A98-31621 ICAS-98-R,4,11

FREE FLEXURAL VIBRATIONS OF COMPOSITE PLATES OR PANELS STIFFENED BY A CENTRAL OR NON-CENTRAL PLATE STRIP --- SOME COMPARISONS

by

U. Yuceoglu and V. Özerciyes
Department of Aeronautical Engineering
Middle East Technical University
Ankara, Turkey .

Abstract

The free bending vibrations of orthotropic base plates or panels with a bonded central and non-central stiffening plate strip are analyzed and are compared with each other. In both cases, the base plate or panel and the stiffening strips are made of dissimilar, orthotropic Mindlin plates and they are joined together by a very thin, yet flexible adhesive layer. The governing system of partial differential equations are first reduced to a special first order form and then, integrated by the "Modified Version of the Transfer Matrix Method". The effects of the bonded central and non-central stiffening strip on the mode shapes and natural frequencies of the two composite plate system are investigated. It is shown that the position of the stiffening plate strip greatly influence the mode shapes and natural frequencies of the composite plate or panel system.

Introduction

Rapid developments in "Composite Materials" technology, together with the progress in the very strong epoxy-based "Adhesive Bonding" methods are making the "composites" more and more feasible in the aerospace vehicle, marine vehicle and mine sweeper structures, structural panels and components. Also, the composites are gradually finding a place in other areas such as automative engineering technology, too[1,2]. In the design of flight vehicle structures, theoretical as well as experimental knowledge and information are needed on the free and forced vibrations of the stiffened composite plates or panels. This kind of knowledge is extremely important in the studies of panel flutter, sonic fatigue, and sound transmission in panels of the complex structural systems used in aircraft structures.

Some of the most important studies which can be found in the engineering and scientific literature on the free vibrations on the stiffened isotropic or anisotropic, thin plates are in [3,4,5,6,7,8,9]. In [3] and [4], isotropic thin plates with a central stiffener are analyzed. The vibration analysis of anisotropic thin plates with a central stiffener are presented in [5]. The free vibrations of isotropic thin plates with two or more stiffeners are considered in [6,7,8,9]. In some papers such as [3,7,8] the stiffeners are beams with rectangular cross-sections.

In aforementioned studies, either isotropic or anisotropic, Classical Thin Plate Theory (CPT) are employed to develop dynamic equations. These put very severe restrictions on the usefulness of the natural frequencies and mode shapes obtained in these studies. Also, in the deformation of multi-layer composite plates, the importance of the transverse shear deformations are shown in [10,11].

The main objective of this study, therefore, is to analyze free, flexural vibrations, of relatively thick, composite base plate or panel stiffened by a central or non-central, narrow plate strip in terms of "two cases". In both cases, the base plate and stiffening plate strip are to be dissimilar orthotropic plates connected by a relatively very thin, yet deformable adhesive layer. In this study, the dynamic equations of relatively thick, orthotropic Mindlin plates are developed by means of the variational methods. Thus, the dynamic equations are obtained as an extension of the Mindlin Plate Theory of isotropic plates [12] to the case of orthotropic plates. In literature, several Higher Order Shear Deformation Theories of Plates (HSDT) [13,14] are also available. However, the natural frequencies obtained by the First Order Shear Deformation Theories (FSDT) such [12] and by (HSDT) are quite similar [14]. Therefore, in this study, the Mindlin Plate Theory for orthotropic plates is used.

Theoretical Analysis

The "two cases" of the stiffened composite plate or panel system to be analyzed here are given in terms of their two general configurations and longitudinal cuts in Figures 1 a, b and Figures 2 a, b, respectively. In the "First Case", the orthotropic, composite base plate with a

21st ICAS Congress
13-18 September 1998
Melbourne, Australia

central stiffening plate strip and their longitudinal cross-section are given in Figure 1 a, b. In the "Second Case", the orthotropic, composite base plate or panel with non-central stiffening plate presented in Figure 2 a, b. In both cases, the stiffening plate strip is bonded to the base plate by a very thin, elastic adhesive layer. The adhesive layer is considered to be an elastic continuum with transverse normal and shear deformations. The system of coordinate axes are shown in Figures 1 and 2. The entire stiffened, composite plate or panel system of "two cases", are divided into 3 regions in the y-direction. These are Part. I, Part. II and Part. III as shown in Figures 2 a and 2 b, respectively.

For the "First Case", the equations of motion of the orthotropic Mindlin Plates are as follows:

For Part. I region (two-layer plate),

$$\begin{aligned}
&\frac{\partial M_x^{(j)}}{\partial x}+\frac{\partial M_{yx}^{(j)}}{\partial y}-Q_x^{(j)}+\underline{\frac{h_jG_a}{4h_a}\left(h_1\psi_x^{(1)}+h_2\psi_x^{(2)}\right)}=\frac{\rho_jh_j^3}{12}\frac{\partial^2\psi_x^{(j)}}{\partial t^2}\\
&\frac{\partial M_{yx}^{(j)}}{\partial x}+\frac{\partial M_{y}^{(j)}}{\partial y}-Q_y^{(j)}-\underline{\frac{h_jG_a}{4h_a}\left(h_1\psi_y^{(1)}+h_2\psi_y^{(2)}\right)}=\frac{\rho_jh_j^3}{12}\frac{\partial^2\psi_y^{(j)}}{\partial t^2}\\
&\frac{\partial Q_x^{(j)}}{\partial x}+\frac{\partial Q_y^{(j)}}{\partial y}+\underline{(-1)^{(j+1)}\frac{E_a}{h_a}\left(w^{(2)}-w^{(1)}\right)}=\rho_jh_j\frac{\partial^2 w^{(j)}}{\partial t^2}
\end{aligned}$$

$$(0<y<\ell_I),\qquad (j=1,2) \qquad (1)$$

where the underlined terms are the coupling terms between the upper stiffening strip and the lower base plate due to the in-between adhesive layer. And E_a and G_a are the adhesive layer elastic constants.

