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## AN IMPROVED TECHNIQUE FOR FLIGHT PATH AND GROUNDSPEED ANALYSIS USING RECORDED RADAR DATA

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### Abstract

Recorded radar data retrieved from air traffic control facilities are often used in aviation accident investigation and reconstruction for the purpose of determining an aircraft's flight path and groundspeed. Simple averaging methods for smoothing the flight path and extracting groundspeed are shown to yield unsatisfactory results. In this paper, improved methods are presented which allow for a more accurate determination of both the times of the radar returns for long-range data, and more accurately establish the flight paths for either long-range radar data or terminal radar data. Least-squares moving-arc methods are applied in order to 1) adjust for azimuth and range errors that are inherent in recorded radar data, and 2) to smooth the flight path. Groundspeed and true course, as computed from the adjusted and smoothed flight path, are further smoothed by means of multi-point weighted averaging. Weighting functions and end-point handling methods are suggested that have proven successful at minimizing anomalous end-point excursions. Sample sets of actual recorded radar data are used to illustrate the application of the above techniques. For the example of terminal radar data, the groundspeed and true course are compared with the recorded output from the aircraft's digital flight data recorder.

### Introduction

Air Traffic Control (ATC) relies upon its radar facilities and the associated ground-based computer systems for the efficient and safe flow of air traffic. The received radar information is digitally recorded and can later be retrieved by means of "extraction" software. Even though this procedure was never intended for use as a means of accident reconstruction and analysis, it has, nevertheless, become, when available, an important item in the accident reconstructionist's tool chest.

Enroute Air Route Traffic Control Centers (ARTCC) and Terminal Radar Approach Control (TRACON) facilities are able to extract aircraft positions, times, and altitudes at a later time using the National Track Analysis Program (NTAP) and the Continuous Data Recording (CDR) Editor, respectively. This paper discloses some novel methods for calculating and smoothing the aircraft's flight path, groundspeed, and true course from these extracted data. These smoothed values, in combination with the computation of atmospheric winds, temperatures, and pressures, yield the aircraft's true heading, true airspeed, and the calibrated (indicated) airspeed.

Determining the most probable flight path from the recorded radar data requires more than conventional least-squares smoothing or regression analysis. Moreover, using simple or weighted arithmetic averaging to determine the groundspeed of the aircraft from the radar data is unreliable because it tends to "smooth out" real time variations in the aircraft's speed, and it skews the results toward errantly high values, particularly when the aircraft is relatively far from the radar antenna.

For the most part, position and time inaccuracies in the recorded and extracted radar data are not stochastic, but are instead deterministic. With this in mind, techniques will be presented that are effective in specifically determining the magnitude and direction of the inaccuracies and adjusting the recorded positions and times accordingly. One technique is applied to yield more precise time intervals between radar returns. Another technique is used to more precisely determine the aircraft locations along the flight path. The more exact times and positions are then utilized in the calculation of groundspeeds and true courses that are, correspondingly, more accurate. Overall, the ability to correct for deterministic errors that are inherent in extracted radar data is a significant advancement in our ability to deduce useful flight path and groundspeed information from recorded radar data.

Time Corrections for Enroute Radar Data (NTAP)

Extracted data from terminal radar approach facilities usually includes return times that are quite precise (listed to the nearest 0.001 second); consequently, time intervals for groundspeed calculations can reliably be based on the listed times. NTAP radar data extractions, on the other hand, only provide the time of each radar return to the nearest second; as a result, the time interval between consecutive radar returns typically does not remain constant throughout the data listing, but instead will change occasionally in increments of one second. Hence, if the nominal radar sweep interval is 10 seconds, one can expect variations in the groundspeed calculation of about 10% (due solely to the time interval variation) when the time interval changes by one second. In spite of this situation, a more precise radar sweep time interval can, in fact, be calculated. More importantly, the time interval of interest is not the radar antenna rotation interval. Instead, the time interval necessary for more accurate calculations of the groundspeed is the time between radar returns.

The concept of time corrections to recorded radar as presented herein is similar to that proposed by Vermij et. al.<sup>(1)</sup>. We assume first that the aircraft has a groundspeed component in the direction of rotation of the antenna, in which case the time between consecutive radar returns will be greater than the antenna rotation time. Similarly, if the direction of flight is opposite the direction of antenna rotation, the time interval will be less than the antenna rotation time.

