ACTIVE CONTROL OF FLEXIBLE STRUCTURES BASED ON FUZZY LOGIC

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Keywords: *Fuzzy Logic, Flexible Structures, Active Control, Vibration Suppression*

Abstract

This study deals with the development and implementation of an active control law for the vibration suppression of beam-like flexible structures experiencing transient disturbances. Collocated pairs of sensors/actuators provide active control of the structure. A design methodology for fuzzy logic based closed-loop control algorithm of beam vibration is proposed. First, the behavior of the open-loop system is observed. Then, the number and locations of collocated actuator/sensor pairs are selected.

The proposed control law, which is based on the principles of passivity, commands the actuator to emulate the behavior of a dynamic vibration absorber. The absorber is tuned to a targeted frequency, whereas the damping coefficient of the dashpot is varied in a closed loop using a fuzzy logic based algorithm. This approach, not only assures inherent stability associated with passive absorbers, but also circumvents the phenomenon of modal spillover. The developed controller is applied to the AFWAL/FIB ten bar truss. Simulated results using MATLAB show that the closed-loop system exhibits fairly quick settling times and desirable performance, as well as robustness characteristics.

1 Introduction

A major driver that affects the overall system performance of LFS (Large Flexible Structures) involves their sizing for minimum mass, subject to both static strength and dynamic requirements. This need to increase structural efficiency in high performing systems has recently motivated a field of research referred to as *adaptive structures* which involves the control of structural dynamics through interdisciplinary design of an integrated structure and control system. Adaptive structures may be introduced to influence the geometry, shape, apparent stiffness, damping, or inertia of the structural modes. Significant payoffs attainable in aerospace systems by introducing adaptive control surfaces, active acoustic coatings for signature suppression, vibration suppression and twist control, and active structural tuning and damping. The main motivation of using an active control system as opposed to passive means (e.g. dynamic vibration absorber) is weight savings.

Aerospace facilities may generally comprise of repetitive latticed trusses, span large areas with a few intermediate supports, are light in weight and extremely flexible, and consequently are characterized by a large number of high-density low frequency structural modes. These higher order structural systems utilize feedback control laws that are based on system stimulus-response models, embedded sensors to sense their response to operational and environmental stimuli, and actuators to modify their response in such a way as to maintain or optimize structural performance.

In this study, a fuzzy logic controller is proposed and developed for the least settling time of suppression of transient induced vibration in beam-like flexible structures. For such a class of dynamic systems, Juang and Phan [1] presented a robust passive design controller which is based on a virtual second-order dynamic system comprising of virtual mass, spring and dashpot elements. In addition, Juang and Phan [1] showed that overall closed-loop stability was guaranteed independently of the system structural uncertainty and perturbations in the temporal plant of the system. These second-order controllers may also be termed "dissipative", or "collocated", and they consist of compatible pairs of actuators and sensors which may be distributed throughout the structure [2]. The main reason for using the "passivity" approach is its inherent robustness. Furthermore, the virtual mechanisms incorporated into the passive design serve only to transfer and dissipate the energy of the system thereby maintaining the stability of the system.

The above passive design approach is also adopted in the present study, however by treating the virtual system in a manner different than that suggested by Juang and Phan [1], i.e. in compliance with results yielded by non-linear time optimal control analysis. The proposed control law, which is based on the principles of passivity, commands the actuator to emulate the behavior of a dynamic vibration absorber. The absorber is tuned to a targeted frequency, whereas the damping coefficient of the dashpot is varied in a closed loop using a fuzzy logic based algorithm. The purpose of the fuzzy logic based, variable damping strategy, is to provide quicker settling times yielded by non-linear control action. In this paper, the proposed approach is applied only to transient disturbances. Furthermore, the developed strategy was also applied to a laboratory model of a flexible beam-like cantilever the vibrations of which were actively controlled using piezoceramic sensors and actuators [3].