For Part. II and Part. III regions (one-layer plate),

$$\begin{aligned}
&\frac{\partial M_x^{(2)}}{\partial x}+\frac{\partial M_{yx}^{(2)}}{\partial y}-Q_x^{(2)}=\frac{\rho_2h_2^3}{12}\frac{\partial^2\psi_x^{(2)}}{\partial t^2}\\
&\frac{\partial M_{yx}^{(2)}}{\partial x}+\frac{\partial M_{y}^{(2)}}{\partial y}-Q_y^{(2)}=\frac{\rho_2h_2^3}{12}\frac{\partial^2\psi_y^{(2)}}{\partial t^2},\ \begin{Bmatrix}(0<y<\ell_{II})\\(0<y<\ell_{III})\end{Bmatrix} \qquad (2)\\
&\frac{\partial Q_x^{(2)}}{\partial x}+\frac{\partial Q_y^{(2)}}{\partial y}=\rho_2h_2\frac{\partial^2 w^{(2)}}{\partial t^2}
\end{aligned}$$

where in (1) and (2) ℓ_I, ℓ_{II} and ℓ_{III} indicate the length of the two-layer and one-layer plate regions in Figure 1. b.

Similarly, for the "Second Case", the equations of motion of the orthotropic Mindlin Plates are:

For Part I region (two-layer plate),

$$\begin{aligned}
&\frac{\partial M_x^{(j)}}{\partial x}+\frac{\partial M_{yx}^{(j)}}{\partial y}-Q_x^{(j)}+\underline{\frac{h_jG_a}{4h_a}\left(h_1\psi_x^{(1)}+h_2\psi_x^{(2)}\right)}=\frac{\rho_jh_j^3}{12}\frac{\partial^2\psi_x^{(j)}}{\partial t^2}\\
&\frac{\partial M_{yx}^{(j)}}{\partial x}+\frac{\partial M_{y}^{(j)}}{\partial y}-Q_y^{(j)}-\underline{\frac{h_jG_a}{4h_a}\left(h_1\psi_y^{(1)}+h_2\psi_y^{(2)}\right)}=\frac{\rho_jh_j^3}{12}\frac{\partial^2\psi_y^{(j)}}{\partial t^2}\\
&\frac{\partial Q_x^{(j)}}{\partial x}+\frac{\partial Q_y^{(j)}}{\partial y}+\underline{(-1)^{(j+1)}\frac{E_a}{h_a}\left(w^{(2)}-w^{(1)}\right)}=\rho_jh_j\frac{\partial^2 w^{(j)}}{\partial t^2}
\end{aligned}$$

$$(0<y<\ell'_I),\qquad (j=1,2) \qquad (3)$$

where the underlined terms are the coupling terms of the upper stiffening strip and lower base plate due to the in-between adhesive layer.

For Part. II and Part. III regions (one-layer plate),

$$\begin{aligned}
&\frac{\partial M_x^{(2)}}{\partial x}+\frac{\partial M_{yx}^{(2)}}{\partial y}-Q_x^{(2)}=\frac{\rho_2h_2^3}{12}\frac{\partial^2\psi_x^{(2)}}{\partial t^2}\\
&\frac{\partial M_{yx}^{(2)}}{\partial x}+\frac{\partial M_{y}^{(2)}}{\partial y}-Q_y^{(2)}=\frac{\rho_2h_2^3}{12}\frac{\partial^2\psi_y^{(2)}}{\partial t^2},\ \begin{Bmatrix}(0<y<\ell'_{II})\\(0<y<\ell'_{III})\end{Bmatrix} \qquad (4)\\
&\frac{\partial Q_x^{(2)}}{\partial x}+\frac{\partial Q_y^{(2)}}{\partial y}=\rho_2h_2\frac{\partial^2 w^{(2)}}{\partial t^2}
\end{aligned}$$

where, again, in (3) and (4), ℓ'_I, ℓ'_{II} and ℓ'_{III} correspond to the length of the two-layer and one-layer plate regions in Figure 2. b.

Since the boundary conditions at x=0,a are simple support conditions, then, the displacement field (or transverse displacements and angle of rotations) which satisfy the support conditions at x=0,a can be expressed in the classical "Levy Type Solutions" for both the stiffening strip and the base plate in the x-direction. Therefore, in the "First Case", the "Levy Type Solutions" in the x-direction are :

For Part. I region,

$$\begin{aligned}
&\psi_x^{(j)}(\eta,\xi_I,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{\Psi}_{mx}^{(j)}(\xi_I)\cos(m\pi\eta)e^{i\overline{\omega}_{mn}t}\\
&\psi_y^{(j)}(\eta,\xi_I,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{\Psi}_{my}^{(j)}(\xi_I)\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t},\\
&w^{(j)}(\eta,\xi_I,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}h_1\overline{W}_m^{(j)}(\xi_I)\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t}
\end{aligned}$$

$$(0<\xi_I<\ell_I)\qquad (j=1,2) \qquad (5)$$

where "barred" terms are the dimensionless transverse displacement and the dimensionless angle of rotation quantities with (j=1) corresponding to the upper plate strip and (j=2) to the lower base plate. Also, $\overline{\omega}_{mn}$ is the dimensionless natural frequency of the entire composite plate or panel system.

