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NUMERICAL SIMULATION OF TEMPERATURE FIELD DURING FATIGUE PROCESS

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Abstract There is a great amount of plastic strain energy changes to heat due to the intensive internal friction during low-cycle fatigue. Thermal emission is the main form of energy dissipation in fatigue. Thus obvious temperature increment is found in fatigue experiments. To investigate the energy dissipation law during fatigue process, a heat conduction equation in the material is established according to the classic thermodynamics theory and the Gibbs local equilibrium assumption. By analyzing the relation of the post-fatigue toughness and the energy stored in materials during fatigue, the heat generation rate in each cycle is obtained. Using finite difference method to simulate the temperature field of a rectangular plate specimen, the authors obtains the law of temperature variation during the whole fatigue process and compares it with the experimental result. Those factors affecting the temperature variation, including stress/strain level, frequency and load wavetype, are also analyzed.

Keywords Low-cycle fatigue, energy dissipation, thermal emission, finite difference,

Introduction

The fatigue damage process is accompanied with the energy accumulation and dissipation.

Materials absorb a great amount of plastic strain energy during fatigue. Many fatigue criterions based on hysteresis energy have been presented since 1960s[1-4]. Miner[5] presented the well-known linear damage accumulation theory utilizing the cyclic hysteresis energy as damage variable. A physical value named fatigue toughness is defined by the total energy absorbed during fatigue process,

$$W_{fT} = \Delta W^P \cdot N_f$$

where ΔW^P is the hysteresis energy. But it is difficult to utilize the fatigue toughness to predict the fatigue life or evaluate the fatigue damage, since fatigue toughness is not identical and varies with the stress (or strain) level, as found by Halford in 1966[6].

In 1970s, as the rapid development of experimental techniques, advanced non-contact thermometers are used to investigate self-heating process during fatigue. The idea of irreversible energy dissipation based on heat dissipation gradually forms. Mahn[7] measured the temperature distribution during fatigue crack propagation by thermovision. Reifsnider[8] measured the thermal emission of Boron/Epoxy and Boron/Aluminum composite materials under strain controlling and found that the temperature distribution was similar to the stress distribution in the initial period, and then changed gradually. Thus thermal emission showed the development of the fatigue damage. Charles[9] utilized infrared camera to predict pre-fatigue damages and investigate the fatigue crack propagation. The view that the mechanical energy absorbed by materials during fatigue can be divided into dissipated energy and stored energy is extensively accepted. Гуревин, Blotny[10,11] obtained the heat generation during fatigue by direct experiments or approximate calculation. Gross[12-14] deduced the total heat generation during cyclic loading when studied the relation between the heat generation and plastic work in each cyclic, i.e.

$$Q_{tot} = A_0 K_{eff} \left[\int \Delta T dt - N(A + B_I)/k\right] 2b\Delta x$$

where, K_{eff} is the efficient heat conductivity, k is the heat conductivity, b is the thickness of specimen, $N(A+B_I)$ is the thermal background, ΔX is thermosensitive resistence.

Li Hao[15] established a non-linear model of temperature field, by assuming that plastic work totally changes to heat and results in the temperature increment around the crack tip. Tong Xiaoyan[16,17] established a dissipation structure of fatigue damage based on the experimental measurement results and thermodynamics theory of metal deformation, that is

$$\rho c \frac{\partial T}{\partial t} = k\nabla^2 T + q$$

He also pointed that the heat generation rate in a material unit is correlated to the hysteresis energy dissipation rate and anelastic energy dissipation rate, i.e.

$$q = f_1(\Delta \dot{W}^P) + f_2(\Delta \dot{W}_{an})$$

He thought that the energy caused fatigue damage is the total plastic strain energy excluding heat dissipation and then defined the efficient dissipation ratio of fatigue damage to be

$$\eta = \frac{\overline{Q}_{TS}}{\Delta W^P} = m_1\beta + m_2$$

Heat Conduction Equation

Materials absorb great amount of plastic strain energy during fatigue. Plastic deformation is irreversible, it definitely accompanies irreversible entropy increment and continuous temperature variation. So fatigue process is an irreversible and non-equilibrium thermodynamic process. Great care must be taken when using classic thermodynamic theory in fatigue analysis. Under the condition of low stress rate and strain rate, since material structure changes very slowly in a small time interval, it can approximately be treated as an equilibrium process, and the classic thermodynamic relations are still applicable, according to the Gibbs local equilibrium assumption of non-equilibrium thermodynamics.

