OPTIMIZATION OF TAPERED WING STRUCTURE WITH AEROELASTIC CONSTRAINT

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ABSTRACT

A methodology for including maximum flutter speed requirement in the preliminary structural wing design is developed. The problem of minimizing structural weight while satisfying static strength, dynamic characteristic and aeroelastic behavioural constraints is stated in a non-linear mathematical form , with beam width and thickness taken as design variables , and solved using gradient-based optimisation technique. Dynamic characteristics of the structure are calculated using finite element model. Laplace form of the unsteady aerodynamics forces are obtained from Fourier transform of unit pulse aerodynamics response. The frequency-domain p-k method is applied for the calculation of aeroelastic stability boundaries. Based upon constraint values and the required gradients, a first order Taylor series approximation is used to develop an approximation linear programming for weight minimization. A modified feasible direction method is, then, applied iteratively to solve the optimisation problem. Validation of the method are carried out in the design of cantilever straight wing structure with 6% hyperbolic airfoil. It will be shown that the optimised wing design can significantly differ from those obtained without optimisation process.

INTRODUCTION

With advancing design process in which input data for all calculations become more precise and easy to generated, an interdisciplinary design concept, that taking into account all important interdisciplinary mutual effects including aeroelastic stability boundary, must be initiated to provide a basis for design decisions on time. This design concept should have capability in providing the effects of change on each of the design parameter on the behaviour of the structure (stress level, damping, natural frequency, aeroelastic boundary etc.). Furthermore this design concept should also can be used to find optimal design solution by means of mathematical optimisation tools. One such methodology that based upon this interdisciplinary design concept was proposed by Sobieszczanski – Sobieski[1] in the late eighty which explicitly includes the important aeroelastic stability constraints, i.e.: maximum divergence and flutter speed. Since then, there were significant number of research works have been done in the field of optimal design of aircraft structures subject to aeroelastic stability constraints, the so called aeroelastic tailoring. One of important early contribution into this field were made by Haftka and Yates[2] in which an optimisation algorithm was developed for the used in repetitive aeroelastic calculation during the structural design process. The used of advanced composite / smart materials have further improved the possibilities in tailoring the dynamic response characteristics of the structure under unsteady aerodynamic loads. And the availability of powerful computer opened the possibilities for lengthy optimisation calculation. Because aeroelastic tailoring in the design process of an aircraft structure is true

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multidisciplinary analysis and optimisation, design results cannot be achieved independently of the requirements of certain other disciplines.

With regard to the structure modelling, it is important to understand that there is a risk of multiplication in modelling error in each discipline when different models are combined during the optimisation process. Aeroelastic optimisation includes evaluation of non-linear aeroelastic stability constraints, which consist of the solution of eigenvalues of non-linear unsymmetrical and complex matrices. Evaluation of such non-linear and non-differentiable constraints lead to discontinuities in divergence or flutter speed design parameters.

The purpose of present paper is to demonstrate the accuracy of an aeroelastic optimisation procedure, which was developed based upon a simple modified feasible direction method (MFD), used in the design of flexible wing structure which is linear but the external aerodynamic loads are non-linear and are function of structure response, i.e. : transonic unsteady aerodynamics. The unsteady aerodynamics theories used for the aeroelastic optimisation in the past were linear such as doublet lattice and Theodorsen's strip theory for unsteady subsonic flows and Mach box theory for unsteady supersonic flows. Nonlinear dependency of the unsteady transonic aerodynamic forces on structural response makes it difficult for the integration of aerodynamic solution into the solution of the Aeroelastik equation of motion and stability. Few works specifically associated with wing structure design subject to transonic aeroelastic constraints are available at present time[3,4] . In this works, the optimal design of wing structure were obtained using non-linear unsteady transonic aerodynamic theory by solving transonic small disturbance (TSD) flow equations. The unsteady aerodynamic forces were calculated from the unit pulse aerodynamic response in time domain[5]. The structural parameter considered are those found in the aeroelastic equation of motion: mass ratio, static unbalance, first bending and first torsional natural frequencies and free stream Mach number. Finite difference method is applied in the calculation of design sensitivities. The accuracy of this optimisation methodology are validated using the results obtained using other methods and experimental data.

