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NUMERICAL SIMULATION OF FATIGUE CRACK CLOSURE BEHAVIOUR UNDER BIAXIAL LOADING

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Abstract

Fatigue crack closure behaviour of a center-cracked panel under cyclic biaxial loading conditions was investigated using finite element methods. Under the biaxial loading the calculated crack opening and closure stresses are found to correlate closely with the results reported in the literature. To account for the influence of biaxial loading, the Brown&Miller's model is modified to estimate the plastic zone size and the crack-tip opening displacement. Comparison with finite element results demonstrates that the modified model offers a considerable improvement over the original.

Introduction

Many structures in airframes and engines in service experience biaxial/multiaxial loading conditions (proportional or nonproportional). Experimental results have shown that fatigue crack growth rate is strongly influenced by the biaxial loading conditions[1]-[2]. To predict the fatigue crack growth rate under the biaxial loading based on the experimental results generated under uniaxial loading condition, it is important to understand and quantify the influences of biaxial stresses on fatigue crack closure behaviour.

In recent years, there has been increasing interest in finite element simulation of fatigue crack propagation considering plasticity-induced crack closure effect, mainly under the uniaxial loading condition[3]-[5]. The findings are fundamental to the development of the third generation of fatigue crack growth models that are increasingly being used as engineering tools for fatigue life prediction. One difficult aspect of the finite element simulation is the appropriate crack advance/closure scheme. In this regard, a number of techniques have been proposed, such as spring element approach, truss element approach, boundary change technique and contact surface approach. Furthermore, the definitions of crack opening and closure stresses associated with elastic-plastic finite element analysis under the uniaxial cyclic loading have also received a considerable amount of attention to obtain more accurate values.

The main purpose of this study is to investigate fatigue crack closure behaviour of a center-cracked panel under cyclic biaxial loading conditions using finite element methods. The fatigue crack opening and closure stresses were obtained using a spring element release method for elastic-perfect plastic material behaviour. Two typical biaxial stress states have been considered, equibiaxial and shear. The calculated crack opening and closure stresses are compared with those reported in the literature. Especially, based on a strip yielding model, an improved Brown&Miller's theoretical model is developed. The results show a good agreement with the finite element prediction. The study has concluded that biaxial loadings have a significant influence on fatigue crack closure behaviour at low stress ratio with higher maximum applied stress levels. However, the biaxial loading effect at high stress ratio may be negligible. It should be noted that the results presented here all are applied to plane stress condition.

Finite element simulation of fatigue crack growth

After Elber[6] discovered the plasticity-induced crack closure during fatigue crack growth, scientists[7]-[8] started to employ finite element to calculate the crack opening and closure stresses at the crack tip under either constant or variable amplitude uniaxial cyclic loading so that crack growth data can then be correlated properly with the effective stress intensity factor range. The effective stress intensity factor range is the difference between the maximum stress intensity factor and the minimum stress intensity factor at which the crack is just fully open during loading.

To simulate fatigue crack propagation using finite element (FE) methods, a spring element approach was first proposed by Newman[7], and then extensively followed up because of its simplicity. Similarly, the method used in this study involves introducing a series of spring elements along the crack plane. There are a pair of spring elements, one tension-only and one compression-only spring element, for each element node. The tension-only spring elements are used to restrain crack tip during loading, while the compression-only spring elements are used to prevent the crack surfaces penetration during unloading.

Crack growth is simulated by releasing one tension-only spring element per cycle at maximum load.

The material of the center-cracked panel was Ti8-1-1 which is commonly used in the compressor components of aircraft engines. Its Young's modulus, E, is 117.4 GPa and Poisson's ratio, υ, is 0.33. The initial yield stress, σ_y, is 782.0 MPa. The initial crack length, a_i, was 5.0 mm. The half panel width, w, was 101.6 mm.

The panel was meshed with 1754 plane stress four-node quadrilateral elements. To obtain reliable crack opening and closure stresses the element length along the crack advance line should be less than 10 percent plastic zone size for this particular element[4].