In this study, a ten bar truss (Fig. 1) was selected for the numerical application since it is a well known benchmark, developed by AFWAL/FIB, for studies concerning vibration suppression of flexible space structures with several low natural frequencies and a high modal density. This benchmark serves as an ideal platform to demonstrate some of the important features of a control system design for a typical large space structure. Moreover, the main aim of the numerical exercise is pointing out the effectiveness of the fuzzy logic based controller in shortening the settling times. To this end, the results obtained are compared to those reached using LQG/LTR, which serves as a "universal" baseline for comparison.

2 Actuator/Sensor Placement

Consider a large flexible beam-like structure illustrated in Fig. 1. The state space representation for such a large structure, may be written in conventional form (neglecting noise and disturbances) as:

$$dx(t)/dt = Ax(t) + Bu(t) \qquad (1)$$

where x(t), dx(t)/dt, u(t), A and B are the state vector, the state-rate vector, the control vector, a n x n state matrix and a n x m control matrix, respectively. The corresponding measurement equation may be written as:

$$y(t) = Cx(t) \qquad (2)$$

where y(t) is the output vector and C is a l x n measurement matrix.

Hughes and Skelton [4] addressed the important issue of sensor and actuator positioning, for large flexible structures, by proposing criteria for modal controllability (actuator placement) and modal observability (sensor placement). Another approach, based on passive control strategies for the vibration suppression of large beam-like truss structures [5], using dynamic vibration absorbers (DVA), lead to the following observations:

- One DVA, tuned to a specific targeted mode, is sufficient for the vibration suppression of that particular mode.

- A DVA tuned to a certain frequency does not affect the characteristics of the modes above that frequency. Hence, after identifying the targeted modes, the highest mode is first tuned and then the structure is modified. In the next step, the second highest mode is tuned and so on [5].

- The energy absorbing property of the tuned DVA is most effective when placed on the maxima of the respective mode shape.

The damping value of each of the DVAs may be obtained using fuzzy logic control based on the insight presented in Cohen, Weller and Ben-Asher [6]. The above mentioned guidelines will make the task of selection and tuning of the actuator/sensor relatively simpler. Furthermore, Juang and Phan [1], presented sufficient conditions on actuator and sensor that guaranteed overall closed-loop stability for robust controller designs for second-order dynamic systems.

3 Passivity Based Control

Consider a virtual dynamic vibration absorber, attached at a distance "m" from one of the ends of a large flexible beam-like structure with arbitrary boundary conditions. For the sake of convenience, we denote the natural frequency of an undamped vibration of the absorber as "$\sqrt{a}$". Based on the optimal tuning ratio of a 2-DOF system, for a given open-loop frequency of vibration ω, and a mass ratio of the absorber mass to the structure mass μ, "a" is obtained by equating the first two peaks of the steady state response [7], and is given by:

$$a = \left[\frac{\omega}{1+\mu}\right]^2 \tag{3}$$

Jacquot [8] extended Den Hartog's 2-DOF theory to continuous systems. A dynamic vibration absorber was applied at a distance "m" from the fixed end of a cantilever Euler-Bernoulli beam, thereby transforming Equation (3) to:

$$a = \left[\frac{\omega}{1+(\phi_i(m))^2\cdot\mu}\right]^2 \tag{4}$$

where $\phi_i(m)$ is the value of the normalized mode shape of the natural mode "i" at a distance "m" from the fixed end. Following Equation (4), the necessary plant information required to define "a" is an estimate of the targeted frequency ω which may be identified from the displacement-time sensor output of the open-loop system and $\phi_i(m)$. In certain applications, when there is possible lack of information on mode shapes, we may introduce a constant, μ^*, which is used in the tuning process to make up for the loss in performance. The value of μ^* is determined by a "random walk" during simulations. This practice has been found to be effective for beam-like structures [9]. For such cases, the product of the mass ratio of the absorber, μ, and $\phi_i^2(m)$ may be written as a function of the damping factor, δ, and μ^* as follows:

$$\mu\cdot[\phi_i(m)]^2 = \mu^*\cdot\left[1/\delta + (1/\delta)^2\right] \tag{5}$$

