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THREE-DIMENSIONAL ROTOR FLOW CALCULATION

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Abstract

This paper presents the numerical model developed for rotor blade aerodynamics loads calculation. The model is unsteady and fully three-dimensional. Helicopter blade is assumed to be rigid, and its motion during rotation is modeled in the manner that rotor presents a model of rotor of helicopter Aerospatiale SA 341 *"Gazelle"* (the blade is attached to the hub by flap, pitch and pseudo lead-lag hinges). Flow field around the blade is observed in succession of several azimuth locations. Flow field around helicopter rotor is modeled as fully three-dimensional, unsteady and potential. Blade aerodynamics is modeled using a lifting surface model. Rotor wake is generated from the straight elements of constant vorticity, released from the trailing edge, at fixed azimuth angles. These vortices represent both trailed and shed wake components, and are allowed to freely convect along local velocity vectors. Wake is modeled as free one, and its shape at certain moment can be calculated from simple kinematics laws applied on collocation points of the wake. Wake distortion is calculated only in the rotor near-field, i.e. in finite number of rotor revolutions. Vortex elements are modeled with vortex core. The radius of the vortex core is assumed independent of time, and it depends on circulation gradient at the point of vortex element released from the blade.

Introduction

Helicopter rotor aerodynamic flow field is very complex, and it is characterized by remarkably unsteady behavior. The most significant unsteadiness appears during the forward flight. In that case the progressive motion of helicopter coupled with rotary motion of rotor blades causes drastic variations of local velocity vectors over the blades, where the advancing or retreating blade position is of great significance. In first case, the local tip transonic flow generates, while in second, speed reversal appears. Also, in forward flight, blades encounter wakes generated by forerunning blades and so encounter non-uniform inflow. The wake passing by the blade induces high velocities close to it causes changes in lifting force. Besides that, in horizontal flight blades constantly change pitch, i.e. angle of attack at different azimuths. Such angle of attack variations are very rapid, so that dynamic stall occurs, specially in case of retreating blades.

Dynamics

In this paper the rotor of helicopter SA 341 *"Gazelle"* is modeled, at which the blades are attached to the hub by flap, pitch and pseudo lead-lag hinges.

Blade motion in lead-lag plane is limited by dynamic damper, which permits very small maximum blade deflection. Due to such small angular freedom of motion, we can assume that there is no led-lag motion at all.

Pitch hinge is placed between flap hinge and pseudo lead-lag hinge. In derivation of the equations of motion, the following was assumed:

- the rotor does not vibrate, and its rotation velocity Ω is constant,
- the blade is considered absolutely rigid.

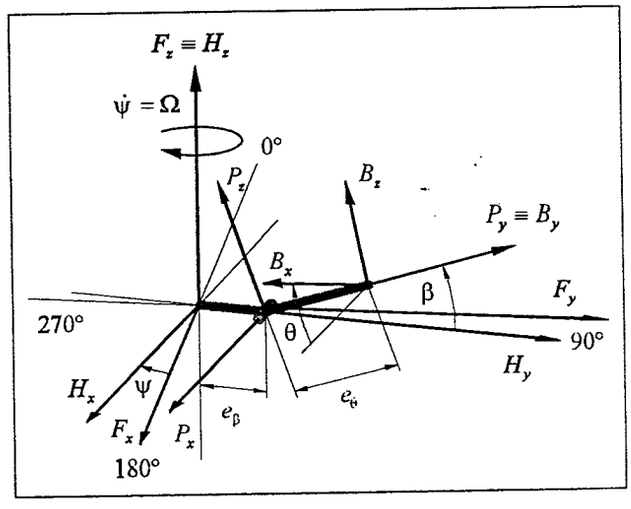

Fig. 1: *Coordinate systems*

According to the above mentioned, the following frames have been selected for use:

- fixed frame F, with x-axis in the direction of flight, and z-axis oriented upwards;

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- frame H, connected with the rotor and rotates together with it; it is obtained by rotating the F frame for a certain azimuth angle ψ, keeping the common z-axis
- frame P, connected to the flapping hinge, so that the y-axis is oriented along the blade; its origin is displaced from the rotating axis for the value e_β, while it is rotated for the angle β with respect to the frame H;
- frame B, connected to the blade, displaced for the value e_θ from the origin of P, and tilted for the value θ (pitch angle) with respect to the P.