GENERAL APPROACH

The problem of minimizing wing structural weight while satisfying static strength and aeroelastic stability constraint of the structure is stated in non-linear mathematical programming form and solved using linearized gradient – based optimisation technique. Wing skin, spars and the other wing structural components are assumed to be built-up and modelled as straight stiffness beam, positioned along the elastic axis of the wing, with linear variation of thickness and width in the span wise directions and concentrated masses at the beam center of mass. The dynamic characteristics of the wing are calculated using finite element modelling in which the stiffness beam is modelled using beam elements with three degree-of-freedom at each of its nodal point. The associated design variables consist of beam width and thickness.

It is assumed that the amplitude of bending and torsional deflection is small that the aerodynamic response of the wing geometry is considered linear with the change in the wing angle of attack. This assumption implies that the pade-approximation method can be used for the generalized unsteady aerodynamic force coefficients curve fitting in Laplace domain. The generalized unsteady aerodynamics forces coefficients in Laplace domain themselves are calculated from the Fourier transform of the time – domain aerodynamic response of the wing geometry due to a unit pulse displacement. Unsteady transonic flow fields are represented using transonic small disturbance (TSD) equations. This method is considered more accurate compared with the indicial response method where a jump in the indicial displacement, even with moderately small amplitude, can generate divergence aerodynamic response. Meanwhile, the continuously unit pulse displacement can avoid inaccuracy in the aerodynamic response.

Aeroelastic stability boundary of the structure calculated using the p-k method and a straight forward finite difference method is used for the approximation of flutter constraints sensitivities, which represented by the rate of change of the damping coefficients. The optimisation problem itself is solved iteratively by linear modified feasible direction method.

MINIMUM WEIGHT OPTIMIZATION

The problem of minimizing the optimum structure total weight can be represented in the form of non-linear mathematical programming as follows :

Minimize the objective function : F (x)

subject to the constraint conditions of :

$$g_j(x) \leq 0 \qquad j = 1, 2, \ldots, n_g$$
$$h_k(x) = 0 \qquad k = 1, 2, \ldots, n_h$$
$$x^l_i \leq x_i \leq x^u_i \qquad i = 1, 2, \ldots, n \qquad [1]$$

where F (x) is the total weight of the structure, and x is the vector of design variables which contain all structural physical properties which are changing during the optimization process , such as dimension of the cross section , skin thickness and the concentrated masses. The g_j and h_k functions contain all of the in-equality and equality constraint, respectively, such as the allowable structural component stresses and strains and aeroelastic damping or rate of change of the aeroelastic damping. In addition to those two constraints, there is a side constraint x_i which specify the upper bounds values, x^u_i , and the lower bounds values, x^l_i , of each of the design variables.

For the present optimization problem of wing structural design with aeroelastic constraint using a flexible beam model divided into (numel) - elements , the objective function is the total structural weight of the wing and is represented by :

$$W(x) = \sum_{1}^{numel} \rho_{mi} L_i A_i = \sum_{1}^{numel} \left[m_o L \frac{x_1 \; x_2}{x_1^o \; x_2^o} \right]_i \qquad [2]$$

where W (x) is the total design weight , ρ_{mi} , L_i and A_i are the mass density , length and cross section area of the i – th beam elements , respectively. Two design variables are the flexible beam width, x_1, and the beam thickness, x_2. Zero subscript or superscript represents values of the variables at the wing root.