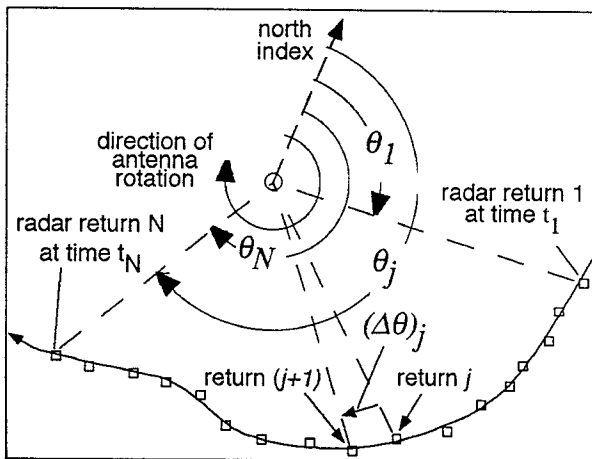


FIGURE 1 - Radar sweep time interval.

Figure 1 depicts a set of NTAP radar returns. If the antenna rotation time interval is  $\tau$ , then the total elapsed time  $T$  from radar return 1 to radar return  $N$  is

$$T = t_N - t_1 = (N - 1)\tau + \left( \frac{\theta_N - \theta_1}{2\pi} \right) \tau \quad (1)$$

where  $t_1$  and  $t_N$  are the listed radar times (to the nearest second) for the first and last returns in the data set, respectively. When the net flight path between the first and last radar returns is clockwise around the antenna,  $\theta_N$  is greater than  $\theta_1$ , and the second term in equation (1) is positive. Conversely, when the net flight path between the first and last radar returns is counterclockwise around the antenna,  $\theta_N$  is less than  $\theta_1$ , and the second term in equation (1) is negative. Solving equation (1) for  $\tau$ ,

$$\tau = \frac{t_N - t_1}{(N - 1) + \left( \frac{\theta_N - \theta_1}{2\pi} \right)} \quad (2)$$

Equation (2) is a satisfactory estimator for the antenna sweep time interval. As illustrated in figure 1, the time interval,  $(\Delta t)_j$ , between consecutive radar returns  $j$  and  $j+1$  can now be written as

$$(\Delta t)_j = \left[ 1 + \frac{(\Delta \theta)_j}{2\pi} \right] \tau \quad (3)$$

The time interval determined by equation (3) can be expected to vary along the flight path as the direction of flight, the airspeed, and the distance from the antenna change. As with the overall flight path,  $(\Delta \theta)_j$  can be either positive or negative.

Using the listed time at point 1 as a reference time, a smoother, more precise sequence of times for the radar returns can be established by the cumulative addition of each  $(\Delta t)_j$ .

Range-Azimuth Convergence and Least-Squares Moving-Arc Smoothing

The application of polynomial least-squares moving-arc techniques to discrete data sets is a classical, conventional method for data

smoothing<sup>(2)</sup>. Recorded radar data typically represent this type of discrete data. Wingrove and Bach<sup>(3)</sup> have used this procedure with recorded radar data to provide smoothed time histories of aircraft position and speed. However, the analysis presented herein differs from previous work in that the least-squares moving-arc procedure is applied independently to the range and azimuth values for the radar returns of a given data set. As a result, the range and azimuth of the data are "converged" on to a smoother flight path that is more representative of the manner in which an aircraft would normally travel; hence the term "range-azimuth convergence"<sup>†</sup>. After convergence, least-squares moving-arc smoothing is again applied to the converged flight path in order to establish the aircraft's most probable flight path. An explanation of these procedures is best presented by example.

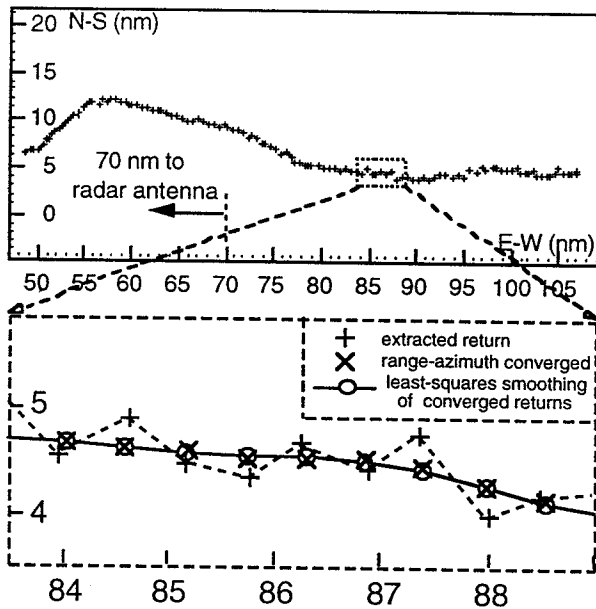


Figure 2 - Flight path based on extracted long-range radar data.