Aerodynamics

Analytical model

The flow field is assumed to be potential (inviscid and irrotational) and incompressible. In that case velocity potential satisfies the Laplace equation:

$$\Delta\Phi = 0$$

The equation is the same, both for steady and unsteady flows. Owing to that, methods for steady cases can be applied for the solution of unsteady flow problems, as well. Unsteadiness is introduced by unsteady boundary condition:

$$\left(\vec{V} - \vec{V}_T\right)\cdot\vec{n} = 0$$

of the Kelvin theorem:

$$\frac{D\tilde{\Gamma}}{Dt} = 0$$

and the unsteady form of the Bernoulli equation:

$$\frac{p - p_\infty}{\frac{1}{2}\rho_\infty} = V_\infty^2 - V^2 - 2\frac{\partial\Phi}{\partial t}$$

where: Φ - is velocity potential, $\vec{V}$ - is absolute fluid velocity, $\vec{V}_T$ - is lifting surface velocity, $\vec{n}$ - is the normal of the lifting surface at a certain point, and $\tilde{\Gamma}$ - is the bound circulation.

In order to define the aerodynamic characteristics of blades, two models should be established: blade model and wake model.

The blade is modeled as a thin lifting surface, which enables a complete 3D modeling around helicopter blades. Unfortunately, it can not deal with compressible and viscous flows.

Numerical modeling of the wake must be done very carefully due to it's high influence on the lift force generation. The *free-wake* model, which is applied in this paper, is one of the most advanced, since it can cover all relevant problems connected with the wake influence.

Unsteady Kutta condition

In case of inviscid problems, it is necessary to satisfy Kutta condition at the trailing edge.

On the basis of unsteady Brenoulli equation, the pressure coefficient for unsteady flow is defined as:

$$C_P = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = 1 - \frac{V^2}{V_\infty^2} - \frac{2}{V_\infty^2}\frac{\partial\Phi}{\partial t}$$

According to that, the difference between upper and lower surface pressure coefficients is:

$$\Delta C_P = C_{P_U} - C_{P_L} = -\frac{V_U^2 - V_L^2}{V_\infty^2} - \frac{2}{V_\infty^2}\frac{\partial}{\partial t}\left(\Phi_U - \Phi_L\right) \quad (1)$$

where subscripts U and L denote upper and lower surface values.

In case of the thin lifting surface, with the assumption that spanwise velocity components are small, the potential difference can be written as an integral from leading edge to a certain point M at the surface:

$$\Phi_U - \Phi_L = \int_{LE}^{M}\left(V_{U_T} - V_{L_T}\right)dl$$

where the tangential velocity difference is the local bound vortex distribution:

$$\gamma = \left(V_{U_T} - V_{L_T}\right)$$

Final equation defining the potential difference is:

$$\Phi_U - \Phi_L = \int_{LE}^{M}\gamma dl \quad (2)$$

If we assume that spanwise velocities are small, the difference of velocity squares can be calculated as:

$$V_U^2 - V_L^2 \approx 2V_\infty\gamma \quad (3)$$

By substituting (3) and (2) in (1), the following equation can be obtained:

$$\Delta C_P = -\frac{2}{V_\infty^2}\left(V_\infty\gamma + \frac{\partial}{\partial t}\int_{LE}^{M}\gamma dl\right)$$

The Kutta condition can be expressed as the uniqueness of pressure coefficients at the trailing edge, which, mathematically expressed, takes the form:

$$\Delta C_p = \frac{2}{V_\infty^2}\left(V_\infty\gamma_{II} + \frac{\partial}{\partial t}\int_{NI}^{II}\gamma\, dl\right) = 0$$

