OPTIMISATION OF CUT-OUTS IN FIBRE COMPOSITE COMPONENTS USING FINITE ELEMENT METHODS

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Abstract

Structural optimisation has been used to reduce stresses associated with cut-outs and produce optimum designs in composite panels. The optimisation utilised the design sensitivity features of MSC.Nastran and reduced stress concentrations by changing cut-out geometries. Reductions in peak failure indices around the hole boundaries of at least 32% were achieved for a variety of bi-axial and shear load cases. It was found that the optimum cut-out shapes in composite laminates could differ significantly from those in an isotropic panel, especially under shear dominated loading. The methodology developed for cut-outs in plates was applied to the optimisation of a wing rib manufactured from laminated composite materials. The shapes of both the stringer cut-outs and access holes were optimised in order to minimise their influence. It was demonstrated that shape optimisation was able to significantly reduce the peak failure indices in the rib, implying that fewer plies would be needed to meet strength requirements.

1 Introduction

Cut-outs introduce stress concentrations that can limit the operational life of an aircraft structure. Consequently, the study of such cut-outs and the development of design methodologies to reduce their impact has been the subject of much research. The development of advanced numerical design and analysis tools, such as the optimisation capabilities of finite element codes, has given renewed impetus to the study of such features. At the same time, the progressive introduction of laminated composite materials into aircraft structures has added complexity to the design process and presents an excellent application for the development of optimisation techniques.

An ideally optimised cut-out, known as a neutral hole, has no variation in the stress field compared to a similar structure without the cut-out. Senocak and Waas [1] showed that a neutral hole could be achieved by changing the shape of the hole and by increasing the thickness around the hole. One means of arriving at such a cut-out design is through the use of structural optimisation, which includes shape and local reinforcement methods [2-4]. The shape optimisation method refers to the process of generating improved local geometry, while the local reinforcement method refers to the process of applying additional material in the region close to the cut-out in order to reduce the stresses.

There exists a variety of methods for shape optimisation, *viz.* biological growth method, Neuber stress reduction method, sensitivity analysis method, evolutionary optimisation method and homogenisation method. The biological growth method proposed by Mattheck [5] and the Neuber stress reduction method are shape-based techniques in which material is added to regions of high stress. The Evolutionary Structural Optimisation (ESO) method used by Falzon *et al.* [6] and homogenisation methods are based on lay-out optimisation theory in which low stressed elements are "removed" from the finite element analysis model.

The design sensitivity technique is a general optimisation method in which the gradients of the design variables with respect to the design responses are calculated [7]. The primary advantage of this approach is that convergence can generally be achieved in a small number of design cycles, however, this is offset by more computational effort required to determine the gradients. Thomson *et al.* [8,9] used this method for shape optimisation of cut-outs in isotropic and composite panels under specific loading conditions. Also investigated in these works was the addition of local reinforcement around the cut-out boundary with the aim of producing a neutral hole. Similar research into local reinforcement and optimum ply orientation has been conducted by Wang and Costin [10] and Senocak [11]. In the present study, the methods developed previously have been extended to include further load cases and are demonstrated in a practical design environment for the optimisation of a composite rib under critical load conditions.

2 Optimisation of Cut-Outs

The optimum shapes of cut-outs in a square panel under a variety of loading conditions have been investigated. The panel configuration considered was a 250 x 250 mm plate containing a centrally located hole of diameter 50 mm, as shown in Figure 1. The sign convention used for the load definition and the composite material definition are also shown in this figure. The finite element mesh used for the analysis consisted of 1,560 QUAD4 elements, as shown in Figure 2. The load cases considered, which were a variety of bi-axial, shear, and combined shear and compression, are presented in Table 1.

Optimisation was carried out for two different laminate configurations, presented in Table 2, as well as a baseline isotropic plate to investigate the material specific behaviour. For the isotropic plate, the properties of aluminium were used and for the composite laminates, carbon/epoxy pre-preg tape was used, as presented in Tables 3 and 4 respectively.

MSC.Nastran incorporates an optimisation capability [12,13] that uses the design sensitivity analysis method. The optimisation procedure consisted of several steps that included available automation and manual manipulation. An analysis model, which is a model for static analysis, was defined using the pre-processor, MSC.Patran, then modified manually to include the design optimisation entries. This design model was then subjected to the MSC.Nastran *optimiser* for optimisation, the results of which were post-processed using MSC.Patran.

