STATIC AEROELASTICITY ANALYSIS IN TRANSONIC FLOW

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Abstract

In this paper the method is suggested for taking account of transonic phenomena for calculations of static aeroelasticity characteristics. This work is implemented as further development of the methods and software of the multidisciplinary computer code KC-2 which is used for the analysis of the problems of static and dynamic aeroelasticity in Russian aviation design bureaus. Elastic characteristics of the structure are computed on the base of polynomial Ritz method. Iteration technique is used to analyze static aeroelasticity characteristics in the transonic regime. Pressure distribution is computed by solving Euler equations by using Godunov's method. Then a numerical linearization is performed for transonic pressure distribution and a linear analysis of static aeroelasticity characteristics is made. The procedure is repeated up to convergence on trim angles and displacements of a structure.

The results of the analysis of static aeroelasticity characteristics in transonic flow for supersonic transport are presented. The comparison with the results of linear analysis is shown.

1. Introduction

Static aeroelasticity analysis in transonic flow is urgent for various types of modern aircraft. Many flying machines have transonic flight as critical regime from the stability and control viewpoint. Therefore, it is important to take into account transonic phenomena when stability and control derivatives of the elastic aircraft are calculated. One of the conditions of raising the economy of high-speed civil aircraft is more accurately prediction of the cruise shape, which depends on the structural elastic displacements in a transonic flow.

Today the practical computational studies of aeroelasticity of airplanes are basically based on linear methods of computation of aerodynamic forces. Doublet-Lattice Method (DLM) [1, 2] and different versions of panel methods [3, 4, 5] are used more often than not.

Advanced computational methods of aerodynamics are founded on the solution of Euler equations and sometimes - Navier-Stokes' equations [6, 7]. They are basically used for aerodynamic design of airplanes, and seldom are used for problems of aeroelasticity. The wide application of these methods for the solution of practical problems of aeroelasticity is limited by the several causes:

- complexity of aerodynamic grid generation for an actual airplane. The successful fulfillment of such calculations without participation of the expert on computational aerodynamics is rather problematic;
- complexity of the fitting of an aerodynamic grid with the mathematical model of a structure;
- necessity of power computational tools.

The urgency of these problems is increased at using of the multidisciplinary approach to structural design, when the calculations of the characteristics of aeroelasticity are executed many times for various variants of a structure [8, 9].

The main difference of transonic aerodynamics is a non-linear dependence of aerodynamic forces on parameters of motion. In this case formulation of the task about finding of the static aeroelasticity characteristics should be considered anew, since concepts of 'derivative of aerodynamic coefficients', 'flexibility correction factor' etc. usually imply linear dependencies. Note that 'the characteristics of a rigid airplane' concept is also ambiguous in a case of a nonlinear aerodynamics.

Recently an essential progress in usage of transonic aerodynamics was reached for multidisciplinary design optimization (MDO) of a structure. It is based on the different approaches to a linearization of aerodynamic forces [10, 11].

Here we shall consider computation of the characteristics of static aeroelasticity in a transonic flow by using our approach, which is founded on a usage of a numerical linearization of transonic pressure distribution and linear analysis of static aeroelasticity. The mentioned formulation is adapted to the methods and software packages which are used in TsAGI, namely:

- the modern version of a polynomial Ritz method;
- founded on Ritz method software package KC-2;
- Godunov's method of the solution of Euler equations for transonic flow (software package TRANS [12]).

The aerodynamic grid for transonic analysis is generated by using the same input data, as for linear case. The problems of the fitting of an aerodynamic grid with the mathematical model of a structure do not arise in a polynomial method. Efficiency of methods and the software allows to perform computations for an actual structure on personal computers for reasonable time.

The main purpose of this approach and method we see in refinement of the stability and control derivatives (flexibility correction factors) and elastic deflections of an airplane in transonic flight. The sequence of computations and application of a suggested procedure for a transonic flight phase of a second generation supersonic transport airplane (SST-2) is reviewed below.