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Since it is impossible to be $V_\infty = \infty$, the relation within the parentheses must be equal to zero:

$$V_\infty \gamma_{TE} + \frac{\partial}{\partial t}\int_{LE}^{TE} \gamma dl = 0 \qquad (4)$$

The integral in the upper equation is, in fact, the contour circulation which covers the lifting surface:

$$\tilde{\Gamma} = \int_{LE}^{TE} \gamma dl$$

so, the equation (4) can be written as:

$$V_\infty \gamma_{TE} + \frac{\partial \tilde{\Gamma}}{\partial t} = 0$$

The expression for unsteady Kutta condition comes out directly as:

$$\frac{\partial \tilde{\Gamma}}{\partial t} = -V_\infty \gamma_{TE}$$

If the right hand-side part is substituted with (3) written for the trailing edge, we obtain:

$$\frac{\partial \tilde{\Gamma}}{\partial t} = -\frac{V_{U_{TE}}^2 - V_{L_{TE}}^2}{2} = -\left(V_{U_{TE}} - V_{L_{TE}}\right)\frac{V_{U_{TE}} + V_{L_{TE}}}{2}$$

From this equation it can be clearly seen that the variation of the lifting surface circulation in time can be compensated by releasing vortices of magnitude $\left(V_{U_{TE}} - V_{L_{TE}}\right)$ at the velocity $\left(V_{U_{TE}} + V_{L_{TE}}\right)/2$.

Discretization and Numerical Solution Procedure

The method for the solution of this problem is based on the coupling of the dynamic equations of blade motion with the equations of aerodynamics. It is not possible to obtain an analytical solution of this problem, so discretization and numerical approach must be accepted.

Dicretization in time is done by observing the flow around the blade in a series of positions that it takes at certain times $t_k (k = 0,1,2\ldots)$, which are spaced by finite time intervals Δt at different azimuths.

Discretization of the thin lifting surface is done by using the panel approach. By this method, the lifting surface is divided in a finite number of quadrilateral surfaces – panels. Vorticity distribution is discretized in a finite number of concentrated, closed quadrilateral linear vortices, whose number is equal to the number of panels, in such a way that one side of the linear vortex is placed at the first quarter chord of the panel, and represents the bound vortex of the corresponding panel. The opposite side of the vortex is always placed at the trailing edge., while the other two sides are parallel to the flow. The wake is represented by quadrilateral vortex in the airflow behind the lifting surface. One side of it is connected to the trailing edge, while the opposite one is at the infinity. The other two sides (trailing vortices), which actually represent the wake, are placed parallel to the airflow. The vorticity of the quadrilateral vortex is equal to the sum of the vorticities of all bound vortices of the panels that correspond a certain lifting surface chord, but opposite in direction. Then the trailing edge vorticity is equal to zero.

Model established in such a manner corresponds to the steady flow case. On the other hand, it can be very easily spread in order to include the unsteady effects.

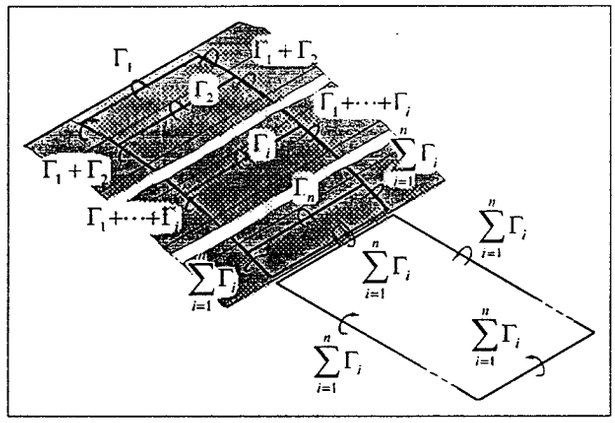

Fig. 2: *The steady panel scheme*

Vortex releasing model

The variation of the lifting surface position in time induces variation of circulation around the lifting surface as well. According to the Kelvin theorem, this variation in circulation must also induce the variation around the wake. According to the unsteady Kutta condition, this can be achieved by successive releasing of the vortices in the airflow.