Figure 2 contains an actual set of extracted long-range radar data. Here, an airplane is flying almost radially away from the radar antenna, with positional irregularities in the flight path becoming more pronounced as distance from the antenna increases. Generally, the azimuth of the target can be expected to be more irregular than the range of

the target. The adjusted flight path that results in order from range-azimuth convergence followed by least-squares moving-arc smoothing is illustrated in figure 2 for a short segment of the flight path.

The groundspeed of the aircraft is calculated using the distance traveled return-to-return divided by the elapsed time between those returns; this is termed "point-pair" analysis. The result of simple point-pair computations of the groundspeed from the "raw" extracted radar data is presented in figure 3. The large and rapid variations in the groundspeed are unrealistic, and they plainly demonstrate the need for smoothing. The conventional approach has been to apply simple multipoint averaging in an attempt to smooth the groundspeed; however, a more sophisticated approach, which is utilized here, is multi-point weighted averaging. Nonetheless, as illustrated in figure 3, even when multi-point

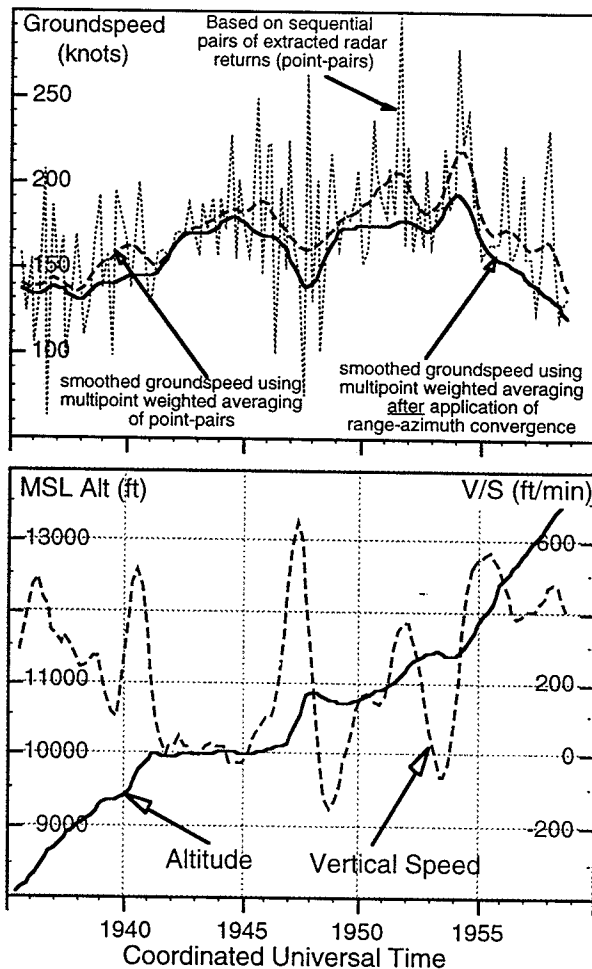


Figure 3 - Groundspeed, altitude, and vertical speed based on the data of figure 2, showing the importance of range-azimuth convergence prior to averaging.

<sup>\*</sup> Conceptually "range-azimuth convergence" has some commonality with the "tolerance box" method suggested by Vermij<sup>(1)</sup>.

<sup>†</sup> A quadratic (second order) moving arc has been adopted for the work reported herein.

weighted averaging is applied, the resulting smoothed groundspeeds are erroneously high. Fundamentally, this is because, as shown in figure 2, the distances between the "raw" extracted radar returns will generally be greater than the actual distance that the aircraft has traveled during that time interval. On the other hand, because the convergence method "adjusts" the returns on to an orderly, more likely flight path, the incorrect bias toward higher values is minimized when the groundspeed is calculated from the converged flight path. Additionally, a comparison of the upper and lower graphs in figure 3 demonstrates that the convergence method provides a groundspeed profile that is more in accord with the variations in altitude and vertical speed as computed from the aircraft's reported altitude.