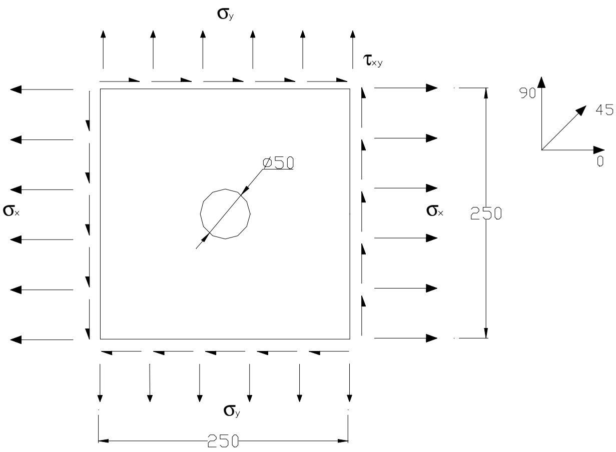


Figure 1 Square plate with central hole (in mm)

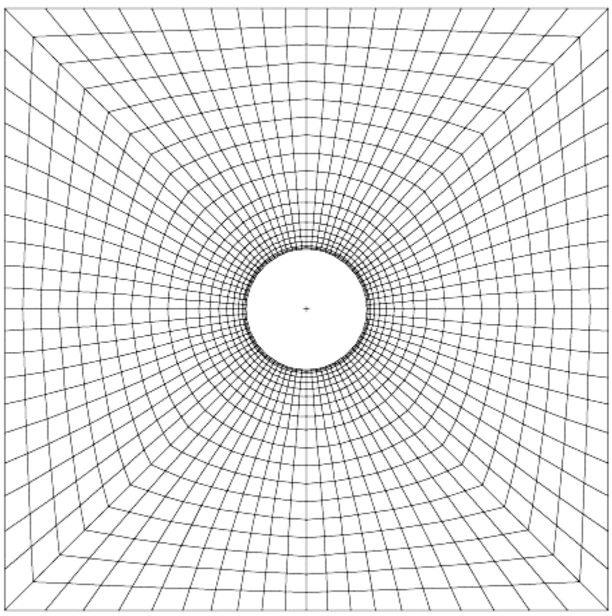


Figure 2 Typical finite element mesh

Table 1 Loading used for optimisation

Load Case	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)
2:1 Bi-axial Tension	240	120	0
4:1 Bi-axial Tension	480	120	0
Shear	0	0	240
1:0.5 Shear/Comp.	-120	0	240
1:1 Shear/Comp.	-240	0	240
1:2 Shear/Comp.	-480	0	240

Table 2 Composite laminates used for optimisation

Panel	Lay-up
Laminate 1	$[45/-45/90/0]_{2S}$
Laminate 2	$[45/-45/90/0_2]_{2S}$

Table 3 Properties for the isotropic plate

Property	Value
Young's modulus, E	69 GPa
Poisson's ratio, υ	0.3
Thickness	10 mm
Density, ρ	2.71×10^3 kg/m^3

Table 4 Properties of T300/914C uni-directional pre-preg carbon fibre / epoxy tape

Property	Value
Longitudinal Modulus, E_1	130.0 GPa
Transverse Modulus, E_2	4.65 GPa
In-Plane Shear Modulus, G_{12}	4.65 GPa
Poisson's Ratio, ν_{12}	0.3
Ply Thickness	0.125 mm
Long. Tension Ult. Strain, X_T	9230 με
Long. Comp. Ult. Strain, X_C	7692 με
Trans. Tension Ult. Strain, Y_T	10750 με
Trans. Comp. Ult. Strain, Y_C	25810 με
In-plane Shear Ult. Strain, S	13980 μγ

3 Optimisation Approach

Shape optimisation in MSC.Nastran requires the definition of an auxiliary model, the objective function and constraints. For shape optimisation of cut-outs using the method detailed in the following sections, no design constraints were necessary.

3.1 Auxiliary Model

It is necessary to define the allowable shape changes in MSC.Nastran, which was accomplished through the use of an auxiliary model. The auxiliary model relates the allowable variations in nodal coordinates through shape basis vectors. These shape basis vectors must be chosen carefully so as to allow free changes to the shape whilst not causing excessive element distortion. The auxiliary model used for the investigation used the geometry and mesh shown in Figure 2, but with the loads and boundary conditions shown in Figure 3. The radial enforced displacements were applied at the nodes two elements away from the boundary so that critical stress values were obtained from undistorted elements throughout the optimisation cycle. This was important since the optimisation capability of MSC.Nastran does not allow removal of elements or re-meshing during shape optimisation.

