INFLUENCE OF AEROELASTIC EFFECTS ON PRELIMINARY AIRCRAFT DESIGN

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Abstract

With the increasing size of modern, conventional transport aircraft aeroelastic effects have become more important for the development of new safe and competitive aircraft. This was the motivation to improve the modelling and analysis capabilities of the in-house aircraft design tool of the Institut fuer Flugzeugbau und Leichtbau (IFL) to take into account the major aeroelastic effect as early as possible in the conceptual and preliminary design phase. Main emphasis was put on the development and implementation of advanced modelling capabilities and analysis methods for both structures and aerodynamics in the integrated Preliminary Aircraft Design and Optimisation Program (PrADO). The finite-element method was chosen for static and dynamic structural analyses. A similar level of accuracy in aerodynamic analysis is achieved by the implementation of the panel method HISSS. All the structural weight estimation and flight simulation routines are based on the results of these advanced analysis tools. Elastic deflections can easily be taken into account in both, the loads and the aerodynamic properties in terms of drag vs. lift. In this paper the methods used and the modelling and analysis capabilities of this newly developed Structural and Aeroelastic Analysis Module (SAM) of PrADO are described.

1 Introduction

The increase in world-wide passenger transport in recent years and the forecast of annual growth rates in air traffic of about 5%, [1], emphasise the necessity of the development of new, large, economically and ecologically competitive civil transport aircraft. One way to increase competitiveness is to increase the size and payload capacity of the aircraft beyond the current limits of existing aircraft.

But with the increasing size of modern, conventional transport aircraft aeroelastic effects become more important for the development of new save and competitive aircraft, [2]. Negative aeroelastic effects like increased drag at cruising speed due to changes in wing twist, aileron reversal or flutter identified at a late stage of the product development, e.g. in ground vibration tests or even in flight tests, will cause severe economic and technological problems. These may only be overcome by considerable changes in structural design or mass distribution, resulting in weight penalty or even unacceptable limitations of the flight envelope. A major redesign to improve the aeroelastic properties with an optimised and least-weight design is impossible because of the huge number of interdependencies of the different disciplines, departments or even companies involved in the design of a new aircraft.

The only way to avoid these problems is to meet the uncertainties of the influences of aeroelastic behaviour on structural weight, flight performance, handling characteristics and finally economy as soon as possible within the aircraft design process. That means, aeroelastic properties have to be taken into account in the conceptual or at least early preliminary design phase. Therefore, it is necessary to implement new, more sophisticated models in the conceptual design which are strictly based on physics and do not only rely on statistical data of real-

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ised aircraft. This option becomes even more attractive, since the introduction of new technologies and materials promise to help reducing the weight of structures and systems. But their benefit and potential risks have to be assessed thoroughly.

The IFL has a long history in the application of numerical methods in conceptual aircraft design: Starting with the implementation of statistical methods, improved by analysis capabilities using simple equivalent beam and lifting line theory to get weight estimations based on physics in the late 1980's [3], and finally using finite-elements for structural analysis and sizing of hypersonic lifting bodies in the 1990's, [4]. This development has gone in parallel with the increasing computational capacities which open today the opportunity to use more sophisticated models at an early stage of an aircraft design project, for extensive parameter variations, and basic research.

The main emphasis of the work described in this article was put on the development and implementation of advanced modelling capabilities and analysis methods for both structures and aerodynamics into the integrated **Pr**eliminary **A**ircraft **D**esign and **O**ptimisation Program (PrADO). For both static and dynamic structural analyses the finite-element method (FEM) was chosen. The structural models used are derived from a limited number of design parameters appropriate for conceptual design level and optimisation.

A similar level of accuracy in static aerodynamic analysis is achieved by the implementation of the **Hi**gher-**O**rder **S**ubsonic/**S**upersonic **S**ingularity Method (HISSS, [8]). All the structural weight estimation and flight simulation routines are based on the results of these advanced analysis tools. Elastic deflections can easily be taken into account in both, the aerodynamic loads and the aerodynamic properties for flight performance prediction.

Finally, a first attempt to include a flutter prediction at the preliminary design level was made. The sub-module is based on the structural model used for weight prediction and linearised unsteady aerodynamics.

In this paper a brief overview of the conceptual design tool PrADO is given and the requirements, methods and tools chosen to achieve the required accuracy and flexibility in structural and aeroelastic analysis and weight prediction are described.

2 Methods and Tools

2.1 The Conceptual Design Tool PrADO

The conceptual aircraft design tool PrADO is a toolbox of all the major disciplines needed to design, analyse, and optimise an aircraft at an early stage of a project. It has been developed and used for basic research at the IFL for many years and in different projects, ranging from conventional aircraft and subsonic flying wing configurations for high payloads [3] to hypersonic two-stage-to-orbit transport systems [4].