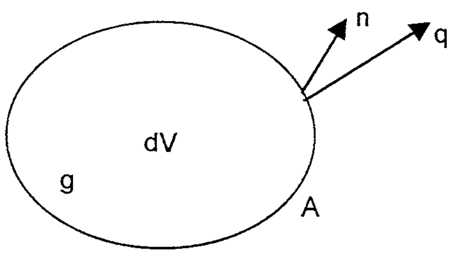

FIGURE 1- Material Volume Unit

Considering a material unit dV (see Figure 1), the energy equilibrium can be described by,

Heat enters in dV a unit time

\+ Energy produced in dV a unit time

= Energy accumulated in dV a unit time

In mathematical form, it can be written by,

$$k \cdot \nabla^2 T(\vec{r},t) + g(\vec{r},t) = \rho C_p \frac{\partial T(\vec{r},t)}{\partial t} \tag{1}$$

Researches show that there is 90 to 95 percent of the plastic strain energy changes to heat during fatigue process. Thus the energy produced per unit time in the above equation can be expressed by $g(\vec{r},t) = \xi \dot{w}(\vec{r},t)$. We have

$$\rho C_p \frac{\partial T(\vec{r},t)}{\partial t} - k\nabla^2 T(\vec{r},t) - \xi \dot{w}(\vec{r},t) = 0 \tag{2}$$

where, ρ is mass density, C_p is specific heat, k is heat conductivity, $\dot{w}$ is plastic strain energy velocity, $\vec{r}$ is position vector of the material element, ξ is heat generation rate of plastic strain energy.

In fatigue researches, we often use a cycle as the unit of time to study the variation of materials. Then the plastic strain energy velocity is the average of plastic strain energy in one cycle, i.e., $\dot{w}_p(\vec{r},n\tau) = \frac{1}{\tau}\Delta W^p(\vec{r},n)$, where $\Delta W^p(\vec{r},n)$ is cyclic hysteresis energy at $\vec{r}$ in nth cycle, n is load cycle, τ is load period. Equation 2 can be rewritten as

$$\rho C_p \frac{\partial T(\vec{r},n\tau)}{\partial (n\tau)} - k\nabla^2 T(\vec{r},n\tau) - \xi \dot{w}(\vec{r},n\tau) = 0 \tag{3}$$

Heat Generation

To make metals rupture, it must overcome the bond force of the atoms. It is easy to believe that the energy needed to overcome the atomic bond strength keeps a constant for the same material. Since the ability to absorb mechanical energy of materials is always defined by toughness, then toughness remains unchanged under different loading conditions. The phenomena found by Halford that fatigue toughness vary tremendously under different stress levels is attributed to the dissipation of energy. That is, the energy measured by Halford is the total energy absorbed by materials, it includes the energy stored in materials and the energy dissipated into the environment, which is mainly in the form of heat.

Experiments have been done to measure the static toughness after a number of cyclic loading. Results show that post-fatigue static toughness decrease monotonicly with increase of the of load cycles under the same stress level[18], that means the ability to absorb energy decreases as the development of fatigue damage. The decreasing part has been stored in the material during the previous fatigue loading.

The variation of the post-fatigue toughness obeys the following equation [18]:

$$U_N = U_0 + (U_0 - U_c)\frac{\ln(1 - N/N_f)}{\ln N_f} \tag{4}$$

where, U_0 is static toughness of the original material, U_N is post-fatigue static toughness after N cycles, N_f is the cycles to failure, U_c is the critical value of toughness and expressed by

$$U_c = \sigma_a(1 + 1.39\varphi_c)\ln\left[\frac{1}{1-\varphi_c}\right] \tag{5}$$

where, φ_c denotes the percentage reduction of cross area after fatigue fracture.