To validate the FE model, the crack opening and closure stresses under uniaxial loading conditions were first calculated. The maximum gross applied stress was 0.4 σ_y. The element length along the crack line was 0.04 mm which was about 3 percent of the plastic zone size. Figure 1 shows the stablized crack-opening and closure stresses normalized to the maximum applied stress as a function of stress ratio R. The stability of the crack growth was normally established after 10 cycles so that crack opening stress levels do not change with further crack advance, and the results used in Figure 1 were obtained at cycle 15. The symbol σ_{op} denotes the crack opening stress and σ_{cl} for closure stress.

Under uniaxial loading, based on numerical calculation, Newman[10] developed two equations (1)-(2) which are able to predict crack opening stresses of a center crack tension specimen under both small scale yielding and large scale yielding conditions.

$$\frac{\sigma_{op}}{\sigma_{\max}} = A_0 + A_1 R + A_2 R^2 + A_3 R^3 \qquad (1)$$

for $R \geq 0$ and

$$\frac{\sigma_{op}}{\sigma_{\max}} = A_0 + A_1 R \qquad (2)$$

for $-1 \leq R < 0$. The coefficients A_1 to A_3 are given by

$$A_o = (0.825 - 0.34\alpha + 0.05\alpha^2)[\cos(\pi\sigma_{max}/2\sigma_o)]^{1/\alpha}$$

$$A_1 = (0.415 - 0.071\alpha)\sigma_{max}/\sigma_o$$

$$A_2 = 1 - A_0 - A_1 - A_3$$

$$A_3 = 2A_0 + A_1 - 1$$

where σ_o is the cohesive stress which normally taken to be the average of the yield stress and the ultimate tensile strength of the material under uniaxial loading condition. The coefficient α is the constraint factor, $\alpha=1$ for plane stress and $\alpha=3$ for plane strain. In practice, α is in between 1 and 3 standing for finite thickness plate.

As can be seen from the Figure 1, the opening stresses from the FE model correlate well with Newman's prediction for $-1 \leq R < 0$. For $R=0$, the FE opening and closure stresses are closer to Budiansky-Hutchinson's theoretical solutions[11]. The difference between the FE prediction and the Budiansky-Hutchinson's theoretical solution is 4.8% for the opening stress, and 13.0% for the closure stress. For $R>0$, the calculated opening stresses are slightly higher than Newman's prediction.

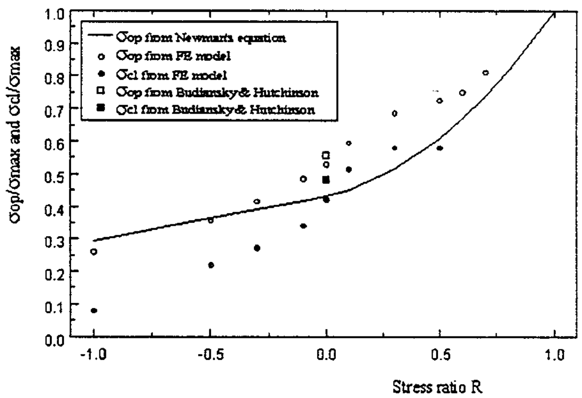

Figure 1 Crack opening and closure stresses vs R

<u>Plastic zone and CTOD under biaxial loading conditions</u>

To predict plastic zone size and crack-tip opening displacement (CTOD), under biaxial loading conditions as shown in Figure 2, Brown&Miller[2] proposed a modification to the original strip yield model by Dugdale[9]. The parameter λ is the ratio of stress parallel to the crack plane to stress perpendicular to the crack plane, for short, the biaxiality ratio. Obviously, the biaxiality ratio $\lambda=0$ stands for uniaxial loading, $\lambda=1$ for equibiaxial and $\lambda=-1$ for shear loading.