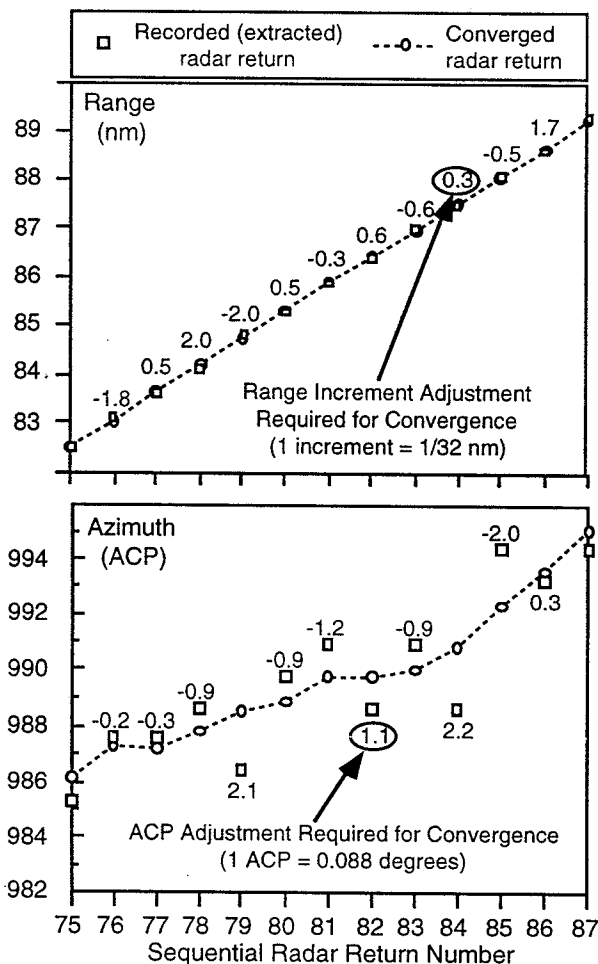


Figure 4 - Seven-point quadratic least-squares moving-arc convergence applied independently to the range and azimuth of the extracted data from figure 2.

Figure 4 illustrates how the least-squares moving-arc procedure is utilized in the analysis of discrete radar data in order to reduce the irregularities

created by range and azimuth errors inherent in the recorded data. In this application, the range or azimuth change pulse (ACP)<sup>‡</sup> is the dependent variable, and sequential return number is the independent variable. Notice that for both the range and the azimuth of the extracted radar returns, the absolute value of the adjustments are less than about 2 increments; in other words, the adjustments are less than 1/16th nm and about 0.18 degrees, respectively. These levels of adjustment are typical, although they may occasionally be larger, especially when the underlying data set is of poor quality.

The converged flight path of the aircraft is represented by X and Y coordinates (true E-W and true N-S, respectively) that are calculated from the converged range and azimuth values. A final smoothing of the flight path is accomplished by again applying quadratic least squares moving-arc smoothing to the X and Y coordinates as dependent variables and time as the independent variable. For long-range (NTAP) radar data, the time will be the corrected time, as developed earlier; for terminal (ARTS) radar data, the time is taken as the recorded time.

#### Multi-Point Weighted Averaging for Groundspeed or True Course

Once the groundspeed and true course have been computed from the converged and smoothed flight path, it is advantageous to use multi-point weighted averaging for additional smoothing. Weighted averaging makes it possible to benefit from trend information that is contained in adjacent values. A pragmatic method for generating weighting functions is to repeatedly apply three-point unweighted averaging until the necessary number of terms have been generated for an m-point application, where m is an odd number (e.g. 5, 7, 9, 11, ...).

Consider a data set for groundspeed or true course as depicted in figure 5. The ordinate values are based on consecutive pairs of converged and smoothed flight path locations. The abscissa values are average or "central" times between locations, also known as "interval" times.

<sup>‡</sup> The azimuth change pulse, ACP, refers to azimuth angle of the radar return based on a 12-bit digitizer that divides 360 degrees into 4096 divisions of 0.088 degrees each.

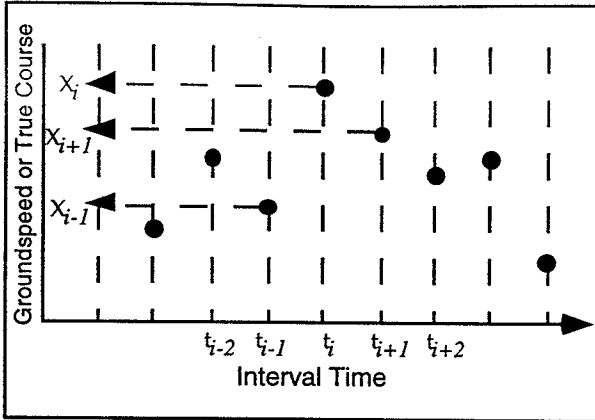


Figure 5 - Generalized depiction of groundspeed or true course values based on consecutive pairs of converged and smoothed flight path locations and corresponding times.