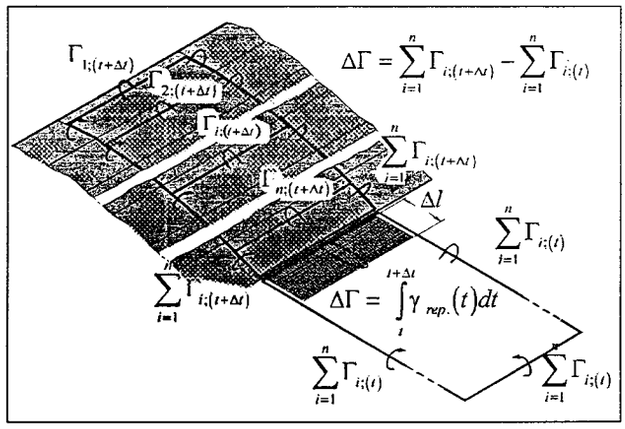

Fig. 3: *Vortex releasing*

Suppose that the lifting surface has been at rest until the moment t, when it started with the relative motion with

respect to the undisturbed airflow. The vortex releasing, as a way of circulation balancing, is done continually, and in such a way a vortex surface of intensity $\gamma(t)$ is formed.

At the next moment $t+\Delta t$, the flow model will look like in Fig. 3. The circulation of the vortex element joined to the trailing edges equal to the difference in circulations at moments $t+\Delta t$ and t.

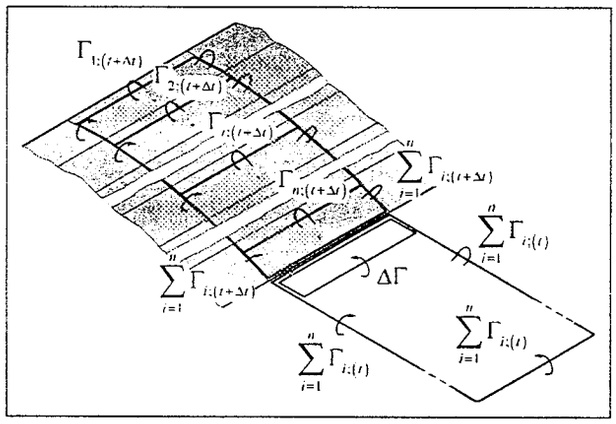

Fig. 4: *Unsteady panel scheme*

We will discretize the vortex "tail" by replacing it with the quadrilateral vortex loop, whose one side is at the trailing edge, and the opposite side is at the finite distance from the trailing edge (shed vortex). By this we can obtain the final model for unsteady case.

Discretization of Wake

The established vortex releasing model is appropriate for the wake modeling using the *"free wake"* approach.

During the time, by continuous releasing of the quadrilateral vortex loops, the vortex lattice formed of linear trailed and shed vortices is created.

The collocation points of the vortex lattice are node points. The wake distortion is achieved by altering the positions of the collocation points in time, by application of a rather simple kinematics relation: $\bar{r}_i(t+\Delta t)=\bar{r}_i(t)+\bar{V}_i(t)\Delta t$. The velocities of the collocation points are obtained as sums of the undisturbed flow velocity and velocities induced by other vortex elements of the flow field. Induced velocities are calculated using the Biot-Savart law. In order to avoid the problems of velocity singularities, line vortex elements are modeled with core.

The core radius varies with the gradient of the bound circulation at the position where vortex is released, from the value $r_c = 0.00275R$ (where R is the rotor diameter) for the elements at the end of the wake (high gradient positions), to the values $r_c = 0.05R$ inside the wake (low gradients). The existence of the vortex core has remarkable influence in blade-wake interactions, since in this way large velocity irregularities on the blade close to the wake are avoided.