AEROELASTIC CONSTRAINT

Since variables that come in the aeroelastic constraints function depend on the solution method used in the flutter analysis of the structure , before the aeroelastic constraint can be defined , the method used in flutter analysis of the wing structure have to be decided first. Considering all the available flutter solution method, the p-k iterative method which is based upon the eigenvalues solution of the stability equation, with flow free stream velocity as the input and motion damping coefficients and frequencies as output , is the most ideal to be used in this optimization problem. Using the p-k method for the flutter analysis, the fundamental wing structure equation of motion can be expressed as[6]

$$\left[M_{hh}\, p^2 + \left(-\tfrac{1}{4k} \rho c V Q^I_{hh} \right) p + \left(K_{hh} - \tfrac{1}{2} \rho V^2 Q^R_{hh} \right) \right] \{u_h\} = 0 \qquad [3]$$

where M_{hh} and K_{hh} are the generalized mass and stiffness of the structure , respectively. The real and imaginary part of the generalized aerodynamic forces are denoted , respectively , by Q^R_{hh} and Q^I_{hh} which are function of flow free stream velocity , V , and motion reduced frequency $k = (\omega b / 2V)$. Meanwhile { u_h } represents the structural generalized coordinate which contains nodal displacements. Eigenvalues of the system is given by complex variable p which is defined as

$$p = \omega(\gamma + i) \qquad [4]$$

where ω is the motion frequency (Hz) and γ is the transient damping coefficient.

The unsteady generalized aerodynamic forces coefficients $[Q_{hh}]$ in the above equation is defined as

$$[Q_{hh}] = [\phi]^T [AFC(ik)][\phi] \qquad [5]$$

in which $[\phi]$ and $[AFC(ik)]$ represents , respectively , the structure natural mode shape and matrix of the aerodynamic response coefficient. This aerodynamic response coefficient are calculated by solving , in time domain , the transonic small disturbance flow equations around the wing structure having a unit pulse displacement. Then, by making used of Fourier transform along with pade approximation function, the aerodynamic response coefficients are calculated from this transformation and can be written in term of p – variable (Laplace variable) as[7]

$$Q_{ij}(p) = Q_o + Q_1 p + Q_2 p^2 + \sum_{m=3}^{6} \frac{Q_m p}{(p + \beta_{m-2})} \qquad [6]$$

where β_{m-2} represents phase-lag parameter. The approximating function coefficients Q_o , Q_1 , Q_2 , … are evaluated by least square curve fitting using complex values of Q_{ij} at discrete number of k or p - values. Before solution of the aeroelastic stability equation, Eq. [3] , as an eigenvalue problem can be carried out , this equation should be written in a state – space form as :

$$[A - pI]\{U_h\} = 0 \qquad [7]$$

where I is a unit matrix , A is a real matrix defined by matrix equation

$$[A] = \begin{bmatrix} 0 & I \\ -M_{hh}^{-1}[K_{hh} - \frac{1}{2}\rho V^2 Q_{hh}^R & M_{hh}^{-1}[\frac{1}{4}\rho c Q_{hh}^I / k] \end{bmatrix} \qquad [8]$$

and the vector of generalized coordinate $\{U_h\}$ contains not only nodal point displacement but also velocity. Because A is not a symmetric matrix, conventional numerical technique can not be applied in the calculation of the eigenvalues and eigen modes of this equation.

Dynamic characteristics of the wing structure required for the analysis, the structural natural frequency , ω_i , and mode shape , ϕ , are obtained using a finite element method. The stiffness beam is divided into several beam element with three degree-of-freedom (d.o.f) at each nodal point , which are : one transverse displacement and two torsional / rotation d.o.f .

The flutter constraint is defined by satisfying requirements on modal damping at a series of velocities, rather than defined straight on the actual flutter speed. Aeroelastic constraints , therefore , can be expressed as[8]

$$\gamma_{ij} \le \gamma_{jreq}$$
$$g_j(x_i) = \frac{\gamma_{ij} - \gamma_{jreq}}{GFACT} \le 0 \qquad [9]$$

in which γ_{ij} is the calculated damping coefficient for the i-th mode at the j-th velocity , and γ_{jreq} is the required damping level at the j-th velocity. Both of those constraints have to be satisfied for all mode shape (i) and velocities (j) used in the analysis. The general normalization factor (GFACT) is used to normalized the constraint values (which can not be normalized with respect to γ_{jreq} because this variable could take a value of zero). As described in ref. 7, the GFACT value used for all calculation in this study is 0.1 .