3.2 Objective Function

MSC.Nastran allows an arbitrary response to be defined as a mathematical relationship between other responses, permitting great flexibility when defining objective and constraint functions. While many possible objectives exist, the two most noteworthy are termed the least squares and the maximum stress [8]. For the optimisation of cut-outs in plates, the least square objective function, which relates the stress at a point on the boundary to the average stress around the entire boundary, was chosen. This objective rapidly results in constant stress around the hole boundary by increasing the lower valued stresses and decreasing the higher valued stresses. The objective for an isotropic material is of the form:

$$\text{Minimise } \frac{\sum(\sigma_i - \sigma_{av})^2}{k^2} \qquad (1)$$

where: σ_i = hole elemental Von Mises stress
σ_{av} = average elemental Von Mises stress
k = number of boundary elements

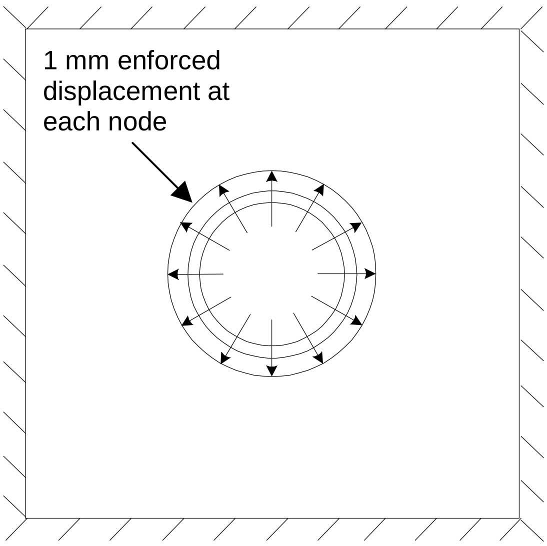


Figure 3 Auxiliary model for shape optimisation showing loads and boundary conditions

For laminated composite materials, the least squares objective function was based on elemental failure indices which were calculated using the maximum strain failure theory. It therefore took the following form:

$$\text{Minimise } \frac{\Sigma(FI_i - FI_{av})^2}{k^2} \qquad (2)$$

where: FI_i = element failure index
FI_{av} = average boundary failure index

The element failure index is the maximum failure index of all plies in the element.

4 Cut-Out Shape Optimisation Results

4.1 Biaxial Loading

The optimum cut-out shape and the corresponding reduction in Von Mises stress or failure index under 2:1 and 4:1 bi-axial loading are summarised in Tables 5 and 6 respectively. The optimum shapes of Laminate 1 are shown in Figures 4 and 5. In all cases, optimisation under bi-axial loading resulted in elliptical or nearly elliptical cut-outs with constant boundary stress or failure index. The effectiveness of the least squares objective function is demonstrated in Figure 6, where the failure indices are seen to have increased or decreased to achieve a constant value at the boundary. The failure index in these graphs has been normalised such that the optimum shape gives a boundary failure index of unity.

It was found that the optimum shape was dependent upon the degree of orthotropy of the material. The quasi-isotropic laminate (Laminate 1) produced shapes very similar to the isotropic case, whereas Laminate 2 generally had a lower aspect ratio. However, similar reductions in failure index were observed in the two laminates. In all cases, convergence was achieved in between six to twelve design cycles proving the efficiency of the design sensitivity approach used in MSC.Nastran and the least squares objective function.

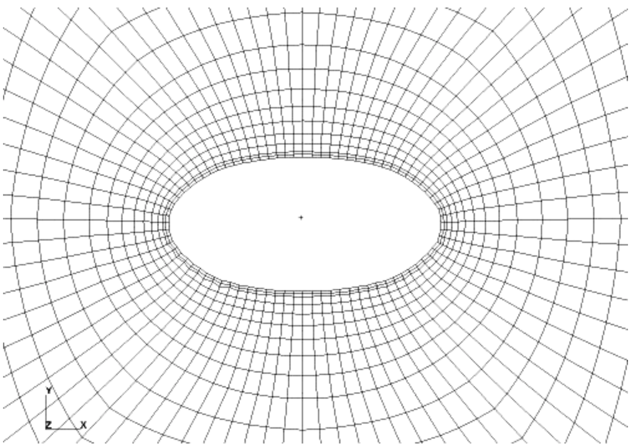


Figure 4 Optimum cut-out under 2:1 uniform bi-axial load for Laminate 1

Table 5 Aspect ratio and the reduction of boundary failure indices for 2:1 bi-axial load

Panel	Aspect Ratio	FI Reduction
Isotropic	2.01	42.4%
Laminate 1	2.05	44.8%
Laminate 2	1.61	35.9%

Table 6 Aspect ratio and the reduction of boundary failure indices for 4:1 bi-axial load

Panel	Aspect Ratio	FI Reduction
Isotropic	4.18	32.4%
Laminate 1	4.26	59.1%
Laminate 2	3.18	56.5%