Eq. (5) represents an empirical relation, whereby a *lightly* damped absorber corresponds to a *large* mass ratio and vice-versa. Inserting the above Eq.into Eq.(4) gives:

$$a = \left[\frac{\omega\cdot\delta^2}{\delta^2+\mu^*\cdot(1+\delta)}\right]^2 \tag{6}$$

Let the damping parameter, "b", be defined as the ratio of the damping coefficient of the absorber to the mass of the absorber, written for a typical second-order system as:

$$b = 2\cdot(\delta\cdot\sqrt{a}) \tag{7}$$

Inserting Eq.(6) into Eq.(7) yields:

$$b = 2\cdot\delta\cdot\left[\frac{\omega\cdot\delta^2}{\delta^2+\mu^*\cdot(1+\delta)}\right] \tag{8}$$

The resulting force (per unit mass) applied by the absorber may be written as:

$$f = a\cdot[x_1 - y(m,t)] + b\cdot\left[\frac{dx_1}{dt} - \frac{dy(m,t)}{dt}\right] \tag{9}$$

where $y(m,t)$ and x_1 are the transverse displacements of the beam and virtual absorber, respectively. x_1 is obtained by solving the following 2-DOF Eq. of motion of the absorber.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a & -b \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix} \cdot \begin{bmatrix} y(m,t) \\ \dot{y}(m,t) \end{bmatrix} \quad (10)$$

where x_2, is the transverse velocity of the absorber.

The approach employed herein is based on MRAC (Model-Reference Adaptive Control, [10]). This method may be used in control of systems whose models possess significant uncertainties, and hence the application of a model-independent control law is a distinct advantage. First the performance errors become the input to a tuner that adjusts the damping factor of the virtual dynamic vibration absorber. The value of δ inferred from the measurements is reached by using a fuzzy control gain weighting adaptation strategy, which will be described in detail by Cohen [9]. In the next step, the parameters that characterize the absorber, $a(\delta)$ and $b(\delta)$, are obtained using Eqs. (5) - (8).

After obtaining the values of a and b, 2-DOF equations of motion of the absorber, given by Eq. (10), are numerically computed to give the displacement and the velocity of the virtual absorber. Then, the input control force, obtained from Eq. (9), is applied to the plant, which consists of the first n modes of a large, flexible, beam-like structure. Finally, the required force is translated into the actuation command.

4 Application to a Ten Bar Truss

The ten bar truss, described in Fig. 1, was introduced by AFWAL/FIB [11] to demonstrate some of the important features of a control system design, without the handling problems associated with typical higher-order large space structures. The mathematical model of the truss, modified by AFWAL/FIB from a similar structural model, is 100 inches in length and 18 inches high [11]. The adaptive fuzzy passive control law, which was proposed and developed in this study, is applied to the problem of the ten bar truss. The evaluation of the developed control system will be based upon the system response to an initial condition. The initial condition vector corresponds to a tip displacement of approximately one-inch and a mid-station displacement of nearly two inches. The initial velocity of the truss is zero. For the initial condition case, results are compared to those presented by Lynch and Banda [11] based on a LQG/LTR design.

The AFWAL/FIB ten bar truss benchmark, illustrated in Fig. 1, is basically 2-dimensional and motion is allowed in the x and y directions only. Force actuators and position sensors are collocated at points 1, 2, 3 and 4 on the truss. The actuators act along the y-axis only whereas the sensors measure physical displacements in the y direction at the above four locations. Further structural details of the AFWAL/FIB ten bar truss benchmark are presented in [11]. The first four modal frequencies for this uniformly damped 8-mode model are shown in Table 1.

Mode	1	2	3	4
Freq.(Hz)	0.500	1.653	3.613	4.702
Freq.(r/s)	3.142	10.39	22.70	29.54

Table 1: Natural Frequencies of the 10 Bar Truss

The transient disturbance involves an initial condition that excites the first, second, fifth and seventh modes of the structure. This vector corresponds to a tip displacement of approximately one-inch and a mid-station displacement of nearly two inches. The initial velocity of the truss is zero. The open loop response of the truss to the above initial disturbance is illustrated in Fig. 2.