The 3-point average values at times  $t_{i-1}$ ,  $t_i$ , and  $t_{i+1}$  are given by

$$\begin{aligned} \bar{x}_{i-1} &= \frac{1}{3}(x_{i-2} + x_{i-1} + x_i) \\ \bar{x}_i &= \frac{1}{3}(x_{i-1} + x_i + x_{i+1}) \\ \bar{x}_{i+1} &= \frac{1}{3}(x_i + x_{i+1} + x_{i+2}) \end{aligned} \quad (4)$$

In equation (4), the terms are equally weighted with coefficients of (1,1,1), and the denominator (the sum of the coefficients) is used to normalize the coefficients. If three-point averaging is applied again to the average values in equation (4), we get

$$\begin{aligned} \bar{\bar{x}}_{i-1} &= \frac{1}{3}(\bar{x}_{i-2} + \bar{x}_{i-1} + \bar{x}_i) \\ &= \frac{1}{9}(x_{i-3} + 2x_{i-2} + 3x_{i-1} + 2x_i + x_{i+1}) \\ \bar{\bar{x}}_i &= \frac{1}{3}(\bar{x}_{i-1} + \bar{x}_i + \bar{x}_{i+1}) \\ &= \frac{1}{9}(x_{i-2} + 2x_{i-1} + 3x_i + 2x_{i+1} + x_{i+2}) \\ \bar{\bar{x}}_{i+1} &= \frac{1}{3}(\bar{x}_i + \bar{x}_{i+1} + \bar{x}_{i+2}) \\ &= \frac{1}{9}(x_{i-1} + 2x_i + 3x_{i+1} + 2x_{i+2} + x_{i+3}) \end{aligned} \quad (5)$$

The average value  $\bar{\bar{x}}_i$  is a five-point weighted average with coefficients of (1, 2, 3, 2, 1). As

before, the denominator is the sum of and normalizes the coefficients.

It can be shown that the above averaging procedure, applied repeatedly, yields the weighting coefficients listed in Table 1.

### End-Point Weighting

Determining the weighted average near the beginning or end of the data sequence requires a slightly modified technique. Clearly, the above distribution of weighting coefficients cannot be applied because there will not be data beyond the end points. By example, a general method can be developed that allows multi-point weighting to be applied near the end points.

Consider a nine-point ( $m=9$ ) weighted average near the beginning of the groundspeed or true course data set as illustrated in figure 6.

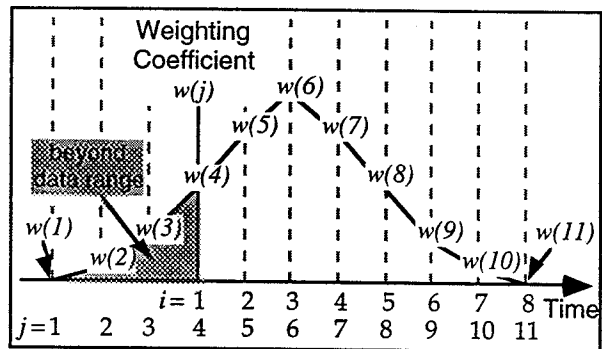


Figure 6 - Use of multi-point weighting functions near the end points of a data set.

The point where the weighted average is to be calculated is the third point where  $i=3$ . The weighting coefficients  $w(j)$  are now a normalized set such that

$$\sum_{j=1}^{j=11} w(j) = 1 \quad (6)$$

Because there is no data for  $j=1$  to  $j=3$ , only the asymmetric weighting distribution from  $j=4$  to  $j=11$  can be applied to the data. In this case, the condition of equation (6) is not met, and the coefficients must be renormalized according to

$$\begin{aligned} w'(j) &= w(j) / \sum_{j=4}^{j=11} w(j) \\ &= w(j) / \left[ 1 - \sum_{j=1}^{j=3} w(j) \right] \end{aligned} \quad (7)$$

# Pts  
in  
Avg

**Table 1**  
**Matrix of Normalized**  
**Weighting Coefficients**

3								0	1	1	1	0					
5							0	1	2	3	2	1	0				
7					0	1	3	6	7	6	3	1	0				
9				0	1	4	10	16	19	16	10	4	1	0			
11			0	1	5	15	30	45	51	45	30	15	5	1	0		
13		0	1	6	21	50	90	126	141	126	90	50	21	6	1	0	
15	0	1	7	28	77	161	266	357	393	357	266	161	77	28	7	1	0
	<i>t<sub>3</sub></i>	<i>t<sub>5</sub></i>	<i>t<sub>7</sub></i>	<i>t<sub>9</sub></i>	<i>t<sub>11</sub></i>	<i>t<sub>13</sub></i>	<i>t<sub>15</sub></i>	<i>t<sub>17</sub></i>	<i>t<sub>19</sub></i>	<i>t<sub>21</sub></i>	<i>t<sub>23</sub></i>	<i>t<sub>25</sub></i>	<i>t<sub>27</sub></i>	<i>t<sub>29</sub></i>	<i>t<sub>31</sub></i>	<i>t<sub>33</sub></i>	<i>t<sub>35</sub></i>