The toolbox is highly modular and can be run in three modes: Analysis of a single, given configuration, parameter survey, and optimisation. The program is organised in five levels. The first level is a pre-/postprocessor used to prepare the input data for the definition and optimisation of an aircraft from a limited amount of input given by the user (e.g. payload-range, Mach-number and cruising altitude, constraints, overall geometry parameters). Graphical and numerical representation of input data and results are prepared as well. In the second level routines different optimisation algorithms are implemented. They are activated if a configuration is optimised with respect to a given objective function (e.g. direct operating costs DOC for civil transport aircraft) by varying the independent design parameters.

On the third level the iterative, multi-disciplinary design process is simulated for every single candidate configuration by consecutively running the analysis tools of the different disciplines (modules MD1 – MD19 in Fig. 1). The input data is checked and completed in MD1. Geometric data in terms of the component size is derived from the input data in MD2-MD7. The fuselage, for example, is developed from inside out, starting with the cabin layout and building the outer, load carrying structure of the fuselage around it. These routines are knowledge based and the methods for the design of completely new or improved concepts have to be added to the source code either by changing existing modules or implementing new routines before these new configurations can be analysed. The aerodynamic properties of the aircraft needed for flight simulation and design of tailplanes is realised in MD8. A "rubber engine model" implemented in MD9 to predict e.g. specific fuel consumption. The fuel consumption of the aircraft for a mission given in the basic requirements for the design is determined in modules MD10-MD13 by a flight simulation. The volume of the tanks is checked and the geometric size of the wing is adjusted if necessary. In MD14 the weight prediction of all the structural components is carried out, followed by the prediction of centre of gravity position for different payload and fuel conditions in MD15. Finally, the constraints and requirements given in FAR for take-off and landing with and without engine failure are checked in MD 16 in MD17.

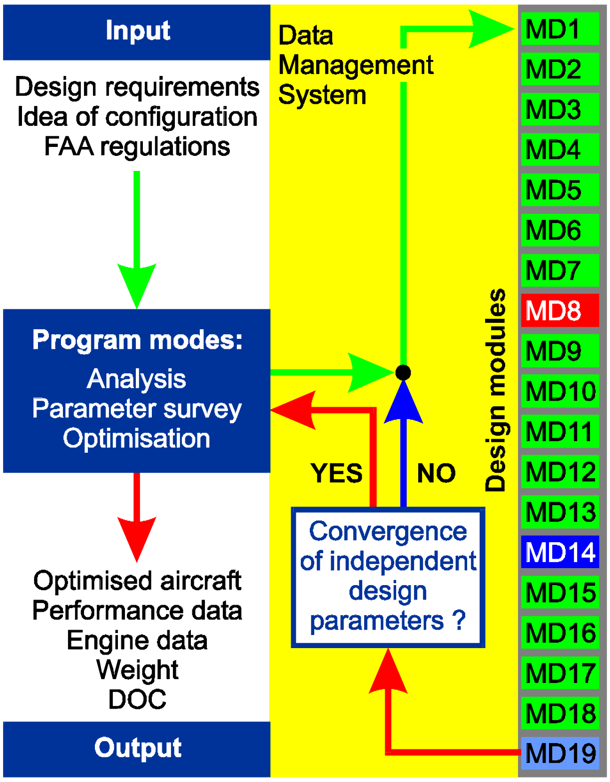

Fig. 1 The concept of PrADO

After completion of the technical analyses of a given configuration the objective function (DOC) is evaluated in MD18. This configuration is not necessarily a valid configuration fulfilling all the requirements and constraints like limits in overall length, span and height, runway length, volume of freight compartment etc. Therefore, the design is checked for theses constraints in module MD19 and if the configuration fails the test, it is ruled out either by a penalty of the objective function or by excluding the configuration directly. The modules MD8 (aerodynamics) and MD14 (weight prediction) are directly affected by the improved analysis and sizing capabilities developed to assess the aeroelastic properties of conventional aircraft configurations.

Level four consists of several libraries of subroutines used for the technical and economic analyses on level three and is not shown in Fig. 1. For data input, transfer between modules, and for data storage a **D**ata **M**anagement **S**ystem (DMS), considered as the fifth level of the toolbox, is used.

2.2 Choice of Methods for Aeroelastic Analysis and Weight Prediction in Conceptual Design

Aeroelastic analysis requires a sound modelling of the aerodynamic loads, the global elastic properties of the load carrying structure, and, if dynamic phenomena are addressed, of additional non-structural mass. On the other hand, only limited information is available in the conceptual design phase of a new aircraft. Furthermore, a great number of configurations has to be analysed and assessed to find the optimum and therefore, the time needed for analysis and sizing of every configuration has to be limited. The challenge of an aircraft model used in conceptual or preliminary design to investigate and assess its aeroelastic properties is to find a compromise between these conflicting requirements.