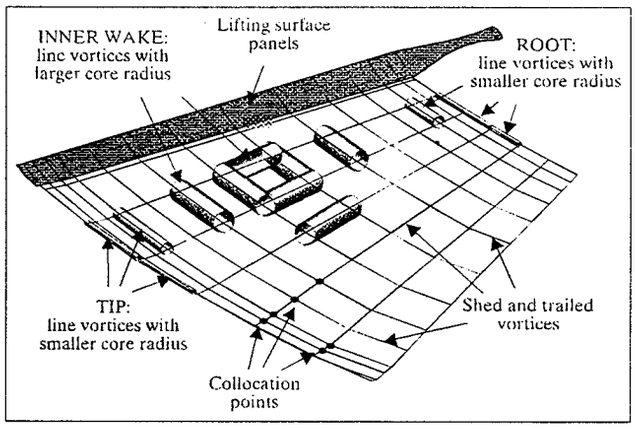

Fig. 5: *Dicretized wake*

The wake influence at large distances from the blade is negligible, so it is possible to neglect the wake distortion at a sufficient distance from the rotor. The area in which distortion is relevant depends on the helicopter flight regime, and it can be determined by the advance ratio μ.:

$$m = \frac{0.4}{\mu}$$

where m – is the number of revolutions for which it is necessary to calculate the wake distortion. After that, the wake shape is "frozen" in achieved state, and it moves through the flow field keeping it for the rest of the time (velocity of collocation points is equal to the flow field velocity).

The "frozen" part of the wake still influences the adjacent area in which it is still being distorted. After some distance, even the influence of the "frozen" part becomes negligible, and then it is eliminated from the model.

In this way, a dicretized wake model, consistent with the panel model of the lifting surface and vortex releasing is obtained.

Discretization in Time of Unsteady Kutta Condition

Let us consider the unsteady Kutta condition from the aspect of the assumed discretized model. The condition can be written as:

$$V_\infty \gamma_{TE} + \frac{\partial \tilde{\Gamma}}{\partial t} = 0 \qquad (5)$$

In case of transition to the discretized time domain, it is necessary to substitute the partial derivative with the finite difference form:

$$\frac{\partial \tilde{\Gamma}}{\partial t} = \frac{\tilde{\Gamma}(t+\Delta t)-\tilde{\Gamma}(t)}{\Delta t} = \frac{\Delta \tilde{\Gamma}}{\Delta t}$$

By substituting this equation in (5), we obtain:

$$V_\infty \gamma_{TE} + \frac{\tilde{\Gamma}(t+\Delta t)-\tilde{\Gamma}(t)}{\Delta t} = 0 \qquad (6)$$

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In case of numerical solutions, it is customary to satisfy Kutta condition in vicinity of the trailing edge. according to that, the intensity of the distributed vorticity at the trailing edge γ_{TE} is treated as equal to the intensity of the distributed vorticity at the trailing edge panel γ_n. The intensity of the distributed vorticity is constant at every panel, so it can be written:

$$\gamma_{TE} = \gamma_n = \frac{\Gamma_n}{l_n}$$

where γ_n is intensity of the distributed vorticity at the trailing edge panel, and l_i is the panel cord length. Substituting this equation in (6), we have:

$$V_\infty \frac{\Gamma_n}{l_n} + \frac{\tilde{\Gamma}(t+\Delta t) - \tilde{\Gamma}(t)}{\Delta t} = 0$$

i.e. the difference of circulations around the lifting surface at the moments $t + \Delta t$ and t can be calculated by:

$$\tilde{\Gamma}(t+\Delta t) - \tilde{\Gamma}(t) = -V_\infty \frac{\Gamma_n}{l_n} \Delta t$$

Definition of equation set

The boundary condition of impermeability of the lifting surface should be satisfied at any moment of time, $t_k (k = 0,1,2\ldots)$ in a finite number of points of lifting surface:

$$\left(\vec{V}_i - \vec{V}_{Ti}\right) \cdot \vec{n}_i = 0; \quad i = 1,2,\ldots,n$$

Points at which this condition must be satisfied are called the control points. One of them is placed on each panel, at the three-quarter chord panel positions. By this, at every moment of time, the number of lifting surface impermeability conditions is equal to the number of unknown values of circulations of bound vortices.

The equations of motion of the lifting surface are known, as well as the velocities $\vec{V}_{Ti}$ of all characteristic points, and their normals $\vec{n}_i$ as well.