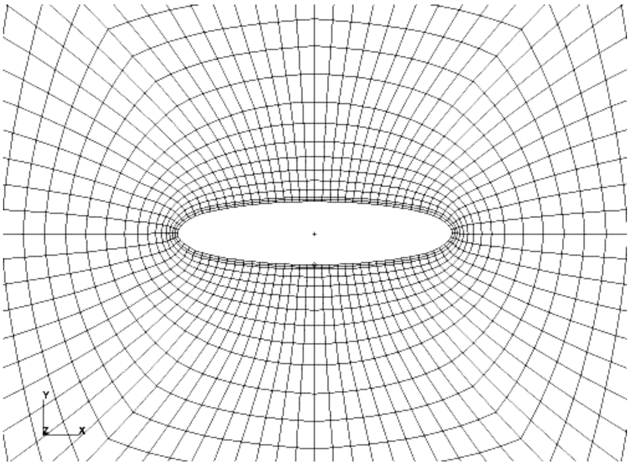


Figure 5 Optimum cut-out under 4:1 uniform bi-axial load for Laminate 1

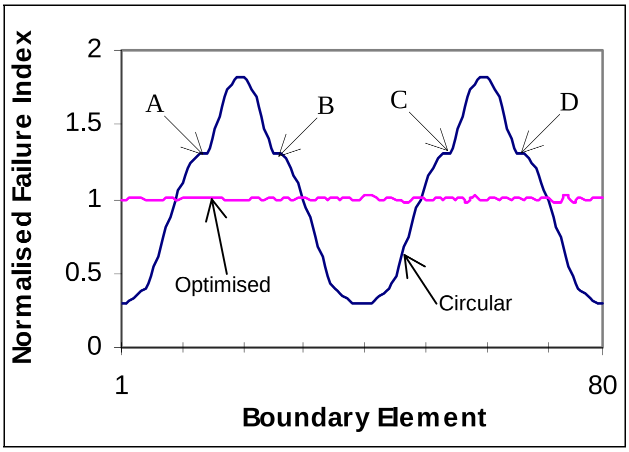


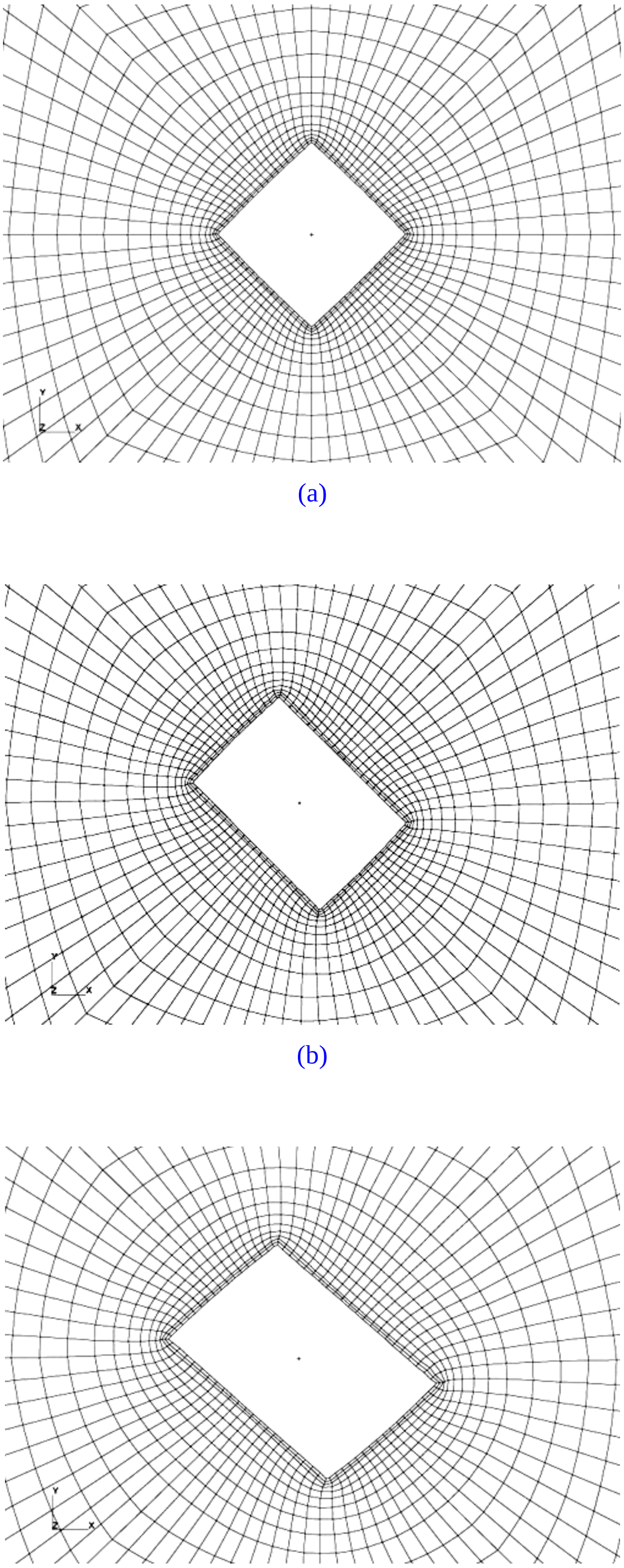
Figure 6 Normalised boundary failure indices for Laminate 1 under 2:1 bi-axial load