Every single configuration considered in the iterative design and optimisation process described above has to be analysed and sized for the acting loads. These loads can be divided into aerodynamic and inertia loads and have to be predicted with appropriate accuracy for all the relevant load cases and all the major components of the aircraft, namely the fuselage, and all the lifting surfaces. The load prediction is followed by the sizing of the structural parts and stiffness and mass distribution can be determined for further static and dynamic analysis. The sizing of the structure has to be done by iteration, since the inertia loads depend on the structural mass.

The aerodynamic analysis has to able to predict the aerodynamic load distribution of three-dimensional configurations properly. It is highly desirable that the displacement effects of the fuselage and the induced downwash of multiple lifting surfaces are appropriately included. The modelling of lifting surfaces with adjustable incidence for trimming is essential. For the flight simulation and prediction of fuel consumption the induced drag of the overall configuration in level flight is of great importance and has to be predicted correctly for the rigid and the flexible configuration. The limitation of required CPU-time is again a major concern. The panel method HISSS [8] meets these requirements. It is kindly supplied by Military Aircraft Division of DaimlerChrysler Aerospace for research purpose.

Different methods are available for structural analysis of an aircraft configuration. Neither of them is capable to analyse the whole aircraft or even a single component of an aircraft like a wing or fuselage analytically in one single step. One way to overcome this problem is to simplify the structure and to use a beam or a plate instead of the real structure of wing and fuselage. The major drawback of this approach is that some information about the structural layout is lost and the properties are reduced to a single point of the cross section (e.g. elastic axis and centre of gravity). In addition, it may be necessary to change the method from the beam-like structure to an equivalent plate if a new configuration like a supersonic transport aircraft is investigated.

The second alternative is to split the components into small parts that can be analysed. This can either be done by using finite-elements or by a decomposition of the structure. In the work presented here, the finite-element approach was chosen because it offers a wide range of applications in static and dynamic analysis and optimisation. The method itself is not limited to a certain class of aircraft and the adaptation of the aircraft analysis and design tool to new configurations is limited to an improvement of the model generator. A similar approach is described in [7], but the main emphasis was put more on the multi-level optimisation than on the aeroelastic capabilities of the program.

In references [5] and [6] a program for weight prediction and aeroelastic analysis based on beam like structural idealisation and structural decomposition is described. This tool is developed in industry for the preliminary design of conventional aircraft only.

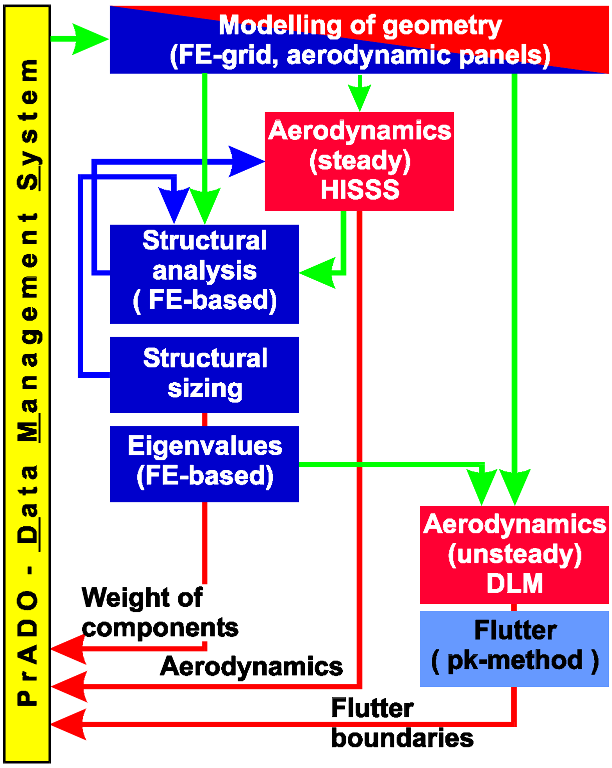

Fig. 2 General concept of the SAM

The general concept of the **S**tructural and **A**eroelastic Analyses **M**odule (SAM) is shown in Fig. 2. The tool consists of four major parts: Model generation, aerodynamic analysis, structural analysis and sizing, and finally a flutter prediction routine. The result of the structural sizing module, namely the weight, is fed back into the loads module of the structural analysis. The elastic deflection of the structure can be applied to the aerodynamic surface and the aerodynamic loads and characteristics of the flexible aircraft can be determined by iteration. The structural FE-mesh and the aerodynamic grid are designed for the specific needs of the respective analysis tools and do not necessarily build the same surface. Therefore, interpolation and extrapolation routines are employed for load and displacement transfer between structural and aerodynamic models.