2. General Outline of Computation

The suggested sequence of computations of the characteristics of static aeroelasticity in a transonic flow can be conventionally devided into six stages:

1. Computation of polynomial matrices of structural stiffness and inertia *G*, *C*, aerodynamic stiffness and damping *B*, *D* by using linear (subsonic or supersonic) aerodynamics.
2. Linear analysis of the characteristics of static aeroelasticity.
3. Computation of structural displacements for two close angles of attack.
4. Computation of a transonic pressure distribution for these two states of structural displacements.
5. Linearization of aerodynamic forces and computation of new (transonic) matrices *B* and *D*.
6. Linear computation of the characteristics of static aeroelasticity with new aerodynamic matrices.

The items 3 - 6 can be repeated (for example, up to convergence on a trim state). Each stage of computation is reviewed more detail below.

3. Computation of Polynomial Matrices.

To analyze an elastic structure in software package KC-2 the Ritz method is used, when the displacements of elastic surfaces (ES) are presented as polynomial functions of the spatial coordinates. A whole structure is modeled by a set thin, originally flat ES, which can be arranged in space arbitrarily. For each of ES the mass and stiffness distribution is specified. The different type elements can be used to model of an actual structure - concentrated masses, bending and torsion beams, plates, panels, etc. The local coordinate systems for each elastic surface are selected so that the plane *x0z* coincides with ES plane. The normal displacements *W (x, z, t)* of an elastic surface are expressed in the following way:

$$W(x,z,t) = \sum_{k=1}^{N} f_k(x,z) u_k(t) ,$$

where $f_k(x,z) = x^{m_k} z^{n_k}$, $m,n = 0,1,...$

For each ES the polynomial can be chosen separately. The factors $u_k(t)$ are used as generalized coordinates of the polynomials method.

The motion of ES in a self-plane is described by a set of additional generalized coordinates.

Further elastic surfaces are integrated in unified computational model with the help of elastic springs, which allow to model different conditions of ES connection among themselves.

For computation of aerodynamic forces the DLM or panel methods are used. Steady-state aerodynamics or unsteady one at small reduced frequency (for more precise estimations for derivative on angular rate) is used.

The equation of motion of an elastic structure in an airflow is represented by the way:

$$C\ddot{u} + (D + D_0)\dot{u} + (B + G)u = Q_0 \qquad (1)$$

Here u - united vector of generalized coordinates;

C, D, D_0, B, G - united matrices of inertia, aerodynamic and structural damping, aerodynamic and structural stiffness;

Q_0 - vector of generalized forces due to initial camber and twist of lifting surfaces.

4. Computation of the Characteristics of Static Aeroelasticity

To solve aeroelasticity problems the special variables are necessary which represent a motion of a structure as a whole and deflections of control surfaces. Such variables are generated from components of united vector of generalized coordinates. The matrices of an equation of motion also are transformed to a special form, in which whole rigid body motion variables and relative control displacements are explicitly separated. The transformation of equations of motion to the static aeroelasticity problem form is performed as a set of linear transformations. The resultant transformation is presented as:

$$\tilde{u} = X_{st}\, u \qquad (2)$$

where $\tilde{u} = (u_{rig}, u_{el})^T$ is a new vector of generalized coordinates of dimension N_{st};

u_{rig} - vector including rigid body motion variables and relative control displacements;

u_{el} - vector corresponding to relative elastic displacements of a structure;

X_{st} - transformation $N_{st} \times N_u$ matrix;

N_u – dimension of initial united matrices.

The matrix X_{st} will be used for recovery of a vector of polynomial generalized coordinates of initial structure after the solution of equations of static aeroelasticity.

The selection a component of vector u_{rig} is one of important points in problems of aeroelasticity. For example, the influence of structural elasticity on aerodynamic coefficients can hardly depend on the manner of definition of the angle of attack. The angle of attack is usually defined as an angle between undisturbed airflow and tangent to one of elastic surfaces near to a center of mass of an airplane (base point). The angle of an elastic control surface deflection is usually defined as an angle between tangents to basic and control surface in section, in which actuator is located.