During a load cycle N, the degradation of post-fatigue toughness can be expressed by

$$\Delta U_N = U_N - U_{N-1} = \frac{(U_0 - U_c)}{\ln N_f} \ln\left[\frac{N_f - N}{N_f - (N-1)}\right] \quad (6)$$

As discussed above, the degradation of toughness stores in the material, while the left part of the plastic strain energy changes into heat. Assume that the heat generation rate remains unchanged in one cycle, then the heat generation rate of Nth cycle can be expressed by:

$$\xi(N) = 1 - \frac{\Delta U_N}{\Delta W^p(N)} \quad (7)$$

Plastic Strain Energy

The accumulating velocity of plastic strain energy is defined to be

$$\dot{w}_p = \sigma \cdot \dot{\varepsilon}_p \quad (8)$$

where, $\dot{\varepsilon}_p$ is the plastic strain velocity.

For Masing materials, after reaching the cyclic stability, the cyclic stress strain relation can be described by

$$\Delta\varepsilon_t = \Delta\varepsilon_e + \Delta\varepsilon_p = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \quad (9)$$

It can be deduced form Equation 8 and Equation 9 that the accumulating velocity of plastic strain energy can be expressed by

$$\dot{w}_p = \frac{\sigma\dot{\sigma}}{n'K'}\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}-1} \quad (10)$$

where, n' is the cyclic hardness exponent, K' is the cyclic strength coefficient. In the case of strain control, the stress velocity is written by

$$\dot{\sigma} = \frac{\dot{\varepsilon}_t}{\frac{1}{E} + \frac{1}{K'n'}\left(\frac{\Delta\sigma}{2K}\right)^{\frac{1}{n'}-1}} \quad (11)$$

where $\dot{\varepsilon}_t$ is the controlling strain velocity. The total accumulated plastic strain energy at any time t in a cycle is

$$w_p(t) = \int_0^t \dot{w}_p dt \quad (12)$$

We often investigate the energy dissipation by treating a load cycle as a unit time, then the mechanical energy in 'unit' time is described by cyclic hysteresis energy. The cyclic hysteresis energy of Masing materials and non-Masing materials are expressed in the following forms [19].

Masing materials:

$$\Delta W^p = \frac{1-n'}{1+n'}\Delta\sigma\Delta\varepsilon_p \quad (13)$$

Non-Masing materials:

$$\Delta W^p = \frac{1-n''}{1+n''}(\Delta\sigma - \delta\sigma)\Delta\varepsilon_p + \delta\sigma\Delta\varepsilon_p \quad (14)$$

where, $\Delta\sigma$ and $\Delta\varepsilon_p$ are local cyclic stress and cyclic plastic strain range respectively, $\delta\sigma$ is the difference in yield limit between the essential hysteresis loop and the normal hysteresis loop, n'' is the cyclic hardness exponent of the essential hysteresis loop.

Numerical Simulation

Numerical Model

A rectangular smooth specimen is used to conduct numerical simulation. Since the thickness-width rate of the specimen is very small, the problem can be simplified to be a two-dimensional thermal conduction problem. The heat flow produced between the specimen surface and the environment due to temperature difference is expressed by q_0, and $q_0 = 2h(T_\infty - T)/B$.