Beside the aeroelastic constraints described above , two side – constraints are also applied in this optimisation process to take into account the maximum and minimum values the design variables x_1 and x_2 (i.e. the width and thickness of the elastic beam) can take on.

SENSITIVITY OF AEROELASTIC CONSTRAINT

Derivative of the constraint function, $g_j(x_i)$, with respect to design variable , x_i , is defined from Equation [9] as

$$\frac{\partial g_j}{\partial x_i} = \frac{1}{GFACT} \frac{\partial \gamma_{ij}}{\partial x_i} \qquad [10]$$

Since this sensitivity derivative is represented in term of γ_{ij} which is obtained from the non-linear eigenvalue solution of the stability equation, Eq. [7] , solution of the constraint sensitivity as given in the above equation will be efficiently obtained using numerical approximation. In this study , calculation of the right hand-side of Eq. [10] are carried out using forward difference formulae. The discrepancy of this approach is that , as in any other finite difference approximation , it has a computational error which may be large that the approximation for the sensitivity become inaccurate. In order avoid this problem , each step interval have to be carefully defined.

OPTIMIZATION PROCEDURE

After the engineering and sensitivity analysis are completed, the structure is then optimized by solving the non-linear mathematical programming problem stated in Eq. [1]. In this study, the method of modified feasible direction (MFD) is applied to solve the optimisation problem[8]. The main task in this method is to find an accurate usable-feasible search direction, S^q , which will defined the direction with maximum gradient in the objectives function , F (x_i) but still lie in the feasible domain (no constraints are violated). Once this search direction defined, new vector for design variables is composed as

$$x^q = x^{q-1} + \alpha S^q \quad , q = \text{iteration number} \qquad [\ 11\ ]$$

where α is the scalar displacement parameter. Values of this scalar parameter are estimated at the beginning of each iteration based upon the gradient of the objective function and constraints. Linear Taylor series approximation is applied in the calculation of the required values of the objective function and also constraints at every iteration step

$$F(x^q) = F(x^{q-1}) + \left[\frac{dF(x^{q-1})}{d\alpha}\right]\alpha$$

$$g_j(x^q) = g_j(x^{q-1}) + \left[\frac{dg_j(x^{q-1})}{d\alpha}\right]\alpha \qquad [\ 12\ ]$$

The procedure to perform the optimisation process are as follows:

a. Select a free stream Mach number and the corresponding velocity , altitude and wing configuration.
b. Define the initial values for design variables and initial dynamic characteristics of the wing structure.
c. Calculate static aerodynamic pressure distribution (by solving the steady TSD equation) and calculate static aeroelastic stability of the structure
d. Based upon the static aeroelastic deformation of the structure , calculate the unit pulse response of the wing structure (solve the unsteady TSD flow equations).
e. Once the unit pulse response is calculated , the generalized aerodynamic forces in Laplace domain can be determined and the flutter damping constraint and sensitivities evaluated.
f. With the results from step e , an optimisation process can be started , yielding a new set of design variables. Calculate a new dynamic characteristic of the structure.
g. With the new design variables, repeat step c to f until a converged optimum results is obtained. At the optimum, the normalized values of the constraint must not larger than the defined error of EMIN and the objective function can not have moved by more than FMIN % from the previous iteration.