In the Brown&Miller's model, by introducing the T-stress, $T = (\lambda-1)\sigma$, and assuming there is no effect of the T-stress on the crack tip singularity, the cohesive stress σ_o for plane stress can then be expressed by the following using the von Mises yield criterion.

$$\sigma_o = \frac{1}{2}\,(T \pm \sqrt{4\sigma_y^2 - 3T^2}\,) \qquad (3)$$

Substituting T and σ_o in Eq.(3) into the Dugdale model, therefore, the plastic zone size is

$$r_p = a[\sec(\frac{\pi}{(\lambda-1)+\sqrt{4(\sigma_y/\sigma)^2-3(\lambda-1)^2}})-1] \qquad (4)$$

and the CTOD is

$$\delta = \frac{4a}{\pi E'}\,[(\lambda-1)\sigma+\sqrt{4\sigma_y^2-3(\lambda-1)^2\sigma^2}\,]\;\ln[\sec(\frac{\pi}{(\lambda-1)+\sqrt{4(\sigma_y/\sigma)^2-3(\lambda-1)^2}})] \qquad (5)$$

where E' = E for plane stress and E' = $E/(1-\upsilon^2)$ for plane strain. From Eqs. (4) and (5), under biaxial loading conditions, the plastic zone size and the CTOD are functions of biaxiality ratio and maximum applied load.

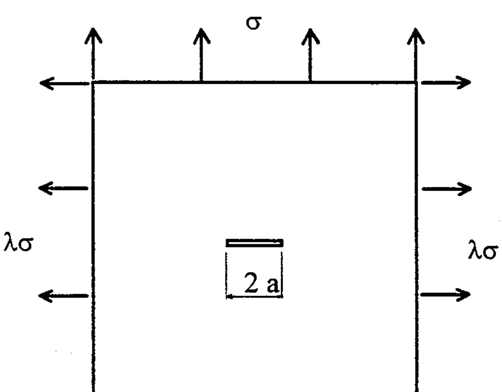

Figure 2 Cracked panel under biaxial loading

The Brown and Miller's model in Eqs. (4)-(5) has some deficiencies. First, the model fails to recover the Dugdale model under uniaxial loading condition. Secondly, the maximum applied stress σ_{max} is limited less than $\frac{1}{\sqrt{7}}\,\sigma_y$ for $\lambda=-1$. In other words, the solution would diverge as the maximum applied stress σ_{max} approaches $\frac{1}{\sqrt{7}}\,\sigma_y$ for $\lambda=-1$.

As shown in Figure 3, the Brown&Miller's model predicts that the plastic zone size is very sensitive to the biaxiality ratios even at the low ratio of σ_{max}/σ_y. However, the FE results show that the plastic zone size is not so sensitive to the biaxiality ratios when the ratio σ_{max}/σ_y is less than 0.4. When the ratio of σ_{max} to σ_y is increased up to 0.5, the FE results did show a strong influence of the biaxiality ratios. The strong influence of the biaxiality ratio at the high ratio of σ_{max}/σ_y (about 0.50) is consistent with the prediction presented in McClung's work[12], which was confirmed by experimental crack growth data.

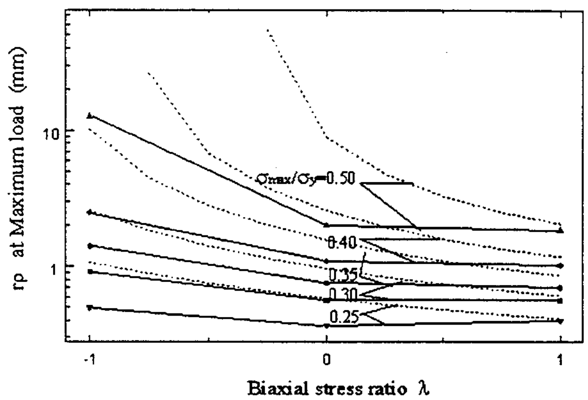

Figure 3 Plastic zone size vs λ
(Dotted lines for B&M model/Line+Symbols for FE)

