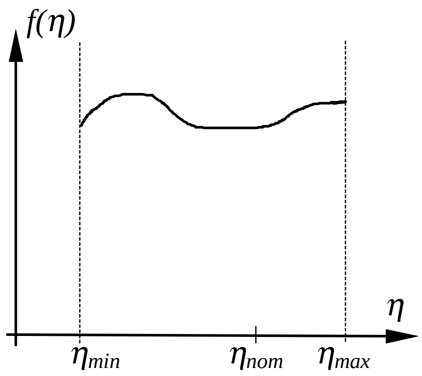

Figure 3.
Investigation of Unknown Distributions

4.1. The Investigation Method

In case of unknown distributions, the linearized diagnostic model — see equation (4) — of the investigated system should be modified. The so-called "positive diagnostic matrix" and "negative diagnostic matrix" should be introduced. Elements of the first one are the positive-sign elements of the original diagnostic matrix (or zero):

$$\mathbf{D}_{+} = \left[d_{ij+} = \begin{cases} d_{ij} & if \quad d_{ij} \geq 0 \\ 0 & if \quad d_{ij} < 0 \end{cases} \right] \quad . \tag{29}$$

Another one's elements are negative-sign elements of the original diagnostic matrix (or zero):

$$\mathbf{D}_{-} = \left[d_{ij-} = \begin{cases} d_{ij} & if \quad d_{ij} \leq 0 \\ 0 & if \quad d_{ij} > 0 \end{cases} \right] \quad . \tag{30}$$

Knowing the above mentioned matrices, the vectors of relative maximum and minimum values of the external parameters:

$$\delta\, \mathrm{y}_{\mathrm{MAX}} = \mathrm{D}_{+}\delta\, \mathrm{x}_{\max} + \mathrm{D}_{-}\delta\, \mathrm{x}_{\min} \tag{31}$$

and

$$\delta\, y_{min} = D_+\, \delta\, x_{min} + D_-\, \delta\, x_{max} \quad . \qquad (32)$$

For the sake of the inverse method, the following hyper-matrix equation should be devised:

$$\begin{bmatrix} \delta \mathbf{y}_{max} \\ \delta \mathbf{y}_{min} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_+ & \mathbf{D}_- \\ \mathbf{D}_- & \mathbf{D}_+ \end{bmatrix} \begin{bmatrix} \delta \mathbf{x}_{max} \\ \delta \mathbf{x}_{min} \end{bmatrix} \quad . \qquad (33)$$

Knowing the relative maximum and minimum external parameter values can be determined by equations:

$$\mathbf{y}_{max} = \mathbf{y}_{nom} + \left(\mathbf{E} \mathbf{y}_{nom} \delta \mathbf{y}_{max} \right) \quad , \qquad (34)$$

and

$$\mathbf{y}_{min} = \mathbf{y}_{nom} + \left(\mathbf{E} \mathbf{y}_{nom} \delta \mathbf{y}_{min} \right) \quad . \qquad (35)$$

4.2. The Inverse Method

If the task and work of the investigated (designed) system limits the output parameter values and their tolerances strictly, the manufacturing tolerances of internal parameters should be determined or estimated depend on required output parameter tolerances. This task can be solved by the inverse method of investigation mentioned above.

For estimation of required maximum and minimum values of internal parameters, firstly the relative maximum and minimum value vectors of output ones should be determined by following equations:

$$\delta \mathbf{y}_{max} = \left(\mathbf{E} \mathbf{y}_{nom} \right)^{-1} \left(\mathbf{y}_{max} - \mathbf{y}_{nom} \right) \qquad (36)$$

and

$$\delta \mathbf{y}_{min} = \left(\mathbf{E} \mathbf{y}_{nom} \right)^{-1} \left(\mathbf{y}_{min} - \mathbf{y}_{nom} \right) \qquad (37)$$

Then, the required maximum and minimum value vectors of internal parameters can be determined on basis of hyper-matrix equation (33). The vector which satisfies the scalar-vector equation

$$\left(\begin{bmatrix} \delta \mathbf{y}_{max} \\ \delta \mathbf{y}_{min} \end{bmatrix} - \begin{bmatrix} \mathbf{D}_+ & \mathbf{D}_- \\ \mathbf{D}_- & \mathbf{D}_+ \end{bmatrix} \begin{bmatrix} \delta \mathbf{x}_{max} \\ \delta \mathbf{x}_{min} \end{bmatrix} \right)^2 = 0 \qquad (38)$$

should be estimated using any search of optimum method.

On the basis of vector estimated above, the required real (measurable) values of internal parameters can be determined by equation

$$\mathbf{x}_{max} = \mathbf{x}_{nom} + \left(\mathbf{E} \mathbf{x}_{nom} \delta \mathbf{x}_{max} \right) \quad , \qquad (39)$$

and

$$\mathbf{x}_{min} = \mathbf{x}_{nom} + \left(\mathbf{E} \mathbf{x}_{nom} \delta \mathbf{x}_{min} \right) \quad . \qquad (40)$$

It is very important to mention that these data have to be investigated from the technological and the manufacturing points of view. If the technological possibilities do not meet the required quality, on the basis of the practicable tolerance zones of the internal parameters should be determined and the base investigation should be performed once more while the external system parameters will meet the requirements [5].

It is also important to mention that this method does not give the unambiguous solution of the above-mentioned technical problem. Because this investigation uses any estimation process. This method is „only" an effective adjuvancy to determine the most practicable manufacturing tolerances of the internal parameters during the design of the system.

5. Summary, Future Work

The writers of this paper would like to arouse readers' interest in possibilities of use of mathematical diagnostic modelling to investigate of new aircraft systems during their design. In this paper the methodology of mathematical diagnostics and investigation of influences of manufacturing anomalies have been shown.

On the basis of this study, methods to investigate influences of manufacturing anomalies to output system parameters should be worked out in following cases:

- manufacturing anomalies of internal parameters are not interdependent random variables;
- manufacturing anomalies of internal parameters do not have normal or other special distributions;
- coefficient matrix of external parameters cannot be inverted;

and combinations of above mentioned cases.

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