EXAMPLE PROBLEM

To demonstrate the preceding optimization derivation , a straight wing model is considered at various flutter speed target and flight conditions. The wing has a rectangular, unswept, untapered planform that uses a stiffness beam representation for the structure's flexibility and concentrated masses for the distributed mass representation. This same rectangular wing was selected to demonstrated transonic flutter prediction in ref. 3. The wing has a moderate aspect ratio of 3.34 with a 6%

hiperbolic airfoil, taper ratio of 0.7 and a tapered cantilever stiffness beam that representing the flexibility of the wing. As shown in Figure 1, the elastic axis of the stiffness beam is placed at 33% chord length from the leading edge, meanwhile its beam mass is position at 43% chord length from the leading edge. Elastic beam is made of aluminium with E = 1.5 E+09 lb/ft^2 and G = 5.5E+08 lb/ft^2. The other structure properties are given in Table 1 below.

Parameter	Values
Chord length	72.0 in
Span length	240.0 in
EI	23.65E+06 lb. - ft^2
GJ	2.39E+06 lb. – ft^2
Mass / span length	0.746 slug / ft
S_α	0.447 slug – ft / ft
I_α	1.943 slug – ft^2 / ft

Table 1. Structural properties of tapered wing

For the determination of structural dynamic characteristics the stiffness beam is divided into 20 beam-elements of equal length with the same number of discrete mass points. Initial values of the design variables are evaluate based upon static load requirements and taken to be x_1 , the stiffness beam width equal to 0.2950 ft and the beam thickness x_2 equal to 0.9221 ft. With this beam initial dimension, the first ten natural frequencies of the structure are in Table 2 below, comparison are made with respect to the uniform model. lt can be seen that , in general , the prediction of the bending frequencies using uniform structural model are more accurate compared to the torsional frequencies.

For aeroelastic analysis it is assumed that the stiffness beam gives no contribution to the 6% hyperbolic airfoil aerodynamics. A cubical spline is applied to accomplished transformation of structural mesh into the aerodynamic mesh required for the solution of TSD flow equation. The aerodynamic response at certain free stream Mach number is obtained using mesh system consist of 100 x 23 x 40 mesh points.

Aeroelastic analysis of the initial structure, carried out using p-k method, shows that the finite element model gives the highest flutter speed for the structure at almost the same flutter frequency, as shown in Table 3. It was shown in the damping versus velocity curve that the initial wing structure undergo a mild flutter in bending mode.

No	Modes	Natural Frequency (Hz)		% Error
		Uniform	Tapered	
1	1st bending	7.872	8.877	12.8
2	1st torsion	12.995	14.667	12.9
3	2nd torsion	38.989	40.199	3.1
4	2nd bending	49.038	51.686	5.4
5	3rd torsion	64.973	66.713	2.7
6	4th torsion	90.962	93.775	3.1
7	5th torsion	116.951	121.480	3.9
8	3rd bending	138.051	141.650	2.6
9	6th torsion	142.940	149.970	4.9
10	7th torsion	168.929	179.400	6.2

Table 2. The first ten natural frequency of tapered wing structure

Flutter parameter	Values		
	Exact	Assumed mode	F E M
U_F (ft/sec)	576.53	564.785	600.00
f_F (Hz)	10.54	11.73	11.04
k_F	0.344	0.358	0.347

Table 3. Flutter properties of the tapered wing

Wing structure optimization are studied for several flutter speed target, which are : 720 , 840 , 960 , 1080 and 1200 ft/sec , or an increase of 20 , 40 , 60 , 80 and 100% in flutter speed from the initial configuration , with maximum error in the constraint and objective function are 0.0001 and 0.001 , respectively. The side constraints of the problem are defined as

$0.01 \text{ ft} < x_1 < 0.5 \text{ ft}$ and

$0.70 \text{ ft} < x_2 < 1.0 \text{ ft}$

The change in design variables from its initial values to the optimum values at various flutter speed target is given in the following Table 4.