The 8-mode model is reduced for design purposes. Following Lynch and Banda [11] examination of the system's second-order modes, the system was found to be most controllable and observable with respect to the first two modes. The resulting design model is as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + Gd(t) \quad (11)$$

$$y(t) = Cx(t) + Du(t) \quad (12)$$

where x(t), y(t), u(t) and d(t) denote the state vector, the measurement vector, the control forces and a scalar external persistent disturbance, respectively.

With

$$A = \begin{bmatrix} -.0314 & 0 & -9.8694 & 0 \\ 0 & -.1039 & 0 & -107.86 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} .3142 & .3142 & .1161 & .1161 \\ -.1040 & -.1040 & .3337 & .3337 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & -.3117 \\ 0 & 0 & 1 & -.3117 \\ 0 & 0 & .3402 & 1 \\ 0 & 0 & .3402 & 1 \end{bmatrix}$$

$$D = 1.0^{-3} \times \begin{bmatrix} 0.4480 & -0.4230 & -0.0263 & 0.0010 \\ -0.4230 & 0.4480 & 0.0010 & -0.0263 \\ -0.0263 & 0.0010 & 0.4239 & -0.3970 \\ 0.0010 & -0.0263 & -0.3970 & 0.4239 \end{bmatrix}$$

$$G = \begin{bmatrix} -0.0393 \\ 0.0822 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

The sensors of the ten bar truss, described in Figure 1, sense displacements in the y-direction only and the actuators act along the y-axis alone. Therefore, the control effort at points 1 and 3 may be identical to those at points 2 and 4 respectively. Based on the strategy developed in the previous section, two virtual dynamic vibration absorbers (DVA) were introduced at points 2 (DVA 1) and 4 (DVA 2). For both DVAs, the damping coefficient is varied in accordance to the rule base described by Cohen [9].

After calculating the force applied by DVA 1, this value was divided by 2 to obtain the force applied at points 1 and 2. Furthermore, DVA 1 was tuned to the fundamental frequency (0.5 Hz.) and DVA 2 was tuned to the second natural frequency (1.6529 Hz.). In addition, the damping coefficients of the two virtual absorbers were varied using the adaptive fuzzy approach based on the rule-Base of the fuzzy logic controller. On the other hand, the membership functions of inputs (displacement and rate at points 2 & 4) and outputs (damping of DVA 1 & DVA 2) required some further fine-tuning. If the velocities at positions 1, 2, 3 and 4 (required for calculating the force applied by the virtual absorbers) are obtained by calculating the change of displacement with time, then the signal may consist of a lot of high frequency harmonics. These harmonics of the signal can affect the closed-loop performance. The transfer function of the estimator may be written as:

$$G(s)= \frac{s}{(\tau_d s+1)} \qquad (13)$$

In Eq.(13), when τ_d is selected as zero, the estimator performs as a real velocity but produces a lot of noise. However, when τ_d is increased, the estimation error of the estimator is increased but the noise is reduced. The estimation error is proportional to the parameter τ_d and when t approaches infinite, the estimation error of velocity approaches zero. Generally speaking, τ_d is selected as a small value to allow the estimated velocity to approach the real velocity in a short time. In this effort, after several tuning attempts, the appropriate value for τ_d was found to be 0.4 sec.

Lynch and Banda [11] developed a LQG/LTR controller based on the 2-Mode design model. The responses at the upper and lower locations are nearly identical as a result of the excitement of only the longitudinal modes. The lateral modes were not excited. For the LQG/LTR controller, the initial vibrations are damped to within 0.1 percent of the initial amplitude in approximately 12 seconds. In comparison, for the adaptive fuzzy controller, the initial vibrations are damped to within 0.1

percent of the initial amplitude in less than 6 seconds (see Fig. 3). This remarkable improvement in the settling times is obtained without exceeding the specified control power limits. The robustness characteristics of the developed controller are examined for the 3 perturbed plants described in Cohen [9]. For all 3 cases, the adaptive fuzzy controller yielded satisfactory results [9].