The failure index distribution for the circular hole in Laminate 1 under 2:1 bi-axial loading shown in Figure 6 has a slight discontinuity at positions A, B, C and D. This is due to a change in critical ply and mode of failure predicted through the maximum strain failure theory. For instance, the 45° ply is critical on the element ahead of A and the 90° ply critical on the element immediately after. The slight unevenness of the optimum boundary failure index was due to difficulty in converging to an optimum solution due to the large number of design responses required for composite laminates and the discontinuous nature of these responses.

4.2 Shear Loading

Optimisation under shear load resulted in quadrilateral cut-outs with constant stresses around the boundary, the results of which are summarised in Table 7. The isotropic plate resulted in a “diamond” shaped cut-out, as shown in Figure 7a. The quasi-isotropic plate resulted in a rectangle of aspect ratio 1.54, oriented at 45° to the horizontal axis, as shown in Figure 7b. The optimum cut-out for Laminate 2 was a parallelogram of aspect ratio 1.48 with the obtuse angle at the corner of 94°, as shown in Figure 7c.

The reason for the unusual optimum geometry is due to the mutually perpendicular stresses that develop under pure shear loading. The rectangular optimum cut-out shape for the quasi-isotropic Laminate 1 was a result of the directional strength of each ply within the laminate. The directional strength is primarily governed by the allowable strains in tension and compression, which are significantly different (refer to Table 2). The aspect ratio of the optimum hole for Laminate 2 was slightly less than that for the quasi-isotropic laminate and was slightly skewed. This was due to the additional 0° plies, which limited the “stretch” in this direction.



(a)

(b)

(c)

Figure 7 Optimum cut-out shape under shear load for (a) Isotropic, (b) Laminate 1 and (c) Laminate 2

Table 7 Aspect ratio and the reduction of boundary failure indices for shear load

Panel	Aspect Ratio	Skew	FI or σ_{VM} Reduction
Isotropic	1.00	0°	33.3%
Laminate 1	1.56	0°	36.7%
Laminate 2	1.48	8°	35.9%

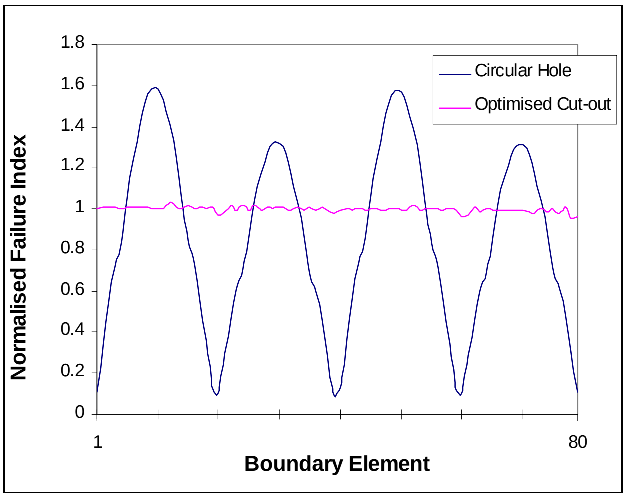


Figure 8 Normalised boundary failure indices for Laminate 1 under shear load

The failure index distribution for the optimum cut-out shape under shear loading demonstrated similar unevenness to the bi-axial loading cases, as shown in Figure 8. The most pronounced unevenness corresponded to the sharp corners of the rectangle and reflects the difficulty in extracting accurate strains at these locations.

4.3 Combined Shear/Compression Loading

To demonstrate the flexibility of the optimisation technique that has been developed, cut-outs in isotropic plates were optimised under combined shear and compression loading. It was found that combining the two load cases resulted in optimum shapes that were between the pure shear case (a square hole orientated at 45°) and the pure compression case (a slit orientated in the direction of the load), as summarised in Table 8. This effect is illustrated in Figures 9, 10 and 11, in which the optimum shape under 1:0.5, 1:1 and 1:2 shear to compression loading are presented. In all cases, a uniform Von Mises stress around the cut-out boundary was achieved, which led to a minimum 33% reduction of stress. It was observed that the reduction became more pronounced as the proportion of compression loading increased.