U_f	Initial Condition			Optimum values		
	Obj. func.	x_1	x_2	Obj. func.	x_1	x_2
720	14.78	0.295	0.922	15.67	0.321	0.899
840	14.78	0.295	0.922	17.65	0.351	0.927
960	14.78	0.295	0.922	19.45	0.383	0.934
1080	14.78	0.295	0.922	21.52	0.416	0.952
1200	14.78	0.295	0.922	22.55	0.470	0.883

Table 4. Comparison between the initial and optimum value of the design variables

The percentage of change in the design variables from their initial values can be summarised in the following Table 5 below. This table shown that the change in x_1 variables to the optimum design is larger compared to the change of x_2 variable, which indicate that the optimization problem is more sensitive to the x_1 design variable. Compared to uniform wing structure model, this change in design variables are smaller.

U_f	% of change	
	x_1	x_2
720	8.81	-2.49
840	18.98	0.55
960	29.83	1.30
1080	41.02	3.25
1200	59.32	-4.23

Table 5. Change in values of the design variables at several flutter speed target

Time history of the objective function and the aeroelastic constraints during the iteration process are given in Figure 2 and 3. Meanwhile, comparison of the variation of damping values with respect to the free stream velocity for initial and optimal design variables is shown in Figure 4. From Figure 2 it is shown that both the objective function and aeroelastic constraints converged in less than 5 iterations regardless of the flutter speed target. The increase in flutter speed from the initial to the optimal design can be seen in Figure 4 , along with the fact that the flutter mechanism does not change from the initial design.

The change in structure natural frequency at the optimal design compared to the initial design values for various flutter speed target is shown in the following Table 6. It is found that the change in the torsional frequency is much higher compared to the change in the bending frequency. This can be explained as that the change of the x_1 design variable from its initial value is higher compared with the change in the x_2 variable . This cause the change in torsional stiffness of the beam will also higher compared to the change in bending stiffness which. In turn, it will cause a higher change in the torsional frequency of the beam.

The variation of the change in the structure total mass with respect to the percentage of change in the flutter speed target is given in Figure 5. Relation between these two parameters is represented as an S curve.

U_f	Initial frequency (Hz)		Optimal values (Hz)		% of change	
	ω_{bendin}	$\omega_{torsion}$	ω_{bendin}	$\omega_{torsion}$	ω_{bendin}	$\omega_{torsion}$
720	8.88	14.67	8.65	16.42	2.52	11.94
840	8.88	14.67	8.92	18.17	0.55	23.89
960	8.88	14.67	9.00	20.20	1.34	37.71
1080	8.88	14.67	9.16	22.18	3.23	51.22
1200	8.88	14.67	8.50	25.69	4.28	75.16

Table 6. Comparison between the initial and optimum natural frequency at several flutter speed target

CONCLUDING REMARKS

A methodology for including flutter speed requirements in the design of a wing structure is developed and tested. The problem of minimizing structural weight while satisfying static behavioural constraints is stated as a non-linear programming which is solved using a modified feasible direction optimisation procedure. The wing structure is modelled as a stiffness beam with discrete masses using finite element method and the associated design variables consist of beam width and thickness. The unsteady aerodynamic generalised forces are calculated based upon TSD flow solution using unit pulse response technique.

For the straight wing test case, the optimisation problem converged in less than 10 iterations. A higher flutter speed constraint / target will gives an optimum design with a larger different between the first two bending and torsion natural frequencies and a larger total mass.

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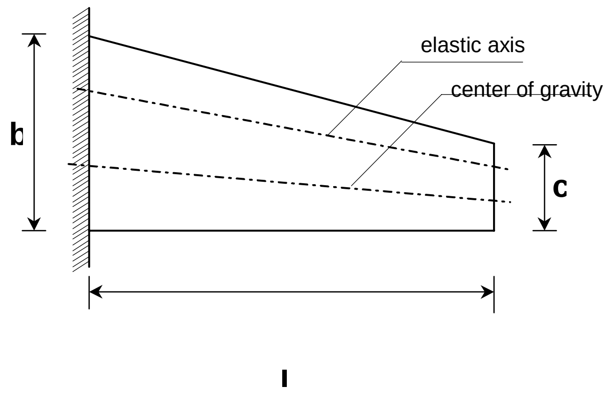