5 Conclusions and Recommendations

The present effort describes the development and application of a fuzzy based controller that emulates the functioning of an adaptive dynamic vibration absorber tuned to the targeted frequencies. The central idea, which drives the developed control law, implies that for large values of system error, the damping effect of the error derivative control is blocked as full control authority is used to quickly drive the system to zero. On the other hand, as the system error tends to zero a progressively greater damping effect is introduced.

The controller is applied to a beam-like ten-bar truss that is subjected to a transient disturbance in the form of an initial condition. Four collocated pairs of sensors/actuators were used to suppress the transient vibrations. MATLAB simulations of the closed-loop transient response, for the nominal and other perturbed plants, demonstrate quick settling times, a high rate of vibrational energy dissipation and no control spillover to the higher modes. The results obtained using the fuzzy based controller compares favorably to designs based on LQG/LTR.

The controller presented may further be developed for the vibration suppression of more complex structures such as 3-D beam-like and plate-like latticed structures, having different boundary conditions and which include coupling between the bending and twisting modes. Future work may include a stability analysis of the closed-loop system and examination of the closed-loop robustness to variations in the location of the sensors relative to the actuators (non-collocated sensors/actuators). Further comparisons with other controllers may include non-linear designs.

6 References

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7 Figures

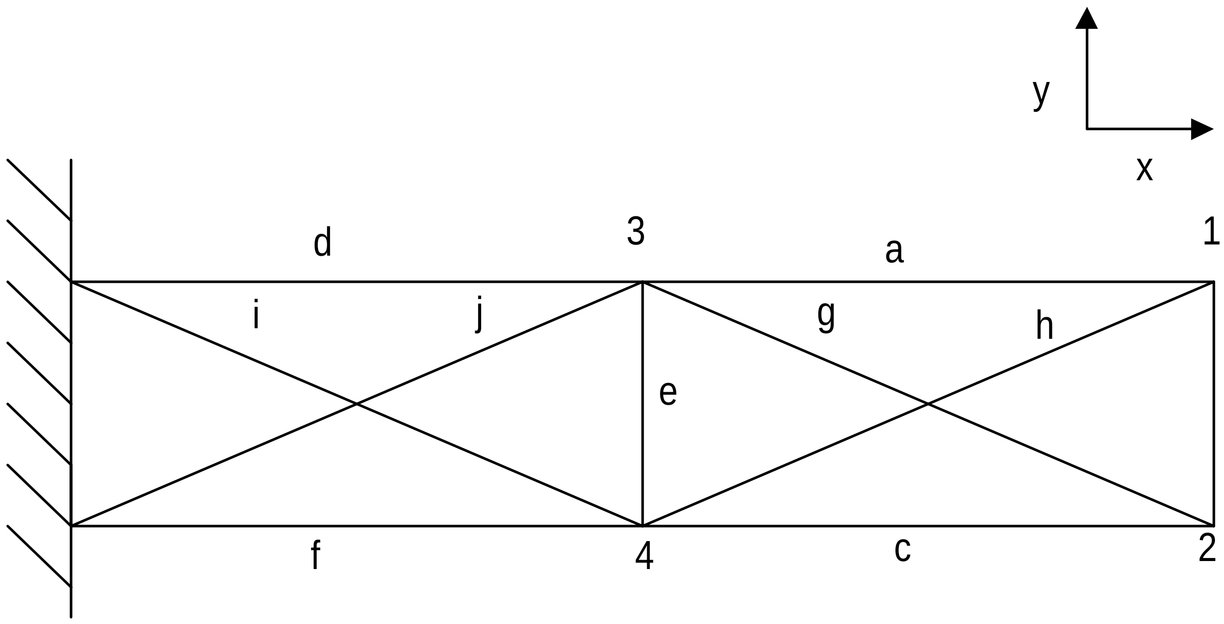
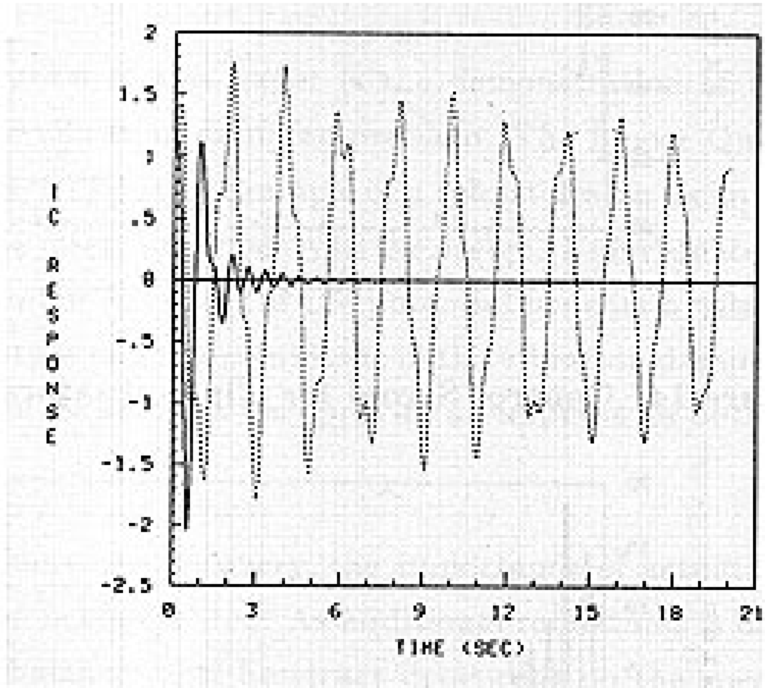
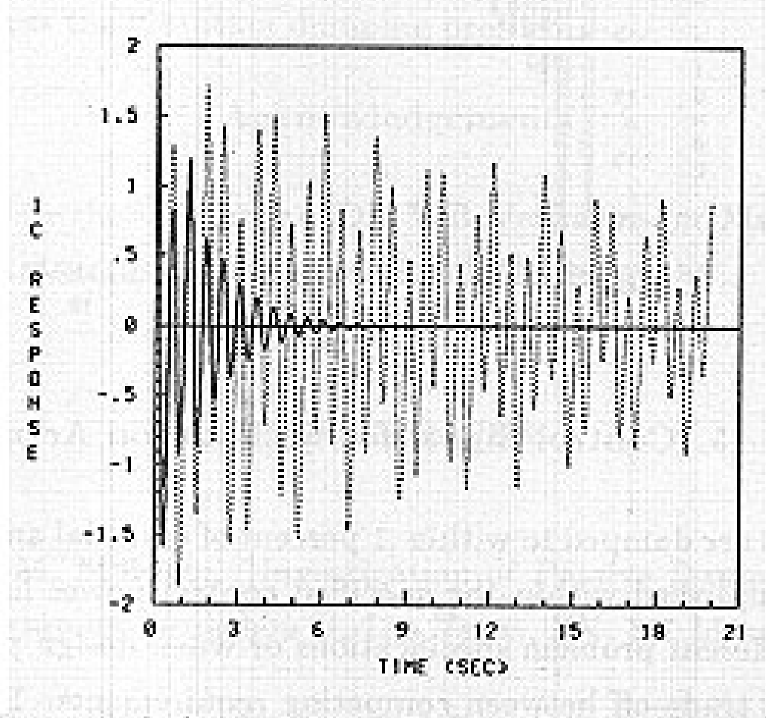


Fig. 1: AFWAL/FIB Two Bay Model as presented by Lynch and Banda [11].



Open and Closed-loop Initial Condition Response (Tip)



Open and Closed-loop Initial Condition Response (Mid)

Fig. 2: Open and Closed-loop response based on LQG/LTR [11].