The equation of aeroelasticity (1) is presented in block's form under quasi-steady assumption when inertial and damping forces due to elastic deformations u_{el} are not taken into account:

$$\left[\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} + \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}\right]\begin{pmatrix} u_{rig} \\ u_{el} \end{pmatrix} = \\ = -\begin{pmatrix} D_{11} \\ D_{21} \end{pmatrix}\dot{u}_{rig} - \begin{pmatrix} C_{11} \\ C_{21} \end{pmatrix}\ddot{u}_{rig} + \begin{pmatrix} Q_{0,1} \\ Q_{0,2} \end{pmatrix} \qquad (3)$$

The structural damping is not considered in static aeroelasticity. The vector u_{rig} has following structure for a case of longitudinal motion:

$u_{rig} = (u_x, u_y, \alpha, \delta)^T$, $\quad \dot{u}_{rig} = (V_x, V_y, \omega_z, \dot{\delta})^T$,

$\ddot{u}_{rig} = (gn_x, gn_y, \dot{\omega}_z, \ddot{\delta})^T$.

Here α, δ are angles of attack and control surface deflection; ω_z is pitch rate; u_x, u_y, V_x, V_y, n_x, n_y are displacements, velocities and load factors of the base point along Ox and Oy axes, g - gravity acceleration.

The blocks of matrices B_{11}, D_{11} contain aerodynamic derivatives of a rigid airplane. To obtain the same blocks under quasi-steady count of structural flexibility, the vector u_{el} from the second equation is substituted to the first:

$$(C_{11} - B_{12}P_0)\ddot{u}_{rig} = -(B_{11} + G_{11} - B_{12}P_2)u_{rig} -$$
$$-(D_{11} - B_{12}P_1)\dot{u}_{rig} + (Q_1 - B_{12}ZQ_{12})$$

Here $P_0 = ZC_{21}$; $P_1 = ZD_{21}$; $P_2 = ZB_{21}$; $Z = (B_{22} + G_{22})^{-1}$. To find an accordance between coefficients of the equation and aerodynamic derivatives, it is necessary to transform an equation to "normal" view, in which there is a term $C_{11}\ddot{u}_{rig}$ in the left side. It can be made by two ways. The first way: to transfer the term $B_{12}P_0\ddot{u}_{rig}$ to the right part. This term defines the influence of inertial forces on aerodynamic coefficients of an elastic airplane; the remaining terms in a right part of the equation correspond to aerodynamic derivatives without inertial forces. The obtained in such a way characteristics are named also as characteristics of a 'weightless' airplane. The second way is a left multiplication of the equation by a matrix $C_{11}(C_{11} - B_{12}P_0)^{-1}$. In this case there are no derivatives on accelerations and the aerodynamic derivatives are obtained with taking account of elastic displacements due to inertial forces; they are named also as characteristics of a 'weighted' airplane. Joint consideration of these derivatives allows to evaluate mass distribution influence on static aeroelasticity characteristics (including other versions of loading).

To estimate divergence and reversal characteristics the derivatives of non-dimensional aerodynamic coefficients are computed in dependence on dynamic pressure at a set of Mach number. The increase of some derivatives (for example, lift slope coefficient c_y^α ,...) shows divergence tendency, and the decrease of control derivatives (for example, roll moment derivatives due to aileron deflection $m_x^{\delta_a}$) shows reversal tendency. An estimation of structural elasticity influence on the aerodynamic coefficient is performed through relative value ξ, which equals to ratio of aerodynamic coefficient derivatives of elastic and "rigid" aircraft ($\xi_{c_y^\alpha} = c_{y\,el}^\alpha / c_{y\,rig}^\alpha$, ...). Relative shift is determined for the aerodynamic center $\Delta\bar{x}_F = \bar{x}_{F\,el} - \bar{x}_{F\,rig}$. The influence of the control surface attachment stiffness (or actuator stiffness) on the stability and control characteristics of the aircraft is also investigated.

To solve a maneuver problem the weighting coefficients of control participation and parameters of maneuver are set. The trim angle and elastic displacements are determined by the solution of an initial (full) equation (3). The right part of the equation is determined through parameters of motion. Thus the strained state in generalized and physical coordinates is determined.

The loads can be further computed through elastic deformations and characteristics of elasticity of a structure.