For Part. II region,

$$\begin{aligned}
&\psi_x^{(2)}(\eta,\xi_{II},t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{\Psi}_{mx}^{(2)}(\xi_{II})\cos(m\pi\eta)e^{i\overline{\omega}_{mn}t}\\
&\psi_y^{(2)}(\eta,\xi_{II},t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\overline{\Psi}_{my}^{(2)}(\xi_{II})\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t},\\
&w^{(2)}(\eta,\xi_{II},t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}h_1\overline{W}_m^{(2)}(\xi_{II})\sin(m\pi\eta)e^{i\overline{\omega}_{mn}t}
\end{aligned}$$

$$(0<\xi_{II}<\ell_{II}) \qquad (6)$$

For Part. III region, a similar equation to (6) may be written by replacing $\xi_{II}\to\xi_{III}$, $\ell_{II}\to\ell_{III}$ in (6). In the above equations (5), (6) for Parts I, II, III appearing $\eta=x/a$, $\xi_I=y_I/\ell_I$, $\xi_{II}=y_{II}/\ell_{II}$, $\xi_{III}=y_{III}/\ell_{III}$ are dimensionless coordinates in the x and y directions.

Similar expressions to (5) and (6) may be written for the "Second Case", too, by replacing

$$\xi_I\to\xi'_I=y_I/\ell'_I,\ \xi_{II}\to\xi'_{II}=y_{II}/\ell'_{II},\ \xi_{III}\to\xi'_{III}=y_{III}/\ell'_{III}.$$

In this way, $\frac{\partial}{\partial x}, \frac{\partial^2}{\partial t^2}$ are eliminated from the governing equations of the "First Case" in (1) and (2) and similarly for the "Second Case" in (3) and (4).

After some algebraic manipulations, the equations (1) and (2) for the "First Case" in combination with the stress resultant- displacement expressions can be put into a special "first order" form in $\frac{d}{d\xi}$. Then, the governing system of coupled ordinary differential equations in the "First Case" are reduced to the following in matrix form or state-vector form (see also Figure 1. b.).

For Part. I region, (two-layer plate)

$$\frac{d}{d\xi_I}\begin{Bmatrix}\bar{Y}^{(1)}\\ \bar{Y}^{(2)}\end{Bmatrix}=\begin{bmatrix}\bar{\mathcal{A}}_{11} & \bar{\mathcal{A}}_{12}\\ \bar{\mathcal{A}}_{21} & \bar{\mathcal{A}}_{22}\end{bmatrix}\begin{Bmatrix}\bar{Y}^{(1)}\\ \bar{Y}^{(2)}\end{Bmatrix}, \quad (0<\xi_I<1) \tag{7}$$

For Part. II region, (one-layer plate)

$$\frac{d}{d\xi_{II}}\left\{\bar{Y}^{(2)}\right\}=\left[\bar{\mathcal{B}}\right]\left\{\bar{Y}^{(2)}\right\}, \quad (0<\xi_{II}<1) \tag{8}$$

For Part. III region, (one-layer plate)

$$\frac{d}{d\xi_{III}}\left\{\bar{Y}^{(2)}\right\}=\left[\bar{\mathcal{D}}\right]\left\{\bar{Y}^{(2)}\right\}, \quad (0<\xi_{III}<1) \tag{9}$$

where the column matrix or the state-vector $\left\{\bar{Y}_m^{(j)}\right\}$ includes "dimensionless fundamental dependent variables" of the problem such that,

$$\left\{\bar{Y}_m^{(j)}\right\}=\left\{\bar{\Psi}_{mx}^{(j)},\bar{\Psi}_{my}^{(j)},\bar{W}_m^{(j)};\bar{M}_{myx}^{(j)},\bar{M}_{my}^{(j)},\bar{Q}_{my}^{(j)}\right\}^T,(j=1,2) \tag{10}$$

corresponding to the upper stiffening plate strip (j=1) and to the lower base plate (j=2). Again, the "barred" quantities are the dimensionless quantities. The coefficient matrices in (7), (8), (9) include dimensionless natural frequency parameter $\bar{\omega}_{mn}$ together with the dimensionless geometric and material constants.

In the "First Case", in the above system of equations in (7), the arbitrary boundary conditions for the stiffening plate strip at ξ_I=0,1 and the continuity conditions for the lower base plate in the y- or ξ-direction at ξ_I=0,1 have to be satisfied. Also, in (8), the arbitrary boundary conditions at ξ_{II}=0 and the continuity conditions at ξ_{II}=1 in the y- or ξ- direction are to be considered. Similarly, in (9), the arbitrary boundary conditions at ξ_{III}=1 and the continuity conditions at ξ_{III}=0 are to be satisfied.

In a similar manner, the governing system of coupled ordinary differential equations (7), (8), (9) with the corresponding arbitrary boundary conditions and the continuity conditions in ξ-direction can be written for the "Second Case".

Solution Method

The above system of the first order ordinary differential equations in the state-vector form for the "First Case" can be integrated by making use a "Modified Version of the Transfer Matrix Method" which is, with some modifications, used by the present authors in [15,16] and in [17,18]. Similarly, for the "Second Case" the same solution technique may be employed.

The "Modified Version of the Transfer Matrix Method" is a combination of "Levy's Method", the "Transfer Matrix Method" and the "Integrating Matrix Method".