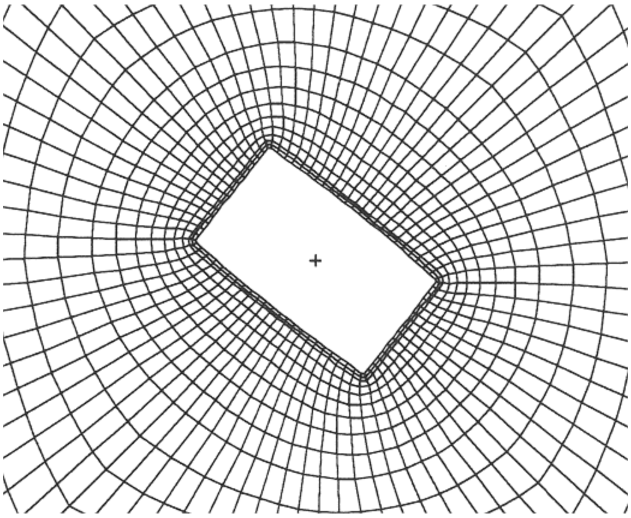


Figure 9 Optimum cut-out under 1:0.5 shear/compression loading for an isotropic plate

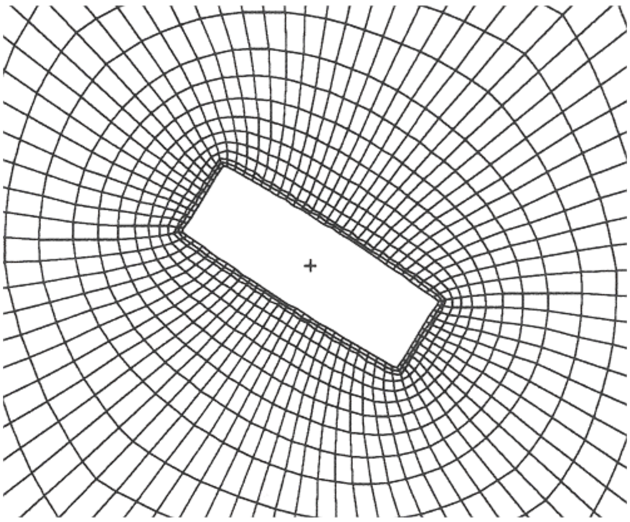


Figure 10 Optimum cut-out under 1:1 shear/compression loading for an isotropic plate

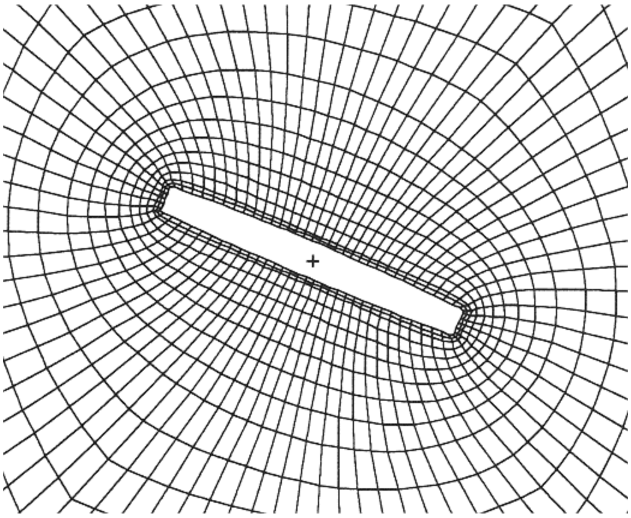


Figure 11 Optimum cut-out under 1:2 shear/compression loading for an isotropic plate

Table 8 Optimum cut-out aspect ratio, orientation and the reduction in Von Mises stress under combined shear and compression loading for an isotropic plate

Loading Ratio		Aspect Ratio	Hole Orientation	Stress Reduction
Shear	Comp.			
1	0	1.00	45°	33%
1	0.5	1.86	39°	41%
1	1	3.11	32°	48%
1	2	9.80	23°	57%
0	1	∞	0°	-

5. Rib Optimisation

The optimisation capabilities developed for cut-outs in simple plates, as described in the previous sections, were demonstrated on a carbon/epoxy composite rib under development for an all-composite wing. This rib, shown in Figure 12, contains four access holes for instrumentation to pass through, as well as numerous cut-outs which provide clearance for the stringers. The rib is fastened to the front and rear spars and each stringer flange using cleats.