In a case of a fixed airplane the values of angles of attack (or side-slip) and deflection of a generalized elevator (aileron and rudder - for lateral motion) are specified, then the deformations are computed in generalized and physical coordinates.

5. Computation of structural displacements for two close angles of attack in specified flight regime.

Alongside with the mentioned above characteristics two generalized coordinate vectors are determined for computations in a transonic flow. They are close in the sense of the angle of attack of the airplane and have following form:

$$\vec{u}_1 = \begin{Bmatrix} 0 \\ \alpha \\ \ddot{a} \\ u_{el} \end{Bmatrix} \qquad \vec{u}_2 = \begin{Bmatrix} 0 \\ \alpha + \Delta\alpha \\ \ddot{a} \\ u_{el} \end{Bmatrix}$$

Here α, δ, u_{el} - angle of attack, deflection of a generalized elevator and elastic deformation vector according to a trim condition of an elastic structure in a given flight regime. For a fixed structure α and δ are considered as specified.

These vectors are converted to initial structure of generalized vector

$$u_1 = X_{st}^T \tilde{u}_1 \qquad u_2 = X_{st}^T \tilde{u}_2$$

and are stored for computation of distribution of local angle of attack in a transonic flow. To analyze a rigid (undeformed) structure the vectors are used which correspond to the same angles of attack without deformations (but not a trim angle of a rigid airplane):

$$\tilde{u}_1 = \begin{Bmatrix} 0 \\ \alpha \\ 0 \\ 0 \end{Bmatrix} \qquad \tilde{u}_2 = \begin{Bmatrix} 0 \\ \alpha + \Delta\alpha \\ 0 \\ 0 \end{Bmatrix}$$

The computations indicated in item 4, 5, are performed in the STAER program from KC-2 package.

6. Computation of transonic pressure distributions for two displacement fields

The further computations are made in the program TRANS. The same geometry of an airplane (aerodynamic trapeziums) is used as in the case of linear aerodynamics. However, dividing into elementary cells differs essentially. The computations are performed using dimensionless coordinates referred to a root chord of a wing b_{o1}. The polynomial matrices of transformation X_p, X_P^x, X_P^z from vector of generalized coordinates u to displacement W_i and angles $\frac{\partial W_i}{\partial x}$, $\frac{\partial W_i}{\partial z}$ in points of a grid x_i, z_i (i=1,...,N_{kk}) are further calculated. Through these matrices the field of displacement and angles is determined for mentioned above vectors u_1 and u_2. The displacement velocities are not considered here, the only static case without angular rate is supposed.

Additional angles due to lifting surface camber and twist are added to the obtained angles; they are determined by interpolation on airfoils, specified in a set of sections of the surfaces.

The initial conditions (velocities, pressure and density) of an undisturbed flow are set in all cells. Then Euler equations integrating starts for 1-st displacement field by using finite-difference Godunov's method [12]. The field of pressure in all cells is determined. The difference of dimensionless pressure between a upper and lower wing surface $\Delta C_{pi}^{(1)}$ (i=1,..., N_{kk}) is calculated for the cells, which are adjacent to the structure. The integrating is executed up to relaxation (for example, on total lift coefficient c_y). Then the computation is repeated for the second displacement field and $\Delta C_{pi}^{(2)}$ is determined.

7. Linearization of aerodynamic forces and computation of transonic matrices *B* and *D*

A linearization can be performed through two computed pressure distributions and a derivative of relative pressure with respect to local angle of attack can be find:

$$\Delta \bar{C}_{pi}^{\alpha} = \frac{\Delta C_{pi}^{(2)} - \Delta C_{pi}^{(1)}}{\alpha_i^{(2)} - \alpha_i^{(1)}} \qquad (i=1,..., N_{kk})$$

Here $\{\alpha_i^{(1)}\} = X_P^x u_1$; $\{\alpha_i^{(2)}\} = X_P^x u_2$.

The derivative of a pressure is obtained by multiplying on the area of cells:

$$\Delta C_{pi}^{\alpha} = \Delta \bar{C}_{pi}^{\alpha} (b_{o1})^2 \bar{S}_i \Delta x \Delta z ,$$

Here $\bar{S}_i$ is relative area of cells, and Δx, Δz are relative sizes of cells.