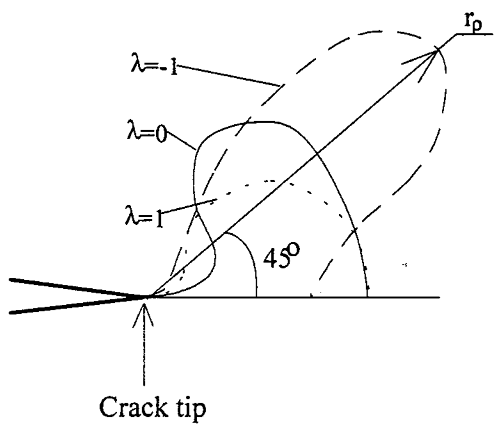

Figure 4 Plastic zones under biaxial loading

In the FE study, it has been found that the plastic zone size and its contour shapes are strongly influenced by the biaxial loading conditions. Figure 4 shows the plastic zone shapes at maximum load for the biaxiality ratios λ=–1, 0 and 1. Plastic zone sizes for λ=0 and λ=1 are equal along the crack line, but different along the 45° direction from the crack tip. The plastic zone for λ=–1 is the largest along the 45° direction, but is smaller than those for λ=0 and λ=1 along the crack line. Because the plastic zone shape are very different for three different biaxiality ratios, the plastic zone size, r_p is herein defined as the maximum radius of the plastic zone contour originating from the crack tip.

Similar to the above discussion on plastic zone size, the CTOD calculated by FE model is also not very much dependent on the biaxiality ratio until the ratio of σ_{max} to σ_y is reaching to 0.5. The value of the CTOD is determined at the intersection of the crack front profile with a 45° line emanating from the crack tip. This operational definition of the CTOD was suggested by Tracey[13]. Figure 5 shows a large degree difference between the Brown&Miller's model and the FE prediction.

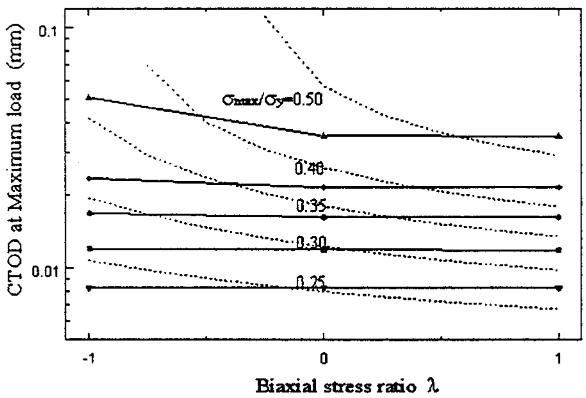

Figure 5 CTOD vs λ
(Dotted lines for B&M model/Line+Symbols for FE)

Improved Brown&Miller's Model

From above results, it is clear that the Brown&Miller's model overestimates the biaxial loading effect. Hence, there is a need to develop an improved model to give better prediction. Based on the Dugdale model, the plastic zone and COD, for the case in Figure 1, can be obtained by superposition of two elastic problems as shown in Figure 6. If applying the von Mises yield criterion for plane-stress condition along the crack plane in Figure 6(b), we have

$$\sigma_o = \frac{1}{2}\left(\lambda\sigma \pm \sqrt{4\sigma_y^2 - 3(\lambda\sigma)^2}\right) \quad (6)$$

On substituting σ_o in Eq.(6) into the Dugdale model, the improved model for the plastic zone size is

$$r_p = a[\sec(\frac{\pi}{\lambda + \sqrt{4(\sigma_y/\sigma)^2 - 3\lambda^2}})-1] \quad (7)$$

and for the CTOD is

$$\delta = \frac{4a}{\pi E'}\,[\lambda\sigma + \sqrt{4\sigma_y^2 - 3\lambda^2\sigma^2}\,]$$

$$\ln[\sec(\frac{\pi}{\lambda + \sqrt{4(\sigma_y/\sigma)^2 - 3\lambda^2}})] \quad (8)$$