Equation (7) can be generalized for an m-point weighted average at the *i*th data point as

$$w'(j) = w(j) / \left[ 1 - \sum_{j=1}^{j=\left(\frac{m+1}{2}\right)+1-i} w(j) \right] \quad (8)$$

for  $1 \leq j \leq (m+2)$  and  $1 \leq i \leq [(m+1)/2]$ . Similarly, for data near the end of the time sequence, with a total of N data points,

$$w'(j) = w(j) / \left[ 1 - \sum_{j=\left(\frac{m+1}{2}\right)+2+N-i}^{j=m+2} w(j) \right] \quad (9)$$

for  $1 \leq j \leq (m+2)$  and  $[N-(m+1)/2+1] \leq i \leq N$ .

The m-point weighted average for either groundspeed or true course at time  $t_i$ , denoted by  $\bar{X}_m(i)$ , is now written as

$$\bar{X}_m(i) = \sum_{j=\left(\frac{m+1}{2}\right)+2-i}^{j=m+2} w'(j) X \left[ j - \left( \frac{m+1}{2} \right) - 1 + i \right] \quad (10)$$

for  $1 \leq j \leq (m+2)$  and  $1 \leq i \leq [(m+1)/2]$ ,

$$\bar{X}_m(i) = \sum_{j=1}^{j=m+2} w(j) X \left[ j - \left( \frac{m+1}{2} \right) - 1 + i \right] \quad (11)$$

for  $1 \leq j \leq (m+2)$  and  $[(m+1)/2+1] \leq i \leq [N-(m+1)/2]$ , and

$$\bar{X}_m(i) = \sum_{j=1}^{j=\left(\frac{m+1}{2}\right)+1+N-i} w'(j) X \left[ j - \left( \frac{m+1}{2} \right) - 1 + i \right] \quad (12)$$

for  $1 \leq j \leq (m+2)$  and  $[N-(m+1)/2+1] \leq i \leq N$ .

Terminal (ARTS III) Radar Example and  
Digital Flight Data Recorder (DFDR) Comparison

Figure 7 presents a plotting of CDR-extracted beacon-reinforced target data for an aircraft during its downwind, base, and final approach to landing.

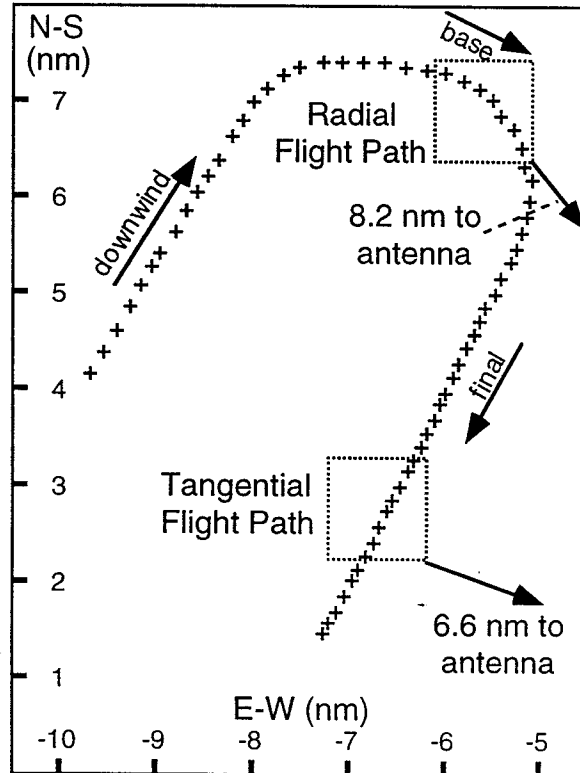


Figure 7 - Beacon-reinforced recorded radar data from a terminal radar facility.

To demonstrate the concepts presented above, it is instructive to consider the two areas of radar returns labeled as "radial flight path" and "tangential flight path" within which the aircraft is traveling radially towards and tangentially around the location of the radar antenna, respectively. Figure 8 reveals the details of the "radial flight

path" denoted in figure 7. The inherent azimuth errors in the recorded data are clearly apparent, and the extracted returns scatter normal to the adjusted and smoothed flight path. Accordingly, the range-azimuth convergence procedure adjusts the locations of the extracted returns by moving them primarily normal to the flight path. As expected, the range corrections are smaller and are in a direction along the flight path.