Figure 1. Rectangular wing planform With taper ratio of 0.7

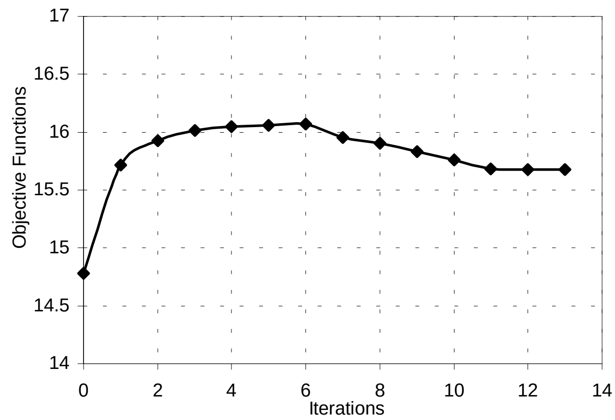

Figure 2. Time history of the objective function at flutter speed target of 720 ft/sec

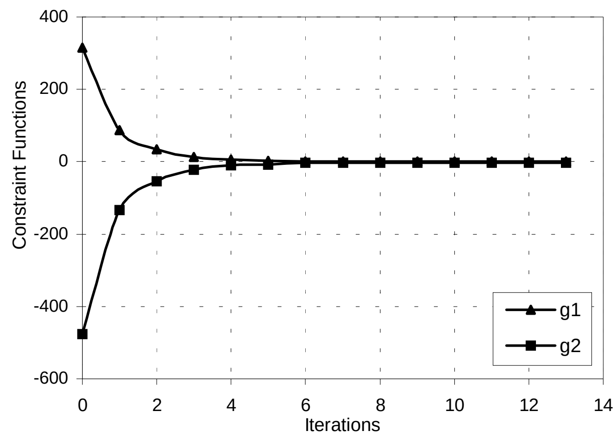

Figure 3. Time history of the constraint at flutter speed target of 720 ft/sec

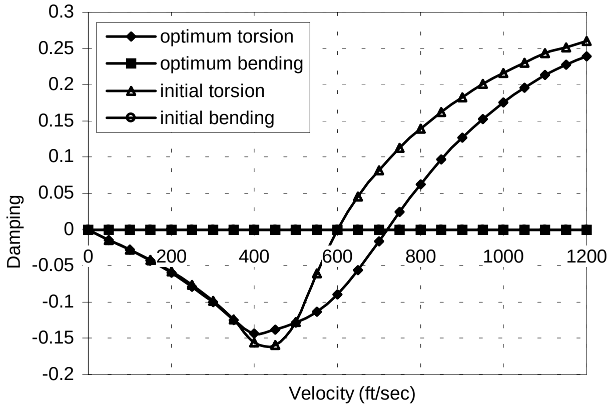

Figure 4. Variation of damping coefficients with velocity for rectangular wing at flutter speed target of 720 ft/sec.

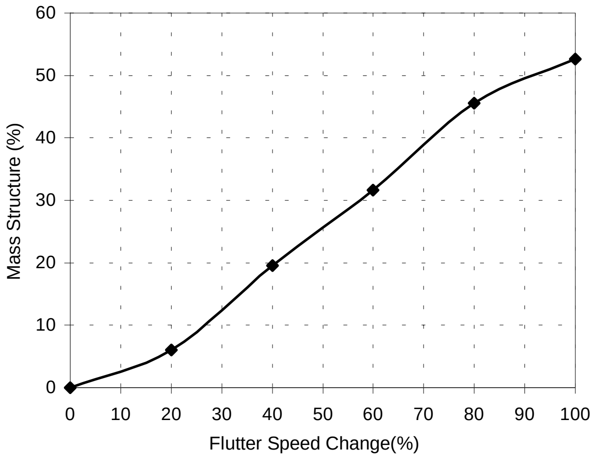

Figure 5. Variation of total mass with respect to change in flutter speed target