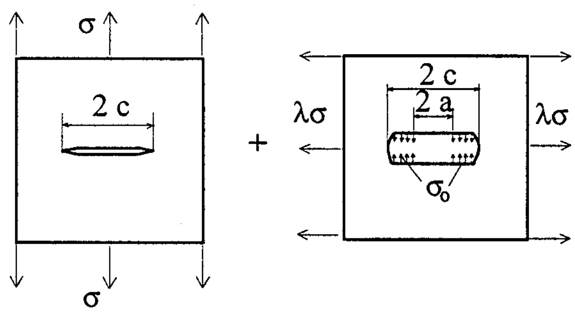

Figure 6 Elastic cracks under biaxial loading

It is evident that the improved model is first able to recover the original Dugdale model under uniaxial loading condition. Secondly, the model is able to calculate the plastic zone size and the CTOD for the maximum applied stress σ_{max} up to $\frac{1}{\sqrt{3}}\sigma_y$ for λ=–1.

Figures 7 and 8 shows the combined effect of the maximum applied load and the biaxiality ratio on the plastic zone size and the CTOD indicating a good agreement with finite element results. The present results reveal that only at the high ratio of σ_{max}/σ_y (about 0.40 and 0.50) do the biaxiality have a significant influence, which is consistent with the McClung's earlier results[12].

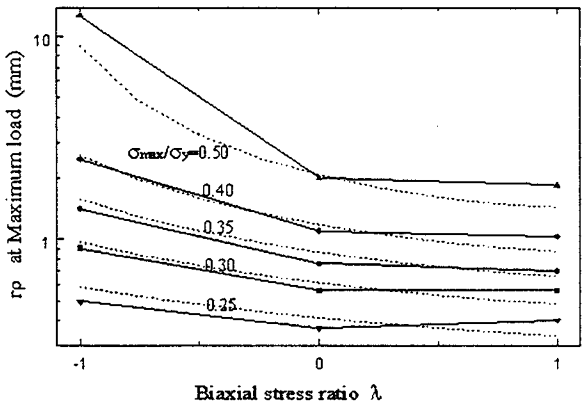

Figure 7 Plastic zone size vs λ by improved model (Dotted lines for B&M model/Line+Symbols for FE)

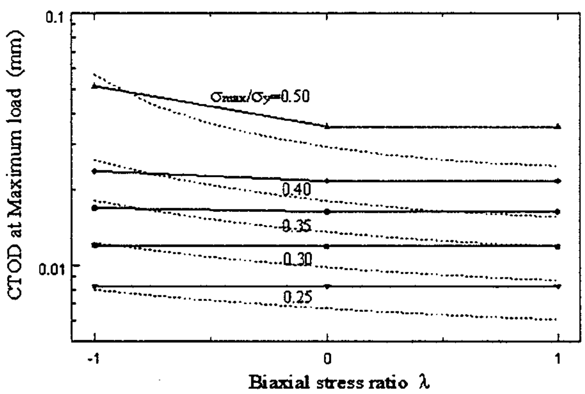

Figure 8 CTOD vs λ by improved model (Dotted lines for B&M model/Line+Symbols for FE)

<u>Crack closure behaviour under biaxial loading conditions</u>

The combined influence of the biaxiality ratios and the maximum applied stress on fatigue crack closure behaviour at various R-ratios in the ranges of $-1.0<R<0.5$ are presented in this section. Generally speaking, there is no definite dependencies of the crack closure on the biaxial stress ratio, λ. It is a combined effect of the biaxiality ratio, the stress ratio and the maximum applied stress level.

Figure 9 presents the crack closure behaviour for $R = -1$ at the biaxiality ratio $\lambda = -1$, 0 and 1. Similar to the McClung's FE results, the σ_{op}/σ_{max} are high for equibiaxial loading and low for shear loading, with an intermediate value for uniaxial loading.