As a preliminary step, the first order governing differential equations in the state-vector form are discritized. For this purpose, the dimensionless dependent variables and the coefficient matrices in equations (7), (8), (9) with respect to the dependent variables ξ_I, ξ_{II}, ξ_{III}, respectively. This is done by dividing Part I, Part II and Part III regions of Figures 2 and 3, into sufficient number of segments along ξ-directions. Then, the governing differential equations are multiplied by the appropriate "Global Integrating Matrices" $[\mathcal{L}]$ which includes 12 square integrating sub matrices $[\ell]$ of dimensions (12n_1 x 12n_1). Here, n_1 is the number of discritizing points along Part I region. Then, this yields for Part-I region,

$$\left\{\dot{\bar{Y}}\right\}-\left\{\dot{\bar{Y}}_1\right\}=\left[\mathcal{L}\right]\left[\dot{\bar{A}}\right]\left\{\dot{\bar{Y}}\right\} \tag{11}$$

where,

$$\left\{\dot{\bar{Y}}\right\}=\left\{\dot{\bar{Y}}^{(1)},\dot{\bar{Y}}^{(2)}\right\}^T, \quad \left\{\dot{\bar{Y}}_1\right\}=\left\{\dot{\bar{Y}}_1^{(1)},\dot{\bar{Y}}_1^{(2)}\right\}^T \tag{12}$$

and the subscripts "m" is dropped from the state-vectors for convenience. The superscripts in (12) as before correspond to the upper stiffening strip (j=1) and the lower base plate (j=2), respectively in Part. I region. Also, the notation "•" indicates the discritization of a particular matrix along the ξ_I-direction. The subscript "1" in the state-vectors in (11), (12) indicates that the state-vectors are evaluated at the "initial end point ξ_I=0" in Part I region.

Furthermore, a relation between the state-vector at a general station along the ξ_I-directions and a state-vector at the "initial end point ξ_I=0" is obtained by rearranging (11) such that,

$$\left\{\dot{\bar{Y}}\right\}=\left[\dot{\mathcal{U}}\right]\left\{\dot{\bar{Y}}_1\right\}, \qquad \left[\dot{\mathcal{U}}\right]=\left([I]-[\mathcal{L}]\left[\dot{\bar{A}}\right]\right)^{-1} \tag{13}$$

where $[\dot{\mathcal{U}}]$ is the discritized version of the "Transfer Matrix" which is obtained from the integrating matrices.

Finally, in this method, the discritized "Transfer Matrix" $[\dot{\mathcal{U}}]$ transfers the state-vectors at the "initial end point ξ_I=0" and an arbitrary station along the Part. I region. Therefore, a relation between the state-vectors at the "initial end point ξ_I=0" and the "final end point ξ_I=1" in Part. I region are obtained. Similar expressions can be obtained for Part. II and Part. III regions. Then, by making use of (13) and similar expressions in combination with the continuity conditions between the regions, and the support conditions at initial and final ends in the y-direction, one can obtain a homogenous matrix equation in which the determinant of the coefficient matrix yields the natural frequencies $\overline{\omega}_{mn}$ as its roots where

$$\left|\begin{array}{l}\overline{\omega}_{mn} = \rho_1 a^4 \omega_{mn}^2 / h_1^2 B_{11}^{(1)} \\ \overline{\Omega} = \overline{\omega}_{mn}\end{array}\right. \qquad (m,n=1,2,3...) \qquad (14)$$

Some Numerical Results and Conclusions

The theorical formulation and the method of solution developed in this study are applied to some numerical cases. These are shown in Figure 1 (for the "First Case") and Figure 2 (for the "Second Case"). The geometric and the material characteristics of the stiffening plate strip and the composite base plate or panel are given in Table 1 and Table 2 for the "First Case" and the "Second Case", respectively. Only the boundary conditions in the y-direction are given on Figures. Thus, the first two letters indicate the conditions for the upper stiffening strip, and subsequent two letters show conditions for the lower base plate, both in the y-direction.

In Figure 3, the first and third mode shapes and the corresponding dimensionless frequencies are presented for the "First Case". Similarly, in Figure 4, the first and third mode shapes and their dimensionless frequencies are given for the "Second Case". It can be seen from Figure 3 and 4 that the mode shapes and the corresponding frequencies are considerably different in the "two cases". Also, in the "First Case", there are symmetric and antisymmetric modes following one after the other, while, in the "second case", the modes do not show this property.

In Figures 4 and 5, the natural frequencies versus the "Joint Length Ratio b_1/b_2" are plotted for the FFCC boundary conditions for the "First Case" and for the "Second Case", respectively. It is obvious that there are drastic differences in the frequencies depending on the noncentrality or the eccentricity of the position of stiffening strip. According to the mode shapes and the natural frequencies for the same plate or panel system, the one with a central stiffening strip is relatively stiffer with consequently higher frequencies.

References

[1] Schwartz, M., 1992, "Composite Materials Handbook", Mc-GrawHill, N.Y.

[2] Hoskins, B.C. and Baker A. A.,1986, "Composite Materials for Aircraft Structures", AIAA Educational Series, New York.

[3] Aksu, G. and Ali, R., 1976, " Free Vibration Analysis of Stiffened Plates using Finite Difference Method", Journal of Sound and Vibration, Vol. 48, No. 1, pp. 15-25.

[4] Kirk, C. L., 1970, "Natural Frequencies of Stiffened Rectangular Plates", Journal of Sound and Vibration, Vol. 13, pp. 259-277.

[5] Lee, D. M. and Lee, I., 1995, " Vibration Analysis of Anisotropic Plates with Eccentric Stiffeners", Computers and Structures, Vol. 57, No. 1, pp. 99-105.

[6] Lin, Y. K., 1960, "Free Vibration of Continuous Skin-Stringer Panels" ASME Journal of Applied Mechanics, Vol. 27, No.4, pp. 669-676.

[7] Wah, T. , 1964, "Vibration of Stiffened Plates", the Aeronautical Quarterly, pp. 285-298.

[8] Long, R. R., 1971, "Stiffness Type of Analysis of the Vibration of a Class of Stiffened Plates", Journal of Sound and Vibration, Vol. 16, pp. 323-325.

[9] Chen, C. J., Liu W. and Chern, S. M. , 1994, "Vibration Analysis of Stiffened Plates", Computers and Structures, Vol. 50, No. 4, pp. 471-480.

[10] Whitney, J.M., 1969, " The Effect of Transverse Shear Deformation on the Bending of Laminated Plates" Jour. of Composite Materials, Vol.3, pp. 534-546.