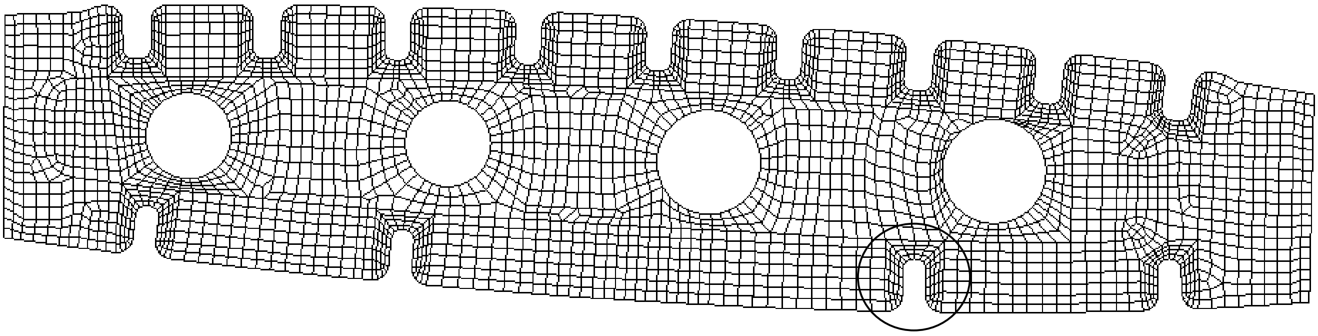


Figure 12 Full rib model (critical stringer cut-out circled)

5.1 Stringer Cut-Out Optimisation

Under the critical load case, it was found that the highest failure index was located at the top of the stringer cut-out identified in Figure 12. This stringer cut-out, the initial shape of which is illustrated in Figure 13, was selected for shape optimisation. To speed up the analysis, a sub-structure model of the stringer cut-out was produced and refined somewhat when compared with the original FE model of the rib (refer to Figure 12). The displacement field resulting from an analysis of the entire rib was applied to the boundary of the stringer cut-out model as enforced displacements.

Similarly to the cut-out optimisation, an auxiliary model was created by applying enforced radial displacements of 1 mm at each of the boundary nodes while fully restraining the model boundaries. The objective function in this case was to minimise the maximum failure index in the elements around the stringer cut-out boundary. This objective, while not as efficient as the least squares objective used in the optimisation of cut-outs, produced better results in this case although more design cycles were required.

The optimised shape, shown in Figure 13, was flattened at the top of the stringer cut-out so as to distribute the peak loading over a greater area. The effectiveness of the optimisation is shown in Figure 14 in which the failure index around the stringer cut-out boundary is plotted. The optimisation reduced the peak failure index by 33% implying that the number of plies in the rib necessary to meet strength criteria could be significantly reduced.

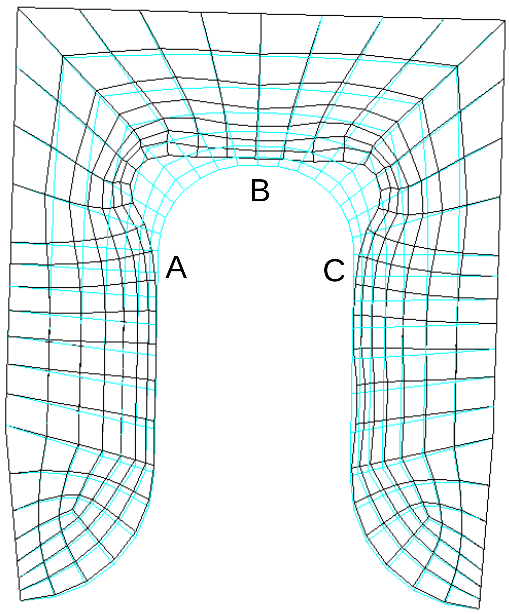


Figure 13 Optimised stringer cut-out shape (original shape shown in light blue)

5.2 Optimisation of Access Holes

The shape and size of the individual access holes were also optimised. The objective in this case was to minimise the mass of the rib while constraints were used to minimise the variation in failure index around each access hole boundary using the least squares criteria (Equation 2). Another constraint was used to prevent an increase in element failure index throughout the rib. To prevent a decrease in rib stiffness, constraints were also placed on displacement at the centre of the rib.