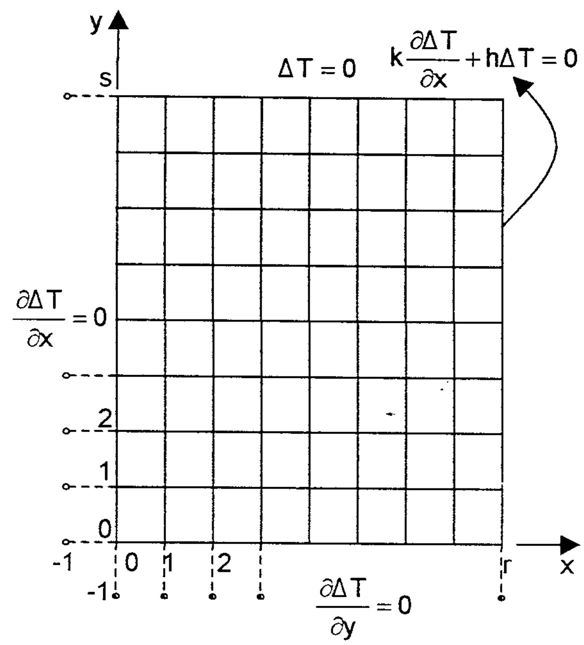

FIGURE 2- Finite Difference Mesh And Boundary Conditions

Define the temperature increment variable

$$\Delta T = T - T_\infty \quad (15)$$

The governing equation 3 changes to

$$\rho C_p \frac{\partial \Delta T}{\partial(n\tau)} - k\nabla^2(\Delta T) - \frac{\xi}{\tau}\Delta W^p + 2h\Delta T/B = 0 \quad (16)$$

where, B is the thickness of the plate specimen.

Because of the symmetry, the specimen is cut off along its symmetric axes and the right-up quarter is selected to construct the numerical model. Choose rectangular equi-spaced mesh, and the numbers of the grids along x direction and y direction are r and s separately.

Because the volume of the test machine is much larger than that of the specimen, it can be proposed that the temperature increment at those nodes connecting the specimen and the grip equal to zero. In the symmetric planes, it satisfies adiabatic condition. While in the free planes (surface planes), it satisfies the third kind of boundary condition, i.e., convection condition. The boundary conditions are given by

$$\text{Symmetric planes: } \frac{\partial \Delta T}{\partial n} = 0$$

$$\text{Grips: } \Delta T = 0 \qquad (17)$$

$$\text{Free planes: } k\frac{\partial \Delta T}{\partial n} + h\Delta T = 0$$

Finite difference solution

According to the Frank-Nilson equation, the difference equation can be written by

$$\begin{aligned} \rho C_p \frac{\Delta T_{i,j}^{n+1} - \Delta T_{i,j}^{n}}{\tau \cdot \Delta m} = & \frac{k}{2} \cdot \frac{\Delta T_{i-1,j}^{n} - 2\Delta T_{i,j}^{n} + \Delta T_{i+1,j}^{n}}{\Delta x^2} \\ & + \frac{k}{2} \cdot \frac{\Delta T_{i-1,j}^{n+1} - 2\Delta T_{i,j}^{n+1} + \Delta T_{i+1,j}^{n+1}}{\Delta x^2} \\ & + \frac{k}{2} \cdot \frac{\Delta T_{i,j-1}^{n} - 2\Delta T_{i,j}^{n} + \Delta T_{i,j+1}^{n}}{\Delta y^2} \\ & + \frac{k}{2} \cdot \frac{\Delta T_{i,j-1}^{n+1} - 2\Delta T_{i,j}^{n+1} + \Delta T_{i,j+1}^{n+1}}{\Delta y^2} \\ & + \frac{\xi_n}{\tau} \Delta W^{P}{}_{i,j}^{n} - 2h\Delta T_{i,j}^{n} / B \end{aligned} \qquad (18)$$

where, Δm denote the number of cycles that each step contains, Δx and Δy denote the step length along x and y direction respectively and are expressed by

$$\Delta x = \frac{L}{2r} \qquad \Delta y = \frac{W}{2s}$$

Here, L is the distance between the two grips of test machine and W is the width of the specimen.

Imaged nodes outside the specimen (see Figure 2) are used to deal the convection boundary and the intersection point of two convection boundaries. The adiabatic boundary is regarded as a special case of the convection boundary.

The temperature of the specimen before the test equals to the environment, so the initial conditions are

$$\Delta T_{i,j}^{0} = 0, \quad i = 0,\cdots,r, \quad j = 0,\cdots s \qquad (19)$$

The linear difference equations are solved by Guass-Sidel iteration. The initial conditions are the initial values of the first step of iteration. After that, the results at each time are the initial values of the next iteration.