2.3 Structural Analysis

The FEM is a well accepted and often used method for structural analysis. Many commercial tools are available that offer a huge functionality. But when these tools are used in an integrated aircraft design environment described in this article, only a small part of their capabilities would be exploited. On the other hand, it is difficult or even impossible to adapt it to the specific requirements of the preliminary design. This was the reason to implement a finite-element code developed in-house into the SAM of PrADO. This tool offers static and normal mode analyses for structures made of isoparametric membrane-, beam- and rod-elements. Additional non-structural mass can be included in the model using rigid body elements. Stringers can be modelled by equivalent membrane elements with orthotropic material properties (regular stiffness in parallel direction, negligible normal and shear stiffness). By this approach, excessive FE-grid refinements are avoided without losing accuracy in the prediction of bending and torsional stiffness. The stiffness matrix **K** is build successively in every iteration of the structural sizing procedure. The system of equations for static analysis is completed by one vector of *n* nodal displacements **u** and one vector **f** of nodal forces for each of the *m* load cases. These vectors can be combined in two *n* by *m* matrices of displacement and loads, **U** and **F,** respectively.

$$\begin{aligned}&\mathbf{U} \quad \mathbf{u}_1 \quad \mathbf{u}_2 \quad \ldots\\&\mathbf{F} \quad \mathbf{f}_1 \quad \mathbf{f}_2 \quad \ldots\end{aligned} \qquad (1)$$

The system of equations (2) describing the static problem is solved by Cholesky factorisation, [13], once for all the considered load cases. Element strains and stresses are determined from the resulting nodal displacements.

$$\mathbf{KU} \quad \mathbf{F} \qquad (2)$$

The SAM applies the fully-stressed design philosophy using the stress envelope of all the load cases taken into account for analysis and design.

The eigenvalue problem is directly based on the finally determined, properly sized structure. For this purpose, the mass matrix **M** of the structural and non-structural mass is calculated. The *n* by *p* matrix **U** represents the *n* nodal displacements of the limited number *p* of eigenvectors belonging to the *p* lowest eigenfrequencies.

$$\mathbf{KU} \qquad \mathbf{MU} \qquad (3)$$

Eq. (3) is solved by simultaneous vector iteration, [14]. The matrices of the eigenvectors are used for a modal transformation of the system matrices **K** and **M** resulting in generalised system matrices $\mathbf{K}_g$ and $\mathbf{M}_g$ used in the flutter prediction module described later.

$$\begin{aligned}&\mathbf{K}_g \quad \mathbf{U}^T\mathbf{KU}\\&\mathbf{M}_g \quad \mathbf{U}^T\mathbf{MU}\end{aligned} \qquad (4)$$

2.4 Aerodynamic Analysis

The panel method HISSS is a higher-order singularity method for the solution of linear potential flow around arbitrary three-dimensional configurations at subsonic and supersonic speeds. Composite source/doublet panels are used on the surface of the configuration. The wake panels have a doublet distribution to carry downstream the vorticity generated by the lifting surfaces of the configuration. The calculation of the drag coefficient is based on the circulation distribution in spanwise direction and the downwash in the Trefftz plane, providing sufficiently accurate results even for coarse surface grids. HISSS is used by different aircraft design and aerodynamics groups in Germany, e.g. [9].