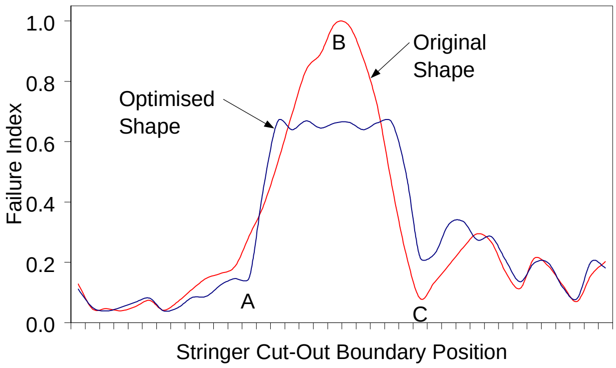


Figure 14 Variation in failure index around the stringer cut-out boundary for the original and optimised shapes

The results of the optimisation, presented in Figure 14, show that the shapes of the access holes were changed significantly. The two outer holes tended towards “diamonds” due to shear loading acting in these regions. The two centre holes become oval due to the bi-axial loading acting in these regions.

The peak failure index around each of the access holes was decreased by at least 19%, as shown in Table 10. Therefore, fewer plies would be necessary in this region to meet strength requirements. However, the overall mass of the rib was not significantly reduced by the optimisation primarily due to a limit on the amount of shape change that was possible. The FE mesh of the rib used for optimisation had been used previously for the design analysis and contained many elements of imperfect geometry. During shape optimisation, the elements became more distorted, eventually resulting in a fatal error. Therefore, to achieve the greatest possible change in shape, the finite element mesh must be carefully constructed. Despite this, the results of the optimisation demonstrated the potential benefits of this approach in detail design of components manufactured from composite materials.

Table 10 Peak failure indices around rib access hole boundaries.

Access Hole	Circular FI	Optimised FI	FI Reduction
Hole 1	0.51	0.40	21%
Hole 2	0.17	0.13	26%
Hole 3	0.19	0.13	29%
Hole 4	0.36	0.29	19%

6 Conclusion

A technique has been developed to successfully optimise the shape of cut-outs in laminated composite plates under various loading conditions. This work has demonstrated the suitability of the design sensitivity method of MSC.Nastran for the optimisation of such structural details.

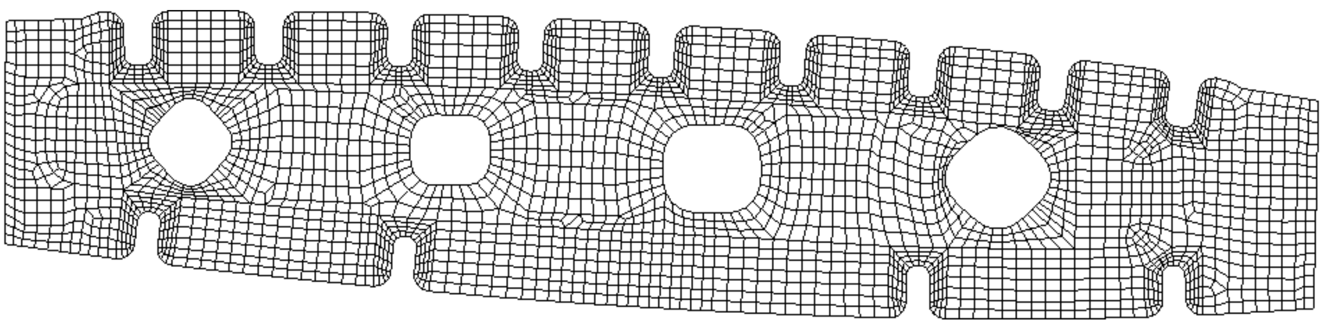


Figure 15 Rib model showing optimised access holes

For the shape optimisation of cut-outs in plates, a least squares objective function based on element Von Mises stress or failure index, for isotropic materials and composite laminates respectively, efficiently produced a constant stress around the cut-out boundary. The optimum cut-out geometry was found to be approximately elliptical for uniform bi-axial loads and quadrilateral for shear loads. The reductions in peak failure index were between 36% and 59% for load cases considered in this study. The results showed that the optimum cut-out shape depends on the degree of orthotropy of the plate. Quasi-isotropic laminates produced cut-outs of very similar shape and aspect ratio to isotropic plates while more orthotropic laminates affected the resulting optimum cut-out aspect ratio and skew (in the case of shear load).

The shape optimisation techniques were demonstrated on a composite wing rib under development for an all-composite wing. Firstly, the shape of the critical stringer cut-out was optimised. On this open boundary, it was found that an objective function which minimised the maximum failure index was most effective. Optimisation resulted in a 33% reduction in peak failure index. The shapes of the four internal access holes were also optimised, which resulted in at least a 19% reduction in peak failure index. This example clearly demonstrated the great potential of shape optimisation in detail design of laminated composite structures.

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