At each flow field point, velocity can be divided to the free stream velocity and perturbation velocity:

$$\vec{V}_i = \vec{V}_\infty + \vec{w}_i$$

The perturbation velocity is induced by lifting surface and wake vortex elements. It is calculated by Biot-Savart law. At every moment, the wake shape and circulations of it's vortex lines are known, and so the wake induced velocity at every flow field point is known as well. On the other hand, the circulations of the bound vortices are unknowns (their positions are defined by the lifting surface shape).

The boundary condition for the i-th control point can be written as:

$$\sum_{i=1}^{n} a_{i,j} \Gamma_{ji}(t) = b_i \quad .$$

where $a_{i,j}$ are the coefficients depending of the blade geometry, and b_i are the coefficients containing the influence of the wake and free-stream flows.

This way, by writing equations for all control points, the equation set of the unknown bound circulations is obtained.

Besides this equation set, the Kutta condition must be satisfied. By adding the Kutta conditions to the equation set, an overdetermined equation set is obtained. It can be reduced to the determined system by the method of least-squares. After that, it can be solved by some of the usual approaches, by which the unknown values of circulations Γ_i at the time t are obtained.

Determination of the aerodynamic force

After unknown circulations Γ_i are obtained, velocity at every point of the flow field is known, and we can use them for the determination of aerodynamic forces that act on the blade. The calculation aerodynamic force is necessary for the defining of the blade position at the next moment of time. The total aerodynamic force is calculated as the sum of forces acting on all panels.

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i$$

Aerodynamic force acting on a single panel can be defined by introducing the Kutta-Joukowsky law in a vector form:

$$\vec{F}_i = \rho \vec{V}_\infty \times \Gamma_{i(ef.)} \vec{BC}$$

where $\vec{BC}$ bound circulation vector, and effective circulation can be defined by using:

$$\Gamma_{i(ef.)} = \Gamma_i + \frac{1}{V_\infty} \frac{\partial}{\partial t} \int_{LE}^{M_i} \gamma dl$$

The integral should be calculated from the leading edge to the quarter-chord position of the ith panel.

$$\Gamma_{i(ef.)} = \Gamma_i + \frac{1}{V_\infty} \frac{\partial}{\partial t} \left(\sum_{k=1}^{i-1} \Gamma_k + \frac{\Gamma_i}{4} \right)$$