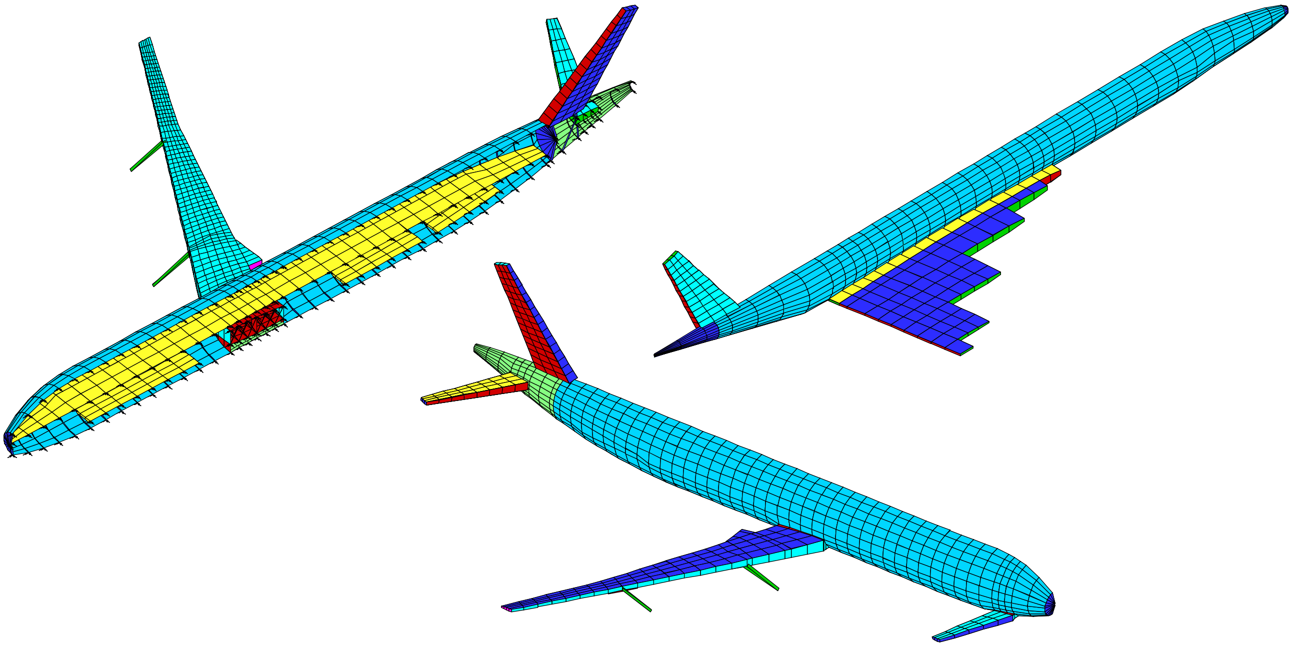

Fig. 3 Example Configurations

2.5 Modelling Capabilities

The numerical methods used for aerodynamic and structural analyses are both based on discretisations of the aerodynamic surface and the structure, respectively. An integrated multi-model generator has been developed to supply the aerodynamic and structural grids.

The application of a single tool and a single input data set for both, the aerodynamic and the structural grid helps to prevent inconsistencies in the models used for the analysis, sizing and weight prediction. A certain variety of configurations can be generated from a limited set of parameters. In Fig. 3 several examples ranging from conventional aircraft with low or high mounted wing to three-surface aircraft and supersonic transport aircraft like a Concorde are shown. Further enlargements to unconventional configurations are under development.

The models are derived from a relatively small set of geometric parameters describing the overall geometry and the structural layout. These parameters are generated by the modules MD2-MD7 of the design program. For the definition of a wing shown in Fig. 4 the following parameters have to be given:

- geometry either by
  - descriptive parameters: reference area, aspect ratio, taper ratio, sweep or
  - leading edge co-ordinate or sweep and local chord at given stations in spanwise direction
- wing twist and dihedral
- wing section geometry

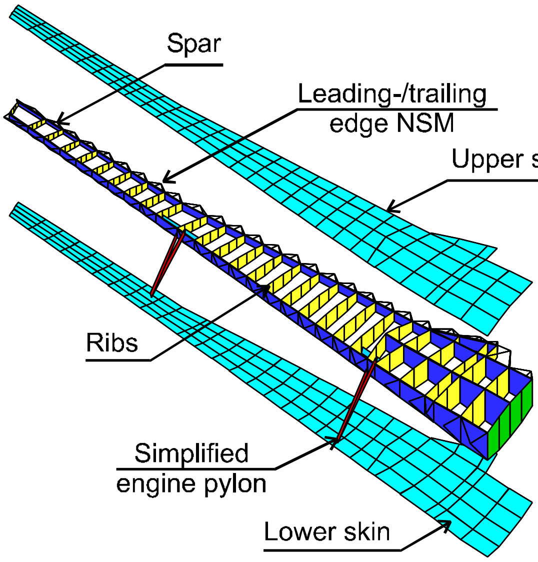

Fig. 4 Structural FE-model of the wing with engines

The load carrying structure consists of upper and lower skin, spars with web and girder, and ribs. The definition of the structural layout is based on

- relative position of spars with respect to leading edge and local chord
- position and orientation of ribs
- number of ribs between given stations
- number of elements between spars

Engines can be mounted on the wing using pylons. The leading- and trailing-edge structure is included by non-structural mass elements (NSM). An example of the structural model of the wing of a conventional aircraft is shown in Fig. 4.

The fuselage is generated from the following input data:

- overall length
- geometry and dimension of cross-sections at given stations
- position of bulkheads
- position of floors (cabin and freight)
- average distance between modelled frames

The finite element model of a widebody fuselage with integrated centre box is shown in Fig. 5.