The aerodynamic stiffness and damping matrices B and D are further calculated as:

$$B = -\frac{\rho V^2}{2} X_P C_P^{\alpha} X_P^x$$

$$D = -\frac{\rho V^2}{2} X_P C_P^{\alpha} X_P$$

Here ρ is air density;

V=M V_s (M is Mach number, V_s is speed of sound);

C_P^{α}=diag(ΔC_{pi}^{α}) is a diagonal matrix of pressure derivative on a local angle of attack.

8. Static aeroelasticity analysis with new aerodynamic matrices

In the case of linear aerodynamics the computation in STAER program is performed for a given set of dynamic pressure values. In the transonic case it is meaningful to perform computation for one value of dynamic pressure, for which the deformations and aerodynamic matrices are obtained.

A computation process in this case doesn't outwardly differ from the computation with linear aerodynamics. Two close structural displacement fields will also be obtained, which however can differ from obtained with linear aerodynamics. Therefore procedure is repeated up to convergence which is defined through structural displacements. The computation tests have shown, that the process converges for 3-4 iterations.

As a result the stability and control derivatives and elastic structural displacements are obtained with taking account of transonic features of flow: airfoil thickness, shock waves, finite angles of attack and control surfaces deflection.

9. Supersonic Transport Airplane

At designing of the SST-2 it is supposed, that there should be two cruise flight regime: supersonic regime at a Mach number M=2.0-2.2, and subsonic one at M=0.9-0.95. For the first (supersonic) regime all panel methods give close results on the characteristics of static aeroelasticity. A first generation SST experience demonstrates that these results are rather well agreed with experimental data. The second regime is more critical from the static aeroelasticity viewpoint - both on stability, and on a controllability and ensuring the demanded characteristics of trimming. Therefore the refinement of the linear static aeroelasticity characteristics is urgent with allowance for of transonic phenomena.

The version of SST-2 with titanium wing (40m span, 3% thickness ratio) is considered; total weight equals to 300 tons. A structural computational model consists of a set of beams, orthotropic panels, concentrated masses and joint springs. The aerodynamic model for DLM and supersonic panel method is presented in a fig. 1; it contains 359 boxes for half structure.

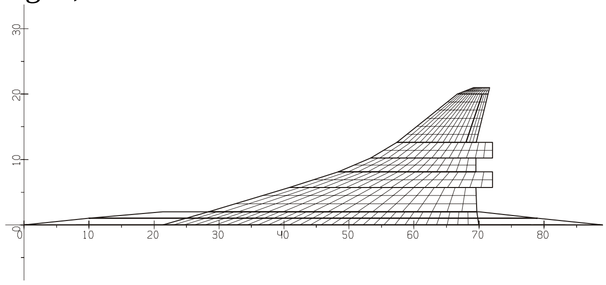


Fig. 1. SST aerodynamic model for DLM and panel methods

The spatial aerodynamic grid for transonic computations contains 60×15×30=27500 cells. From them 361cells are adjacent to a wing surface; ΔC_{pi}^{α} are determined in them. The computations were executed on the personal computer Pentium-2; one iteration took approximately 90 minutes.

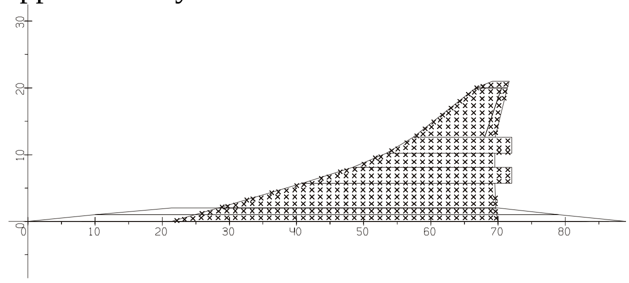


Fig. 2. Aerodynamic cells for Godunov's method

It is difficult to compare among themselves the free airplane characteristics which are obtained with the linear and transonic theory as the trim angles will be different. Therefore the characteristics of an airplane which is fixed near to a center of mass at specified angle α_0 are considered for comparison.