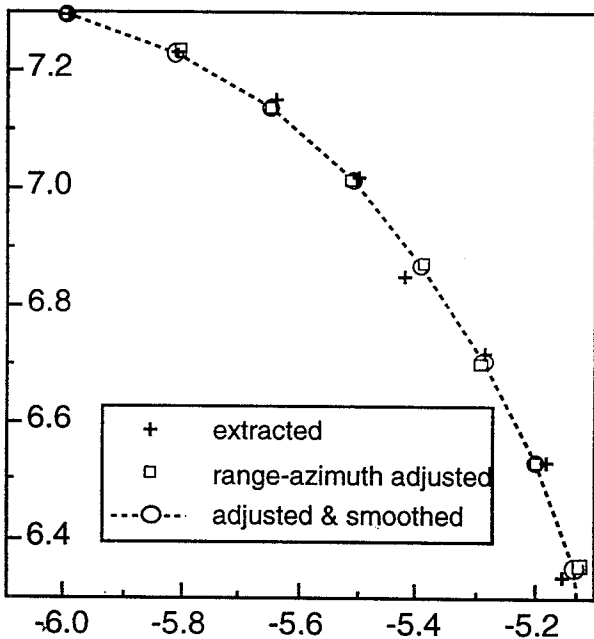


Figure 8 - Details of the "radial flight path" of figure 7.

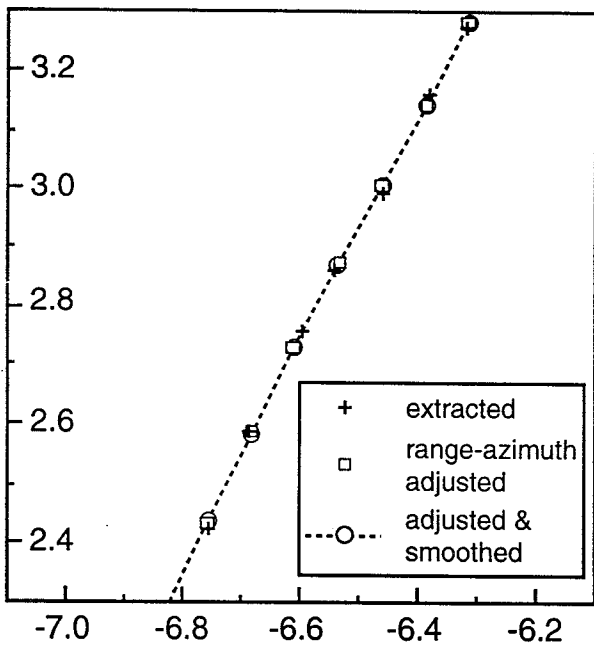


Figure 9 - Details of the "tangential flight path" of figure 7.

Similarly, figure 9 presents the details of the "tangential flight path" denoted in figure 7. Now, however, the dominant azimuthal errors in the recorded radar returns result in scatter of the extracted returns along the flight path rather than normal to it. As before, the range-azimuth convergence procedure is effective in its ability to properly adjust the locations of the returns to locations along a more probable flight path.

Calculation of the true course does not involve using the time interval between flight path locations. As a result, weighted averaging, although typically applied, generally provides little additional smoothing. For recorded data of the quality of the ARTS III example presented here, convergence alone is generally sufficient to produce a fairly accurate description of the true course profile, and little or no weighted averaging is required (e.g., 3-, 5-pt weighted averaging). When the data is of lesser quality, higher levels of weighted averaging may be necessary.

Groundspeed analysis, on the other hand, involves division of the distance between converged, smoothed locations by the smoothed time interval. Hence, significant levels of multi-point weighted averaging are oftentimes necessary to produce an acceptable groundspeed profile (e.g., 11-, 13-, or 15-pt weighted averaging).

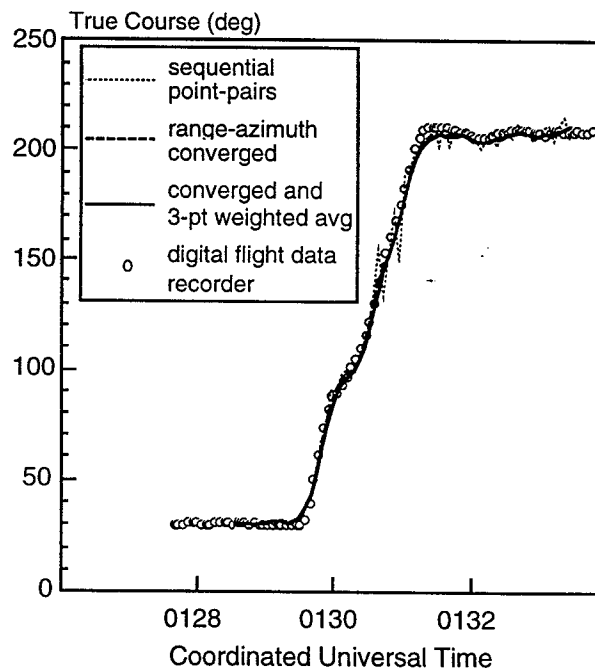


Figure 10 - Comparison of true course from analysis of recorded radar data with true heading from aircraft's digital flight data recorder.