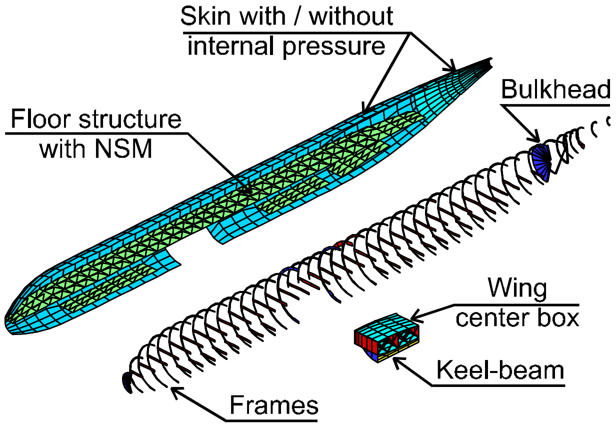

Fig. 5 Structural FE-model of the fuselage with integrated centre box

A half model of the configuration is built from the components that have been generated automatically. The intersections and structural joints are generated without user interaction and the modular concept of the structural layout is kept as shown in Fig. 6. Additional displacement constraints are supplied at the joints for each component. The constraint forces of e.g. the wing are determined in the final iteration of the analysis and are applied on the appropriate nodes the counterpart of the joint. Considerable computational resources, both in terms of required memory and time, can be save by this approach compared with the analysis and sizing of a single, integrated structural model of the whole configuration.

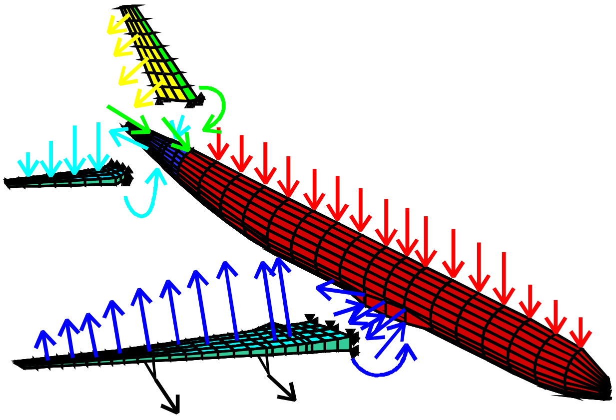

Fig. 6 Modular structural concept

2.6 Flutter Prediction

Finally, another new module was included in the SAM to look into the possibilities to predict the flutter behaviour of new aircraft configurations with the methods, tools, and models available at an early design stage. The model used for flutter prediction was limited to the final wing design with engines. Both, the unsteady aerodynamics and the flutter calculations, are based on the eigenmodes and generalised system matrices of the structure derived directly from the FE-model used for the static analysis enlarged by the mass matrix of the system. The first few eigenfrequencies of the structural system are determined by simultaneous vector iteration. The unsteady aerodynamic loads in terms of generalised aerodynamic coefficients are calculated for different reduced frequencies and Mach numbers using a code based on the subsonic doublet-lattice method (DLM) that was developed and kindly supplied by the DLR Goettingen, [10]-[11]. The generalised, complex flutter equation is solved using a standard routine of the LAPACK, [13].

The results of a flutter calculation, the flight level dependent velocities of flutter onset, are taken as additional constraints in module MD19 of the design program. An optimisation of the structure with respect to its flutter behaviour is not implemented.

First parameter studies investigating the influence of the main geometric parameters of the wing and the influence of engine position on flutter behaviour have been carried out. The general trend of the results agrees with the results published in [12]. But it still has to be proven that a change in flutter behaviour due to a change in geometry or mass distribution identified with the linearised methods and simplified and reduced models used within the SAM is a realistic estimate of the real behaviour of an aircraft in transonic cruise. A certain scepticism is justified not least due to the accumulation of uncertainties throughout the analysis, design, and sizing procedure. A more promising approach is to use the whole model of the aircraft for further, more detailed analyses of the aeroelastic behaviour of the finally optimised configuration. For this purpose an interface for the generation of NASTRAN input files was implemented.

2.7 Load Cases for Structural Sizing

The results of the structural sizing and weight prediction modules depend on the choice of load cases taken into account. FAR 25 defines several ground and flight load cases. The latter being divided into symmetrical and asymmetrical gust and manoeuvre load cases and taken basically as balanced load cases. The whole flight envelope has to be taken into account. Due to the limitations of computational time within the conceptual or preliminary design phases, the number of considered load cases has to be reduced to the most important flight conditions. But these load cases are not known in advance when a totally new configuration is investigated and have to be determined before the optimisation is started. This can be done with the SAM by considering a certain number of candidate load cases, limited only by the size of the load vector of the FE-system, in a single run of the analysis and sizing module and ruling out those load cases which do not contribute to the sizing of the structure. The computational time required for this study is unacceptably long from the point of view of the conceptual design but it is reasonably short to be carried out in advance for every new concept. Only symmetrical load cases can by considered since the geometry used in the SAM of PrADO is limited to a half model of the configuration. In consequence the vertical tailplane cannot be included in the global model used for weight prediction but has to be analysed and dimensioned separately.