[11] Noor, A.K. and Burton, W. S., 1989, "Assesment of Shear Deformation Theories for Multi-Layered Composite Plates", Applied Mechanics Reviews, Vol. 42, No. 1, pp. 1-13.

[12] Mindlin, R. D., 1951, "Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic, Elastic Plates", ASME Journal of Applied Mechanics, Vol. 18, pp.31-38.

[13] Reddy, J. N. and Robbins, D. H., 1994, "Theories and Computational Models for Composite Laminates", Applied Mechanics Reviews, Vol. 47, No. 6, pp. 147-169.

[14] Khdeir, A. A. and Librescu, L., 1988, "Analysis of Symmetric Cross-ply Laminated Elastic Plates using Higher Order Theory - Part II", Journal of Composite Structures, Vol. 9, pp. 259-277.

[15] Yuceoglu, U. and Özerciyes, V., 1996, "Free Bending Vibrations of Partially-Stiffened, Stepped-Thickness Composite Plates", an ASME Publication "Advanced Materials for Vibro-Acoustic Applications NCAD- Vol. 23", pp. 191-202.

[16] Yuceoglu, U., and Özerciyes, V. , 1996, "Free Vibrations of Adhesively Bonded, Partially Stiffened, Stepped- Thickness, Composite Plate", Proceed. of the 3rd. Inter. Conference on Composite Engineering (ICCE/3), Vol. 2.

[17] Yuceoglu, U., Toghi, F., and Tekinalp, O., 1995, "Free Bending Vibrations of Multi-Layer, Bonded, Orthotropic, Composite Plate System", in "Dynamic Response and Behaviour of Composites, AD-Vol. 46", pp. 83-94, an ASME Publication.

[18] Yuceoglu, U., Toghi, F., and Tekinalp, O., 1993, "Free Vibrations of Adhesively Bonded, Composite Orthotropic Plates", in "Composite Materials and Structures AD-Vol. 37/AMD-Vol. 179", pp.297-313, an ASME Publication.

TABLE 1. MATERIAL PROPERTIES AND DIMENSIONS ("First Case")

Graphite - Epoxy Upper Plate	*Kevlar - Epoxy Lower Plate*	*Adhesive Layer (soft)*	*Adhesive Layer (hard)*
E_x =11.71 Gpa E_y =137.8 GPa G_{xy} =5.51 GPa G_{xz} =2.50 GPa G_{yz} =3.00 GPa ν_{xy} =0.0213 ν_{yx} = 0.25 ρ_1 =1.6 gr/cm^3 h_1=h_2=h=0.007 m. a= 0.5 m. b_1=0.3 m. $\tilde{b}$ = 0.5 m.	E_x = 5.50 GPa E_y = 76.0 GPa G_{xy} =2.10 GPa G_{xz} =1.50 GPa G_{yz} =2.00 GPa ν_{xy} = 0.024 ν_{yx} = 0.34 ρ_2 =1.3 gr/cm^3 h_1=h_2=h=0.007 m. a= 0.5 m. b_2=1.0 m.	$E_a/B_{11}^{(1)}=10^{-5}$ $G_a/B_{11}^{(1)}=10^{-5}$ h_a =0.15x10^{-3} m.	$E_a/B_{11}^{(1)}$=0.340 $G_a/B_{11}^{(1)}$=0.119 h_a =0.15x10^{-3} m. E_a =4.0 GPa G_a=1.4 GPa

Upper plate (j=1) =Stiffening Plate Strip
Lower plate (j=2) =Base Plate or Panel

TABLE 2. MATERIAL PROPERTIES AND DIMENSIONS ("Second Case")

Graphit - Epoxy Upper Plate	*Kevlar - Epoxy Lower Plate*	*Adhesive Layer (soft)*	*Adhesive Layer (hard)*
E_x =11.71 GPa E_y =137.8 GPa G_{xy} =5.51 GPa G_{xz} =2.50 GPa G_{yz} =3.00 GPa ν_{xy} =0.0213 ν_{yx} = 0.25 ρ_1 =1.6 gr/cm^3 h_1=h_2=h=0.007 m. a= 0.5 m. b_1=0.3 m. $\tilde{b}$ = 0.35 m.	E_x = 5.50 GPa E_y = 76.0 GPa G_{xy} =2.10 GPa G_{xz} =1.50 GPa G_{yz} =2.00 GPa ν_{xy} = 0.024 ν_{yx} = 0.34 ρ_2 =1.3 gr/cm^3 h_1=h_2=h=0.007 m. a= 0.5 m. b_2=1.0 m.	$E_a/B_{11}^{(1)}=10^{-5}$ $G_a/B_{11}^{(1)}=10^{-5}$ h_a =0.15x10^{-3} m.	$E_a/B_{11}^{(1)}$=0.340 $G_a/B_{11}^{(1)}$=0.119 h_a =0.15x10^{-3} m. E_a =4.0 GPa G_a=1.4 GPa