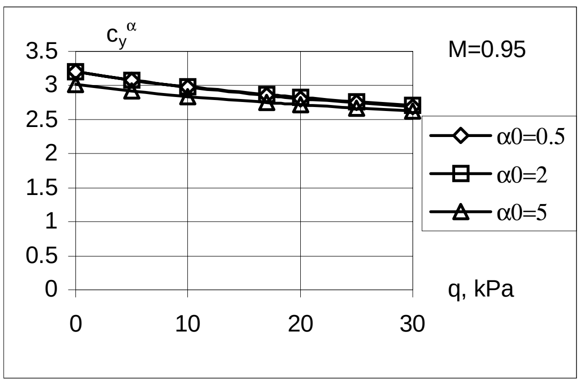


Fig. 3. Flexibility influence on a lift slope coefficient.

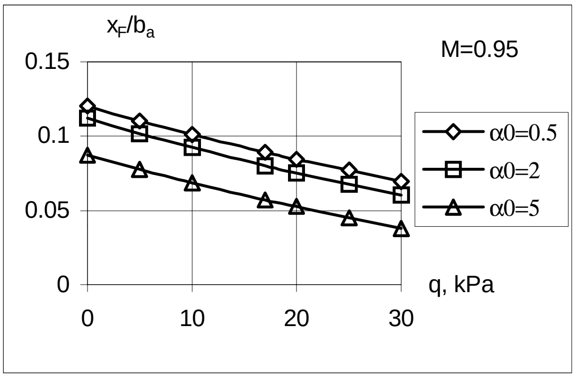
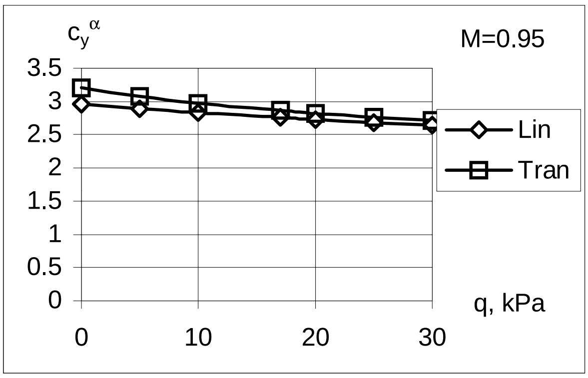


Fig. 4. Flexibility influence on aerodynamic center position

The characteristics which are obtained by the linearization of pressure distribution at various α_0 demonstrate that they differ insignificantly up to values α_0=2°-3°. Fig. 3, 4 show a lift coefficient slope and position of an aerodynamic center in parts of a mean aerodynamic chord versus dynamic pressure at values α_0=0.5°, 2°, 5°. However these linearized characteristics essentially differ from the characteristics which are obtained by using DLM even at the small α_0 (fig. 5).

a)



b)