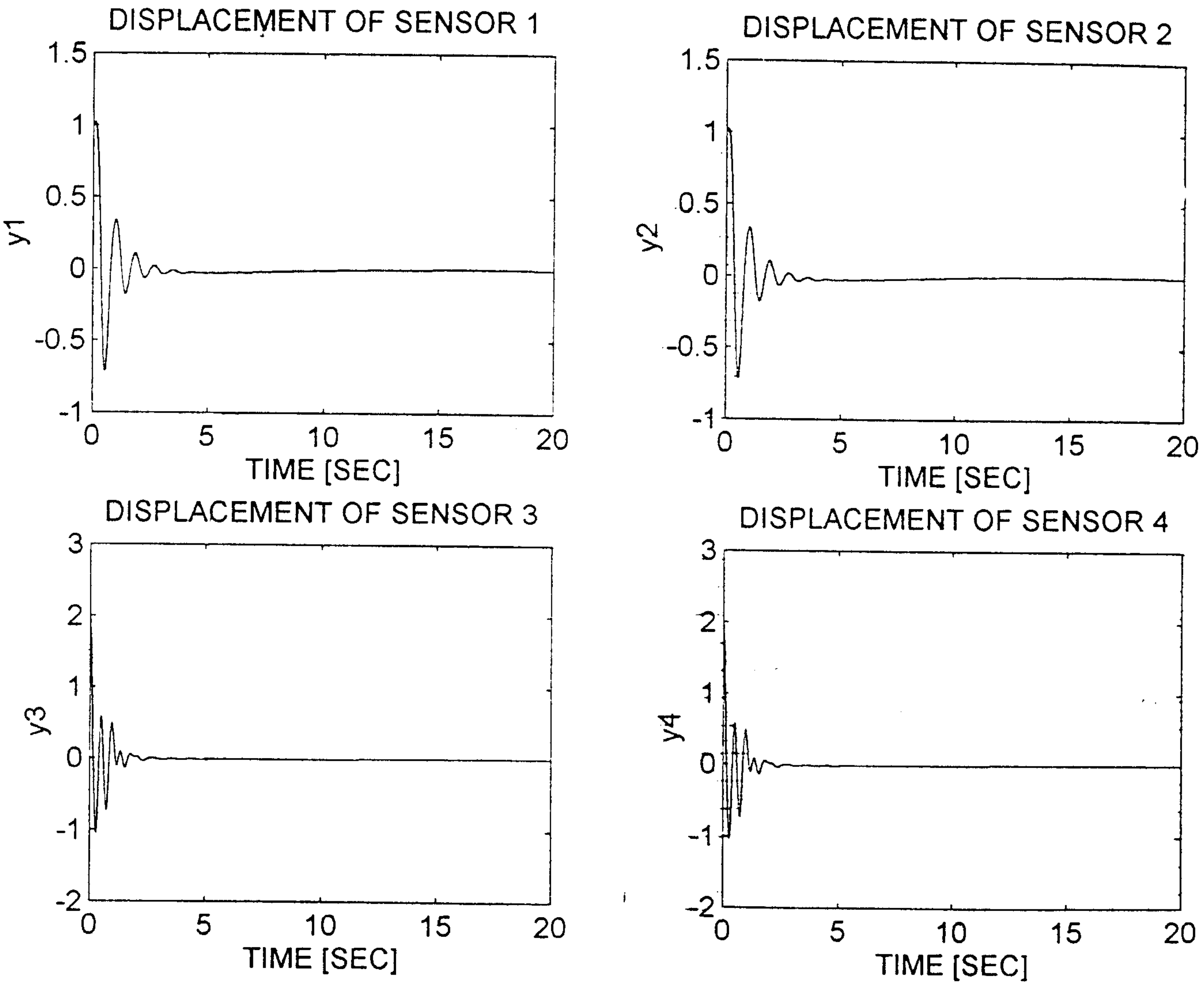


Fig. 3: Closed-loop response using fuzzy based strategy.