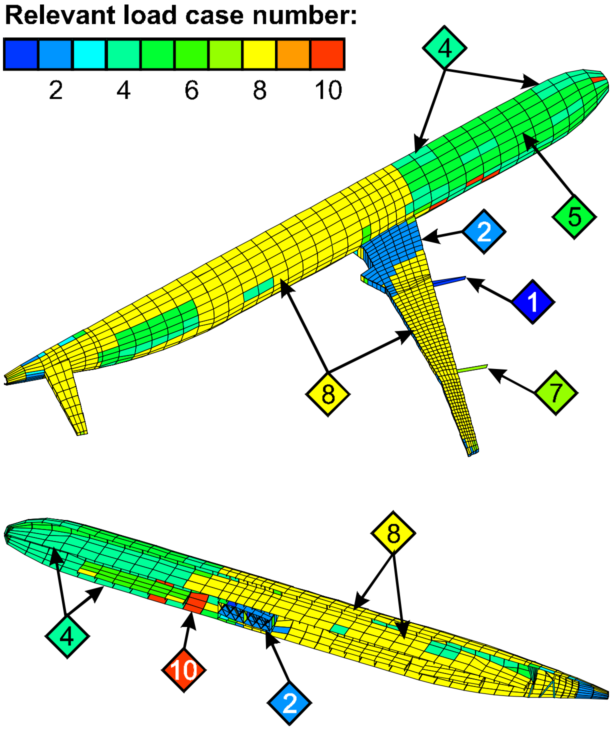

Fig. 7 Example survey of relevant load cases

	1	2	3	4
No. in Fig. 7	2	4	5	8
m / t	1)	1)	1)	MTOW$^{2)}$
altitude / km	8,0	12,5	12,5	8,0
load factor n	2,5	2,5	2,5	2,5
Mach number	0,93	0,93	0,93	0,93
point in V-n	V_D	V_D	V_D	V_D

1) Operational empty weight + max. payload + fuel in centre tank
2) max. takeoff weight, fuel in wing and centre box

In Fig. 7 the relevant load cases determined by such a survey are shown for a conventional passenger aircraft. The definition of the respective flight conditions is given in Table 1. The analysis was run with several other load cases, e.g. gust and flight with V_D at sea level, which do not occur in the plot and consequently do not have any influence on the predicted weight.

3 Example Design Case

For validation of the weight prediction routine of the SAM, two Airbus aircraft have been analysed: A340-300 (twin aisle, long-range) and A320-200 (single aisle, short range). The results of the weight prediction carried out with the SAM are shown in Fig. 8. The total mass of the major components, taken from [16], is shown in the upper part of the diagrams. The structural FE-model does not cover all the parts of the components and consequently does not reach its total mass. Therefore, a more detailed mass breakdown is given in the lower parts of the diagrams. Maximum agreement of the results is achieved if the FE-model covers the structural and non-structural mass completely. A complete, consistent mass breakdown is not generally available and the data used in this example was taken from [16] and several other data sheets supplied for the partners of the German research project DYNAFLEX (see *Acknowledgements* below).

Looking at Fig. 8 it becomes clear, that an accurate prediction of wing and fuselage weight is essential for the overall weight estimation, since these two components make up almost 50% of the operational empty weight of an A340-300. In terms of total mass, a 1% error in the predicted weight of the wing equals a 30% error in the weight of the vertical tailplane. On the other hand, the weight of the landing gear, which is still determined by statistical methods, is significantly higher compared to the tailplanes. These observations have been the guidelines for priorities established in the development of the SAM. The results obtained by the weight prediction module of the SAM agree very well with the weight of realised aircraft. Parameter surveys have shown, that the influence of the main geometric parameters on weight is predicted correctly. Furthermore, the global deformation of the real structure is predicted with adequate accuracy if the finally sized structure is used for aeroelastic analysis.