Upper plate (j=1) =Stiffening Plate Strip
Lower plate (j=2) =Base Plate or Panel

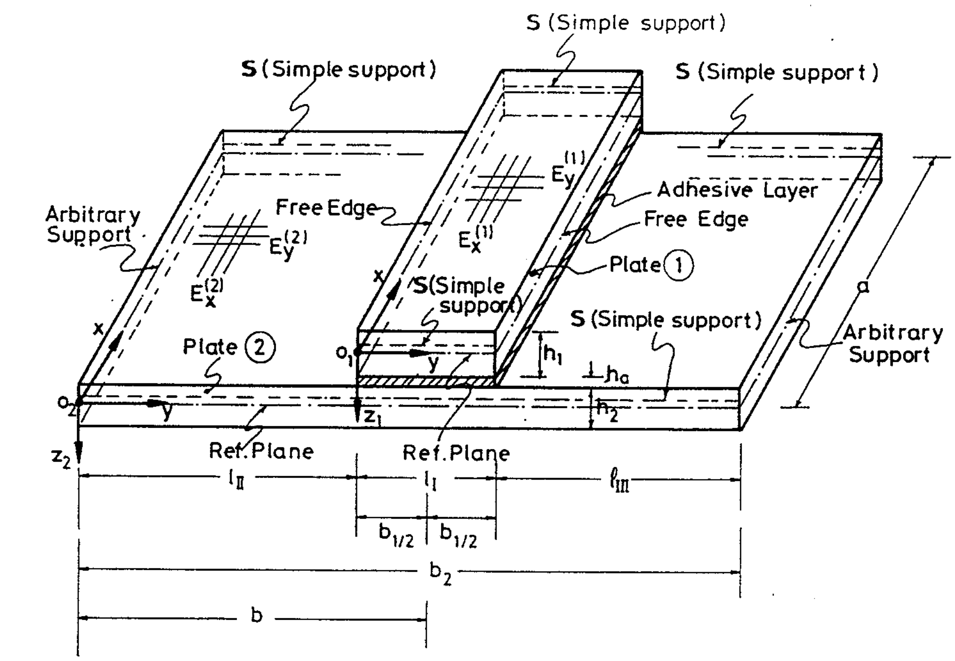


Figure 1. a. General Configuration and Coordinate System of Orthotropic, Composite Plate or Panel with <u>Central</u> Stiffening Plate Strip ("<u>First Case</u>")

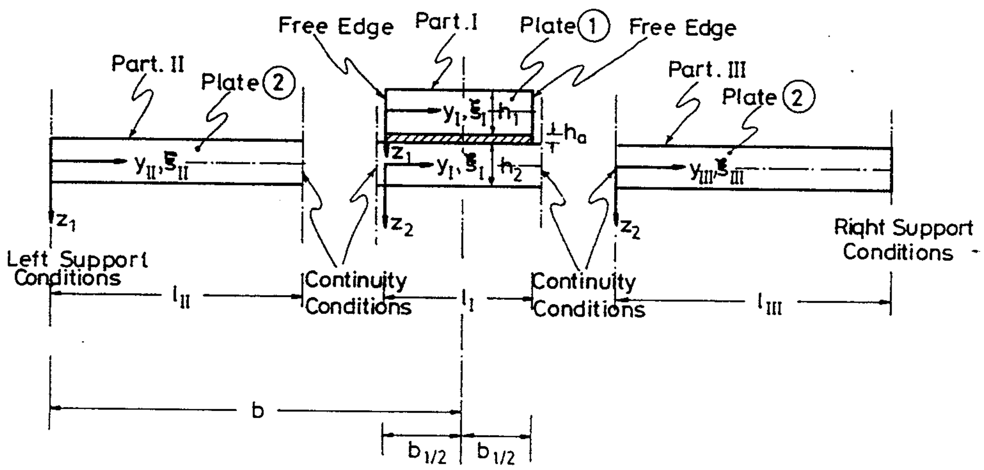


Figure 1. b. Longitudinal Cross-Section and Coordinate System of Orthotropic, Composite Plate or Panel with <u>Central</u> Stiffening Plate Strip ("<u>First Case</u>")

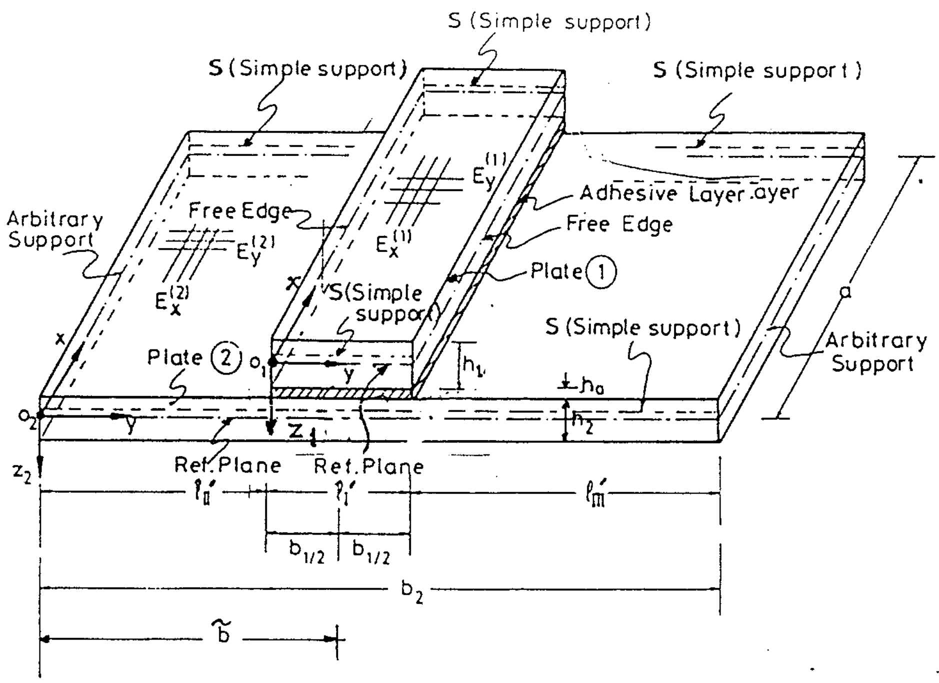


Figure 2. a. General Configuration and Coordinate System of Orthotropic, Composite Plate or Panel with <u>Non-Central</u> Stiffening Plate Strip ("<u>Second Case</u>")