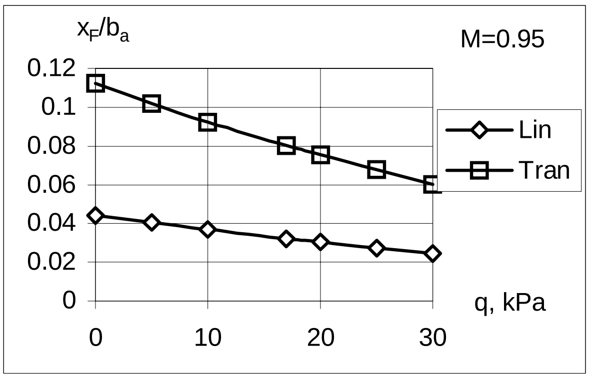


Fig. 5. Comparison of DLM and TRANS (α_0=2°) results

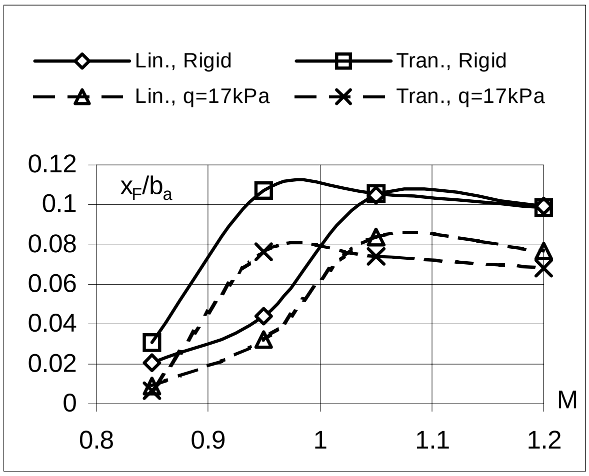
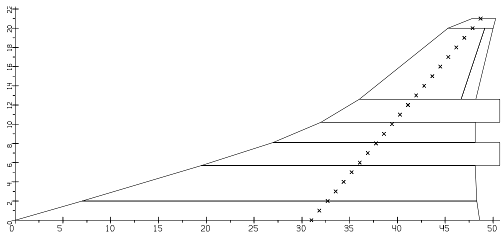


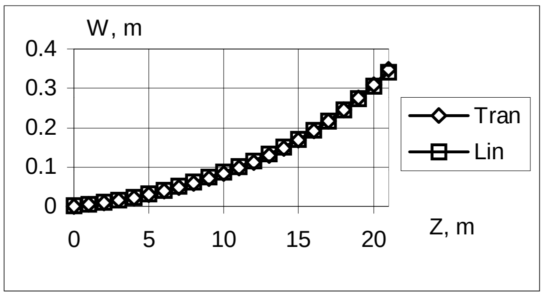
Fig. 6. Aerodynamic center position versus Mach number for rigid and elastic structures.

One of the principal features of transonic flow is the downstream shift of the aerodynamic center in comparison with linear methods (fig. 6).

a)



b)



c)

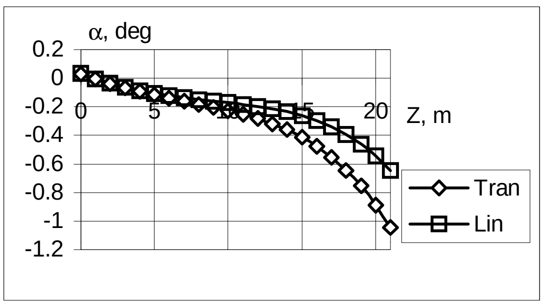


Fig. 7. Elastic displacement (W) and streamwise twist angle (α) along a center of the wing box for DLM and TRANS (M=0.95, α_0=2°).

This phenomenon causes substantial growth of stream-wise torsion angles (fig. 7, c). Elastic displacements along wing box center differ slightly (fig.7, b).

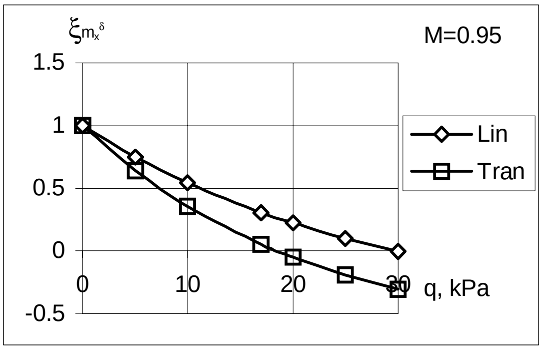


Fig. 8. Relative aileron effectiveness.

The shift of the aerodynamic center downstream causes also some decrease of dynamic pressure of aileron reversal (fig. 8).

10. Concluding Remarks

The considered above method of static aeroelasticity analysis in the transonic flow is not perfect and it has several limitations:

- only a set of lifting surfaces (without body configuration) are considered;
- the linearization of pressure distribution is performed on angle of attack only;
- a calculation of aerodynamic derivatives on angular rate is not considered.

Further studies must be performed for many aspects of the task. But in our opinion the approach allows to take into account principal features of transonic flow for aeroelasticity analysis. An important advantage of the approach is that it is well coordinated with linear analysis and it is adapted and integrated to software package KC-2. We hope that transonic analysis of static aeroelasticity will come into practice at stage of aircraft design and analytical support of wind tunnel tests.

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