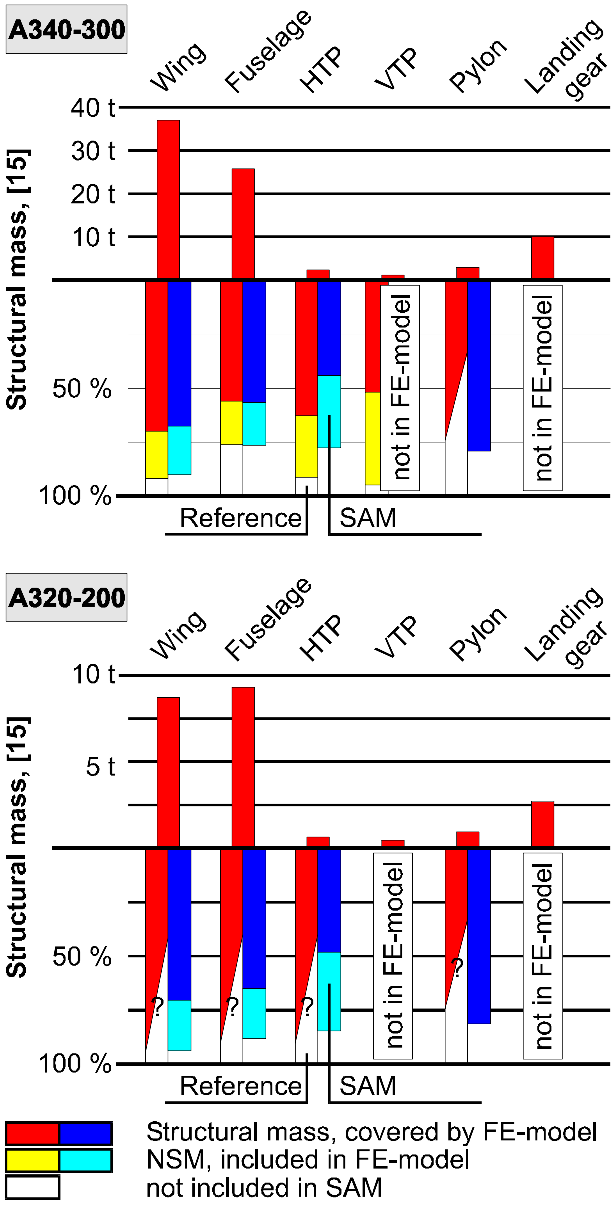

Fig. 8 Weight prediction with the SAM

4 Conclusions and Outlook

The tools and methods described in this paper have proven to be capable to predict weight and the static aerodynamic and aeroelastic properties of given conventional aircraft configurations within the conceptual or preliminary aircraft design phase. The models derived from the preliminary design offer a good starting point for more detailed structural and aeroelastic analyses with more sophisticated tools.

Further design and optimisation studies will be carried out in the near future to exploit the full potential of the developed tools. An extension of the modelling capabilities to analyse and assess unconventional configurations has just been started.

Acknowledgements

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References

[1] IATA. Passenger Forecast 1999-2003, Quebec, 1999

[2] Försching H. New ultra high capacity aircraft (UHCA) – Challenges and problems from an aeroelastic point of view. *Zeitschrift fuer Flugwissenschaften und Weltraumforschung*, Vol. 18, pp 219-230, 1994

[3] Heinze W. Ein Beitrag zur quantitativen Analyse der technischen und wirtschaftlichen Auslegungsgrenzen verschiedener Flugzeugkonzepte für den Transport großer Nutzlasten. *ZLR-Forschungsbericht 89-01*, Braunschweig, 1989

[4] Heinze W, Bardenhagen A. Waverider aerodynamics an preliminary design for two-stage-to orbit missions, Part 2. *Journal of Spacecraft and Rockets*, Vol. 35, No. 4, pp. 459-466, 1998

[5] Schmidt A, Läpple M, Kelm R. Advanced fuselage weight estimation for the new generation of transport aircraft. *57th Annual Conference of the Society of Allied Weight Engineers*, Seattle (WA), SAWE Paper No. 2406, 1997

[6] Kelm R, Grabietz M. Aeroelastic effects on the weight of aircraft in the pre-design phase. *57th Annual Conference of the Society of Allied Weight Engineers*, Seattle (WA), SAWE Paper No. 2407, 1997

[7] Bil C, v. Dalen F, Rothwell A, Arendsen P, Wiggenraad J F M. Structural optimisation in preliminary aircraft design: A finite-element approach *18th Congress of ICAS*, ICAS-92-6.7R2, Beijing (China), 1992

[8] Fornasier L. HISSS – A Higher-Order Subsonic/ Supersonic Singularity Method for Calculating Linearized Potential Flow. AIAA Paper 84-1646, 1984

[9] Strohmeyer D, Seubert R, Heinze W, Österheld C, Fornasier L. Three Surface Aircraft – A concept for future transport aircraft. *38th AIAA Aerospace Science Meeting and Exhibit*, Renov (NV), AIAA-00-0566, 2000

[10] Albano E, Rodden W P. A doublet-lattice method for calculating lift distributions on oscillating surfaces in subsonic flows. *AIAA Journal*, Vol. 7, No. 2, 1969

[11] Lu S, Voß R. TDLM – A Transonic Doublet-Lattice Method for 3D unsteady flow calculation. *DLR-FB 92-25*, 1993

[12] Zimmermann H, Vogel S. Influence of main design parameters on flutter behaviour for aircraft configurations with heavy concentrated masses. AGARD-CP-354, 1986

[13] Schwarz H R. *Methode der Finiten Elemente*. Teubner Studienbuecher Mathematik, Stuttgart, 1991

[14] Zienkiewicz O C, Taylor R L. *The finite element method*. Vol. 2, 4th edition, McGraw-Hill Book Company, 1998

[15] Anderson E, et.al. *LAPACK – Linear Algebra Package User’s Guide*, SIAM, Philadelphia (PA), 1992

[16] anon. *LTH Masseanalyse*. IABG, 1999