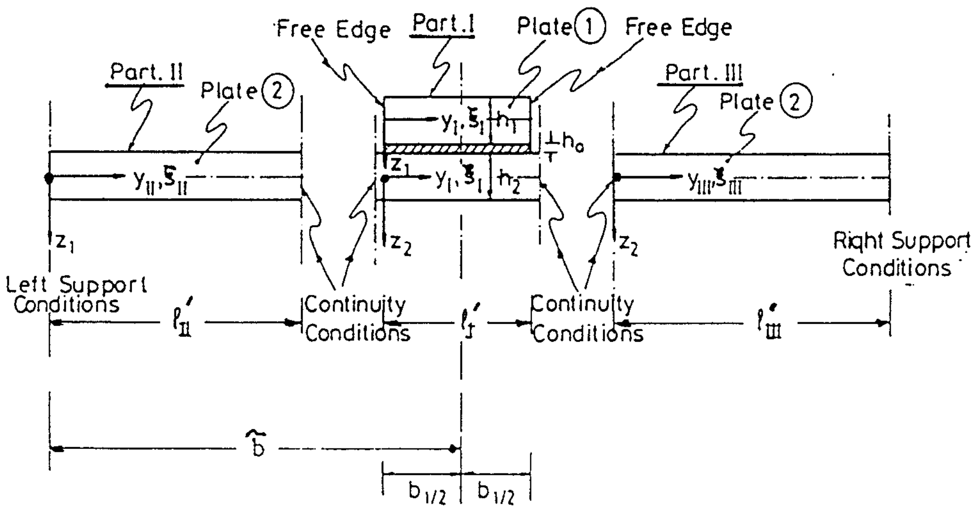
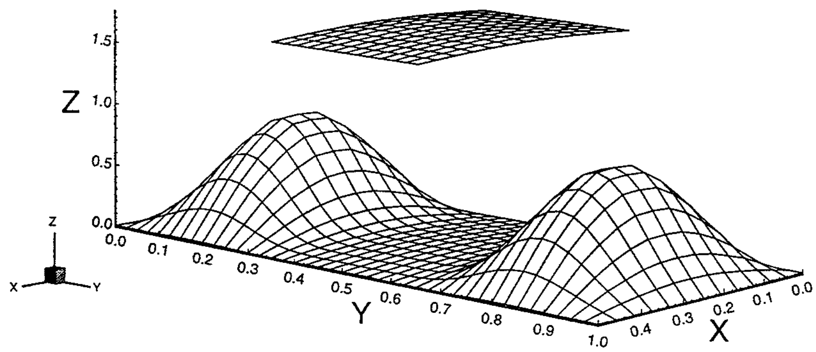
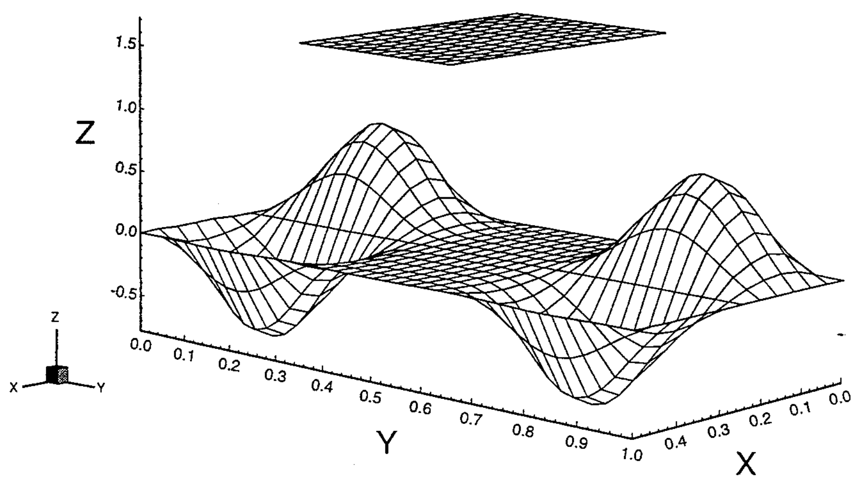


Figure 2. b. Longitudinal Cross-Section and Coordinate System of Orthotropic, Composite Plate or Panel with <u>Non-Central</u> Stiffening Plate Strip ("<u>Second Case</u>")



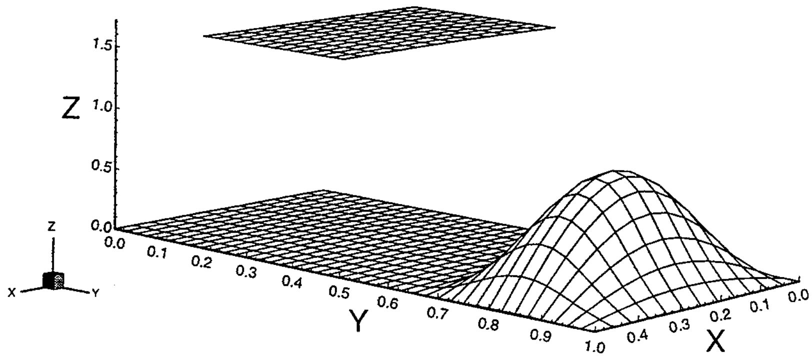
a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 1097.508$