Results and Analysis

The experimental material is high strength steel 40CrNiMoA. The longitudinal distance between the two grips is 100mm. The gauge length is 30mm. The width and the thickness of the specimen are 25mm and 9mm, respectively. The size change of the mesh due to the deformation of the specimen is negligible. The test is strain controlled.

It can be easily imagined that stress (or strain) level is a main factor affecting the temperature variation, since the plastic strain energy accumulated in one cycle changes positively with the stress (or strain), as indicated in Equation 13 and 14. Under high stress (or strain) level, temperature increases rapidly and the maximum value is much larger than that of the low stress (or strain) level, as seen in Figure 3. In this case, fatigue life is short because much more energy accumulated in material during each cycle, although also much more energy dissipates.

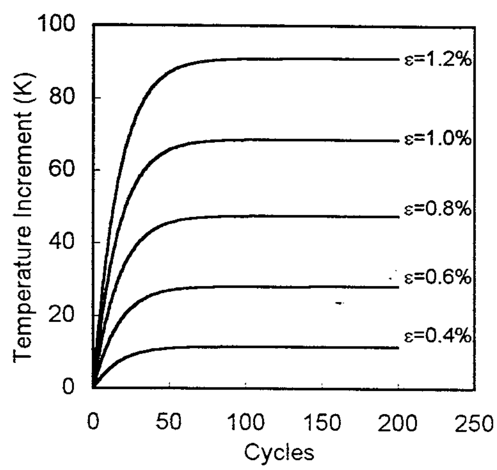

FIGURE 3- Variation Of Peak Temperature With Strain Amplitude

Load frequency also has great influence on the change of temperature field. Everyone has the experience that metals will feel burned if they are loaded very quickly. The calculating result also proves this fact (see Figure 4). But different to the stress (or strain), frequency has little influence on the fatigue life, if the frequency is not too high or too low. It is explained in this way. If other conditions are all the same, then material will consume the same amount of mechanical energy during a cycle. The temperature increase more rapidly under high frequency, heat energy also dissipates more rapidly since the higher temperature difference between material and environment. But to load one cycle, it consumes less time, the total heat energy dissipated maybe is approximately the same as that of the low frequency. From this point of view, it can be concluded that the heat generation rate is nearly the same under different load frequencies.

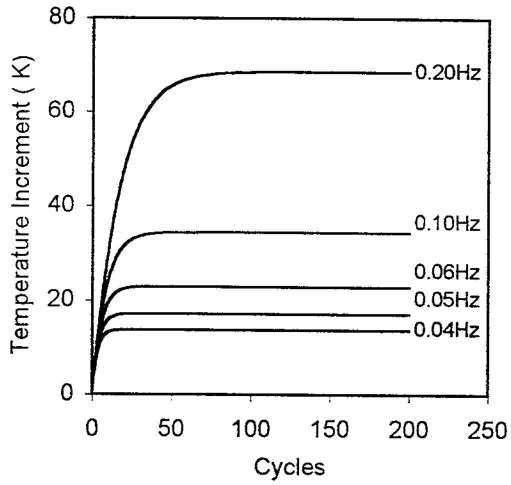

FIGURE 4- Variation Of Peak Temperature With Loading Frequency

Figure 5 and Figure 6 show the influence of load wave on the accumulation of plastic strain energy. One the whole, load type has much little influence than the stress (or strain) and frequency. Their influence on temperature variation needs accurate calculation.

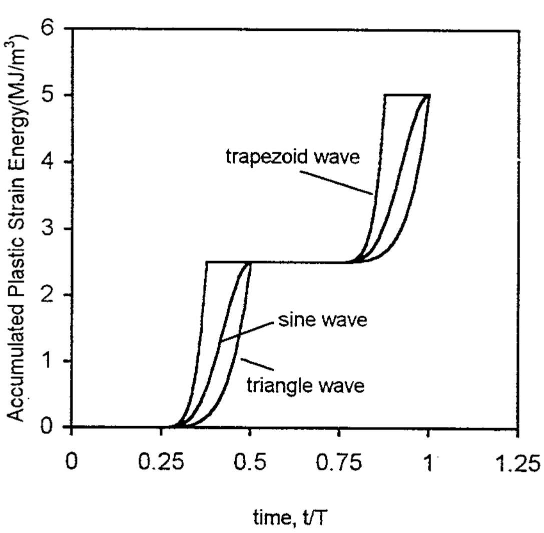

FIGURE 5- Accumulated Plastic Energy In A Cycle (Stress Control)