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Results

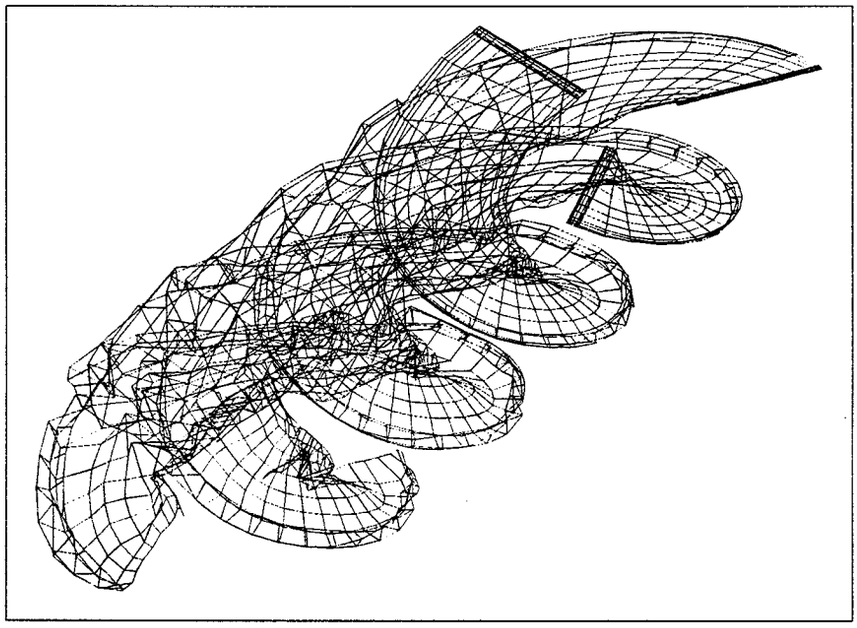

Fig. 6: *Calculated free wake geometry; $\mu = 0.35$ (3D view)*

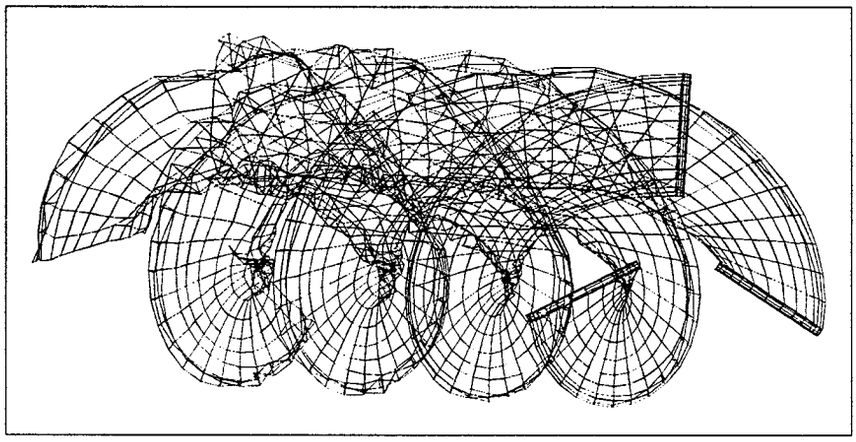

Fig. 7: *Calculated free wake geometry; $\mu = 0.35$ (top view)*

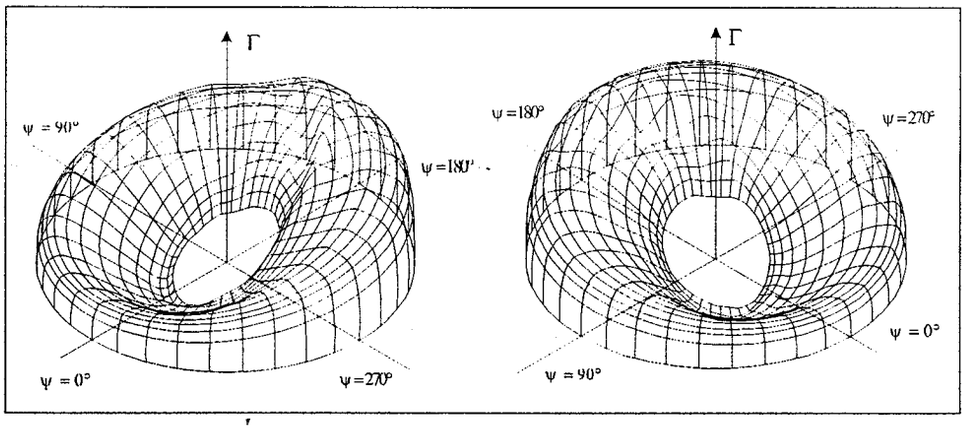

Fig 8: *Circulation distribution ; $\mu = 0.35$ (different points of view)*

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Conclusion

By analyzing the drawings of the blade wakes, it can be concluded that model applied in this paper gives reasonable simulation of actual wake behavior, specially in the domain of wake boundaries, where wake roll-up occurs (although it is slightly underestimated compared with existing experimental data). Also, larger wake distortion in the domains of the forerunning blades or their wakes is noticeable.

The program results show the difference in circulation distributions at different azimuths, as well as the disturbances caused when blades are passing the wakes of other blades, and the characteristic reversal flow domains.

At the same time, these results can define the suggestions for the future model upgrading. Firstly, by incorporating the transonic flow calculations, the advancing blade tip simulation would be more appropriate. Secondly, viscous interaction should be included as well, which would improve the wake roll-up simulation. The viscous vortex core simulation in time would improve the results concerning the wake vanishing effects far enough from the blade. Finally, introduction of the curvilinear vortex elements would give better results from the aspect of wake self-induction.

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