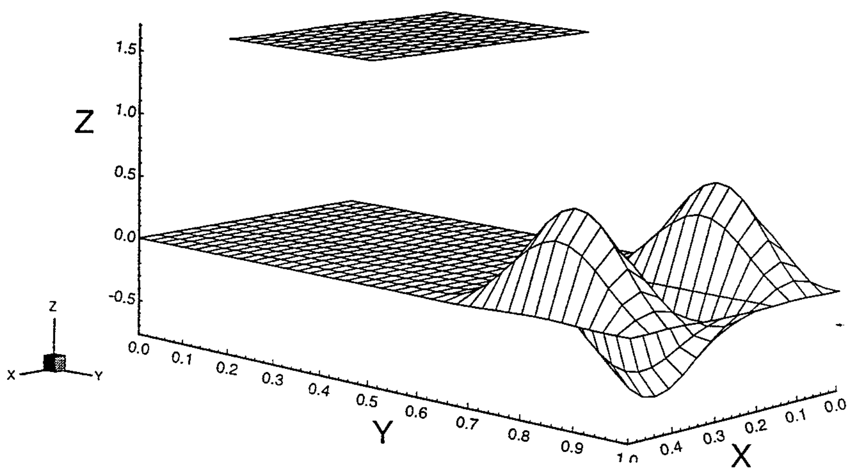
b) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{21} = 1265.070$

Figure 3. Mode Shapes and Natural Frequencies of Orthotropic, Composite Plate or Panel with Central Stiffening Plate Strip ("First Case")

(Plate 1= Graphite-Epoxy, Plate 2= Kevlar-Epoxy)
("Hard" Adhesive, Joint Length=0.30 m.)
(Boundary Conditions in y-direction FFCC)



a) First Mode with $\overline{\Omega}_1 = \overline{\omega}_{11} = 306.736$



b) Third Mode with $\overline{\Omega}_3 = \overline{\omega}_{13} = 773.662$

Figure 4. Mode Shapes and Natural Frequencies of Orthotropic, Composite Plate or Panel with Non-Central Stiffening Plate Strip ("Second Case")

(Plate 1= Graphite-Epoxy, Plate 2= Kevlar-Epoxy)
("Hard" Adhesive, Joint Length=0.30 m.)
(Boundary Conditions in y-direction FFCC)

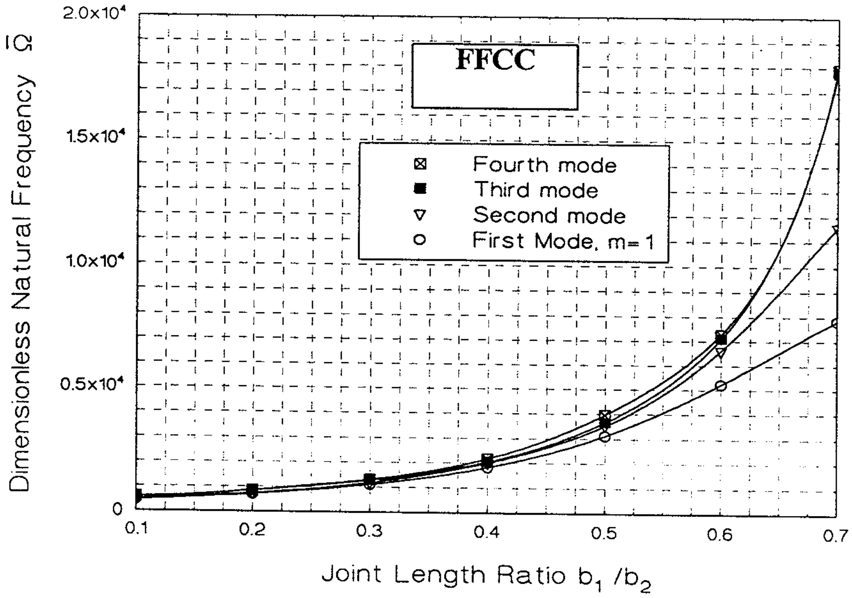


Figure 5. Influence of "Joint Length Ratio b_1/b_2" on Frequencies of Orthotropic, Composite Plate or Panel with Central Stiffening Plate Strip ("First Case")

(Plate 1= Graphite-Epoxy, Plate 2= Kevlar-Epoxy)
("Hard" Adhesive, Joint Length=0.30 m.)
(Boundary Conditions in y-direction FFCC)

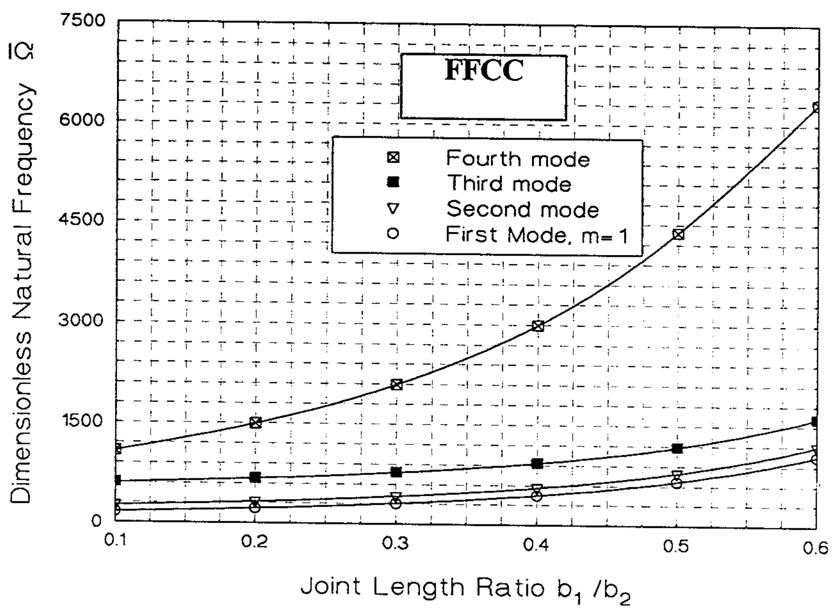


Figure 6. Influence of "Joint Length Ratio b_1/b_2" on Frequencies of Orthotropic, Composite Plate or Panel with Non-Central Stiffening Plate Strip ("Second Case")

(Plate 1= Graphite-Epoxy, Plate 2= Kevlar-Epoxy)
("Hard" Adhesive, Joint Length=0.30 m.)
(Boundary Conditions in y-direction FFCC)