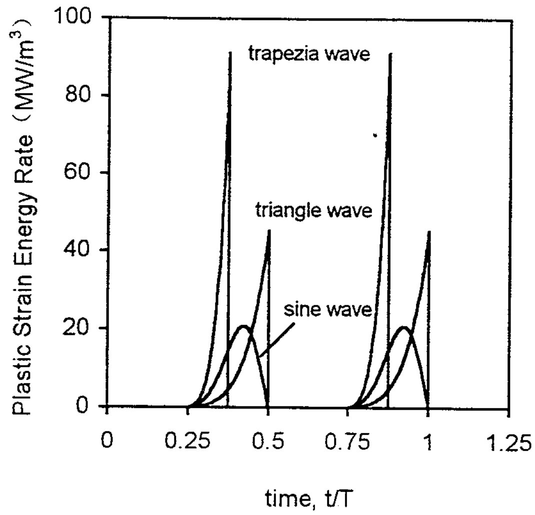

FIGURE 6- Plastic Energy Rate In A Cycle (Stress Control)

Figure 7 shows the contour curves of the temperature field. The temperature distribution shows an ellipsoid in space, as shown in Figure 8. The peak value posits at the middle point of the specimen, as predicted by experience. Figure 9 and Figure 10 show the experimental result.

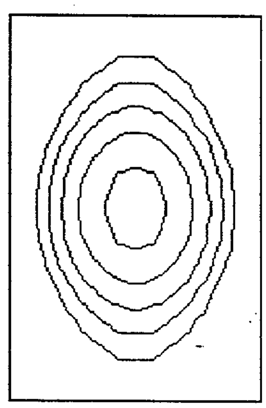

FIGURE 7- Contour Curves Of Temperature Field (Simulation)

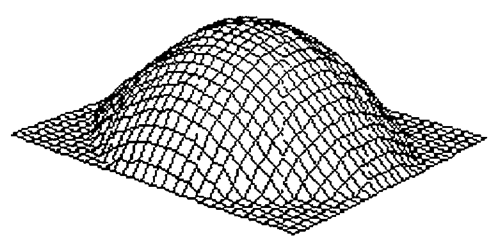

FIGURE 8- Spatial Distribution Of Temperature (Simulation)

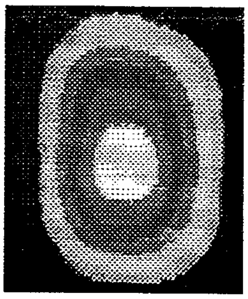

FIGURE 9- Contour Curves Of Temperature Field (Experiment)

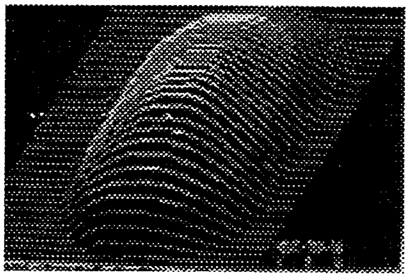

FIGURE 10- Spatial Distribution Of Temperature (Experiment)

It is shown in Figure 3 and Figure 4 that the peak value of the temperature increases as the load cycles increase. At the initial stage of fatigue, the temperature changes rapidly, because the most part of the plastic strain energy is stored in the material and only a little part dissipates into the environment. Some cycles later, the ratio of the dissipation energy increases because of the high temperature difference between the specimen and the environment, and the speed of temperature increment slows down consequently. When the heat dissipation and the heat production due to the plastic strain energy reaches balanced, the temperature becomes nearly unchanged. The temperature increment is very sensitive to the strain amplitude and the loading frequency, since they significantly affect the dissipation rate of the plastic strain energy.