For the sample data set of figure 7, the true course and groundspeed results can be compared with the true heading and groundspeed as stored in the aircraft's digital flight data recorder. Figure 10 shows that the calculated groundspeed based

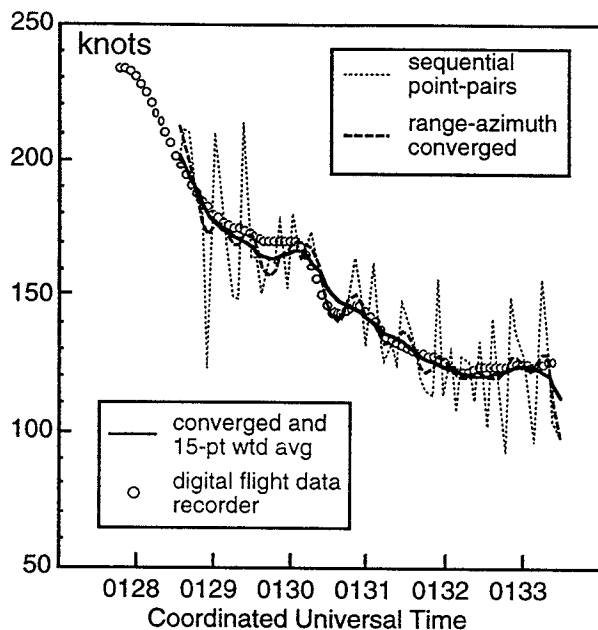


Figure 11 - Comparison of groundspeed from analysis of recorded radar data with groundspeed from aircraft's digital flight data recorder.

only on convergence and smoothing of the flight path compares well with the DFDR data; clearly, in this case, little additional smoothing of the true course is realized from multi-point weighted averaging.

Similarly, figure 11 presents the groundspeed results and a comparison with the DFDR data. The calculated groundspeed correlates well with the DFDR data; in this case, however, it is evident that a higher level of multi-point weighted averaging is necessary to achieve an acceptable groundspeed profile. It is also noteworthy that the groundspeed is in agreement with the DFDR groundspeed at the beginning of the profile, but it differs as the end of the profile is approached, an inevitable consequence of the fact that there is obviously no information on magnitude or trend beyond the endpoints of the data set.

#### Summary and Conclusions

A technique has been presented that provides the aviation accident investigator or reconstructionist with the an improved method for utilizing recorded

ATC radar data to determine the true course and groundspeed of an aircraft. With this method, uncertainties in the true course profile can be expected to be small because the computation requires only the difference between two smoothed flight path locations. However, relative uncertainties in the groundspeed will generally be larger due to the fact that groundspeed is computed by taking the difference of two smoothed flight path locations divided by the difference of two time values. In view of this, the correlation between the calculated groundspeed profile and the DFDR profile that has been presented in this paper suggests that this new approach can provide a reliable tool for the analysis of recorded radar data.

It should be emphasized that without an endpoint handling procedure for the multipoint weighted averaging, as presented above, endpoint groundspeed results would likely not be well behaved and could deviate substantially from the aircraft's actual groundspeed. Hence, endpoint groundspeeds are inherently more uncertain, with accuracy improving as one moves toward intermediate values. The weighting procedure presented in this paper has proven to be successful at moderating end point excursions so that the end point uncertainties are minimized.

The need to smooth both the flight path and the groundspeed when analyzing recorded radar data has been demonstrated. Unfortunately, a consequence of any averaging process is that genuine groundspeed or true course changes that take place within just a few radar sweeps may not be clearly evident in the final results. Nonetheless, given the erratic nature of point-pair calculations of groundspeed and true course, it is likely that those changes would oftentimes go undetected even if averaging were not applied.

#### References

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- <sup>2</sup> Jaluria, Y., *Computer Methods for Engineering*, Allyn and Bacon, Massachusetts, 1988, pp. 267-283.
- <sup>3</sup> Wingrove, R.C., and Bach, R.E., "Analysis of General Aviation Accidents Using ATC Radar Records," *J. Aircraft*, Vol. 20, No. 10, 1983, pp. 872-876.