During experiments temperature fluctuation in a cycle are found (see Figure 11). To investigate these phenomena, we directly use Equation 2 to simulate the temperature field. In this case, time interval is much less than that by treating one cycle as a unit time, since several time points are calculated within one cycle. The results show that the average of the peak temperature increases monotonicly, while temperature fluctuation exists in a cycle, as shown in Figure 12 and Figure 13. This can be explained by Figure 6. In Figure 6, it is found that the plastic energy velocity is not always

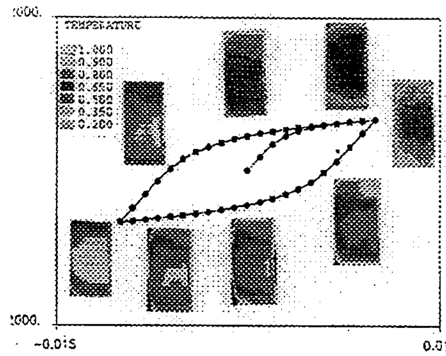

FIGURE 11- Temperature Fluctuation (Experiment)

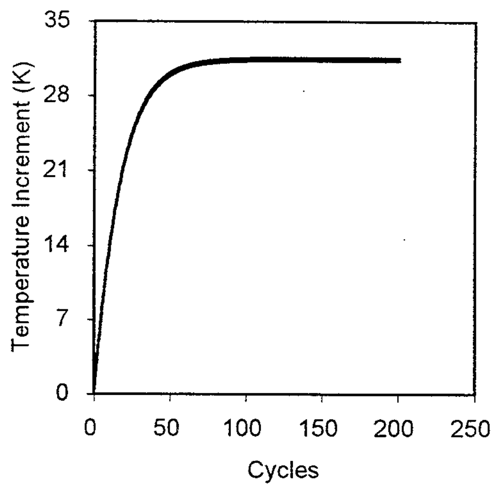

FIGURE 12- Peak Temperature Curve

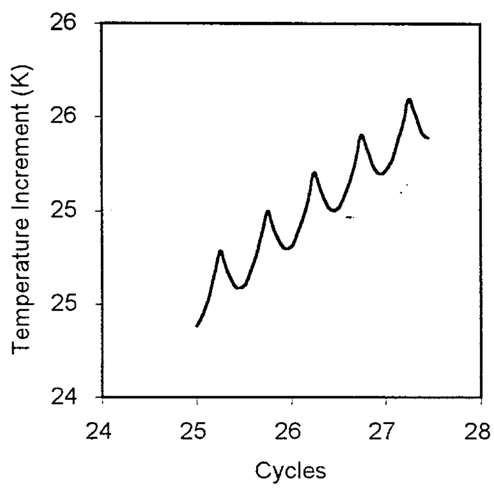

FIGURE 13- Temperature Fluctuation (Simulation)

positive. There is a period that plastic energy velocity is zero, which corresponding to the elastic loading period. In this period, no heat generated, but heat flows into environment due to the temperature difference. Thus the temperature decreases. When entering plastic, great amount of heat is generated and the temperature begins to increase. The amplitude of the fluctuation is about several tenth centigrade and is correlated to the strain amplitude, loading frequency, loading wave.

Conclusion

To investigate the heat emission behavior of metals during fatigue process, a heat conduction equation according to the classic thermodynamics theory and the Gibbs local equilibrium assumption and finite difference method is used to simulate the temperature field of a rectangular 40CrNiMoA specimen. It is concluded that

(1) Temperature increases rapidly at the beginning of fatigue, then the velocity slows down. When the heat dissipation and the heat generation reach balance, the temperature remains nearly unchanged, which corresponds to the stable damage development period.

(2) Stress/Strain level and frequency have great influence on temperature variation. Stress/Stain level directly affects the fatigue life since more energy stored in a cycle at higher stress/strain level. Frequency has no obvious influence on fatigue life if frequency is in a adequate scope. At ultra-high frequency, equation 2 and 3 are not applicable anymore. The influence of load wavetypes on temperature variation and fatigue life needs more accurate calculation.

(3) Temperature doesn't increase monotonically in a cycle. There exists temperature fluctuation. The amplitude is several tenth centigrade, which is correlated to the stress/strain level and frequency.

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