From the Figures 10-12, the effect of biaxiality ratio on the crack closure seems decrease with the increase of the stress ratio R. In extreme case of R=0.5, the biaxial loading effect is diminishingly small and hence may be negligible.

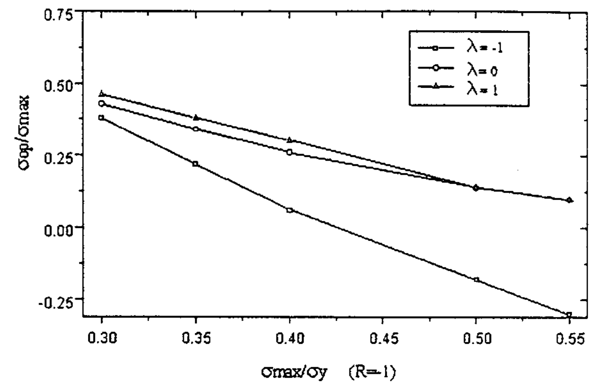

Figure 9 Maximum stress and biaxial loading effect on crack opening stresses for R=-1

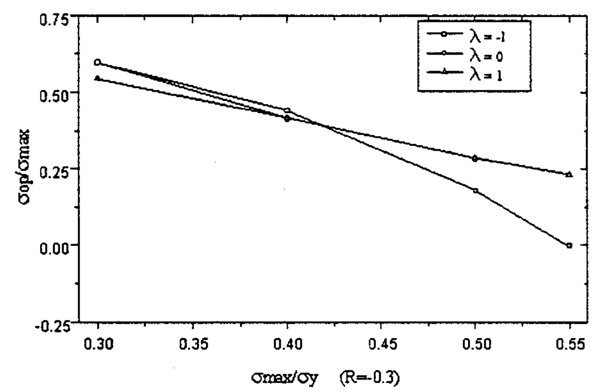

Figure 10 Maximum stress and biaxial loading effect on crack opening stresses for R=–0.3

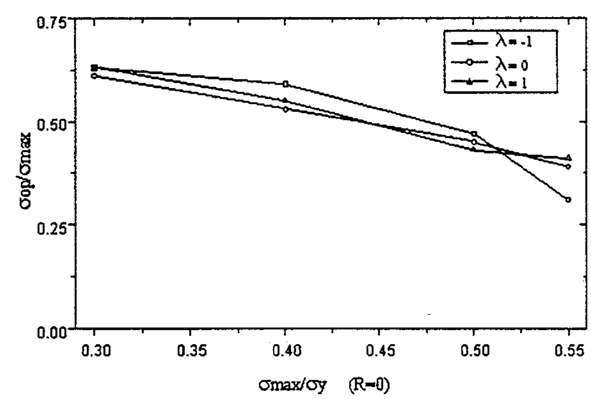

Figure 11 Maximum stress and biaxial loading effect on crack opening stresses for R=0

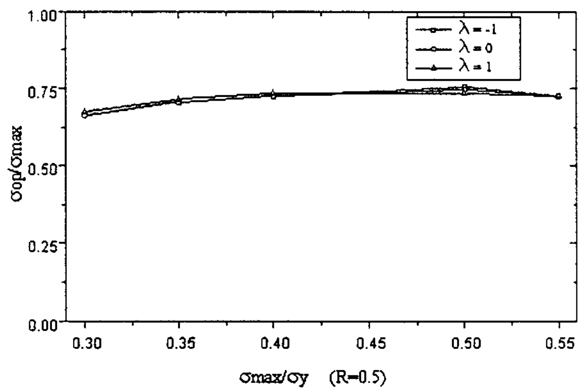

Figure 12 Maximum stress and biaxial loading effect on crack opening stresses for R=0.5

Conclusions

Fatigue crack closure behaviour of a center-cracked panel under cyclic biaxial loading conditions are investigated using finite element methods. An improved Brown&Miller's model is developed for the estimation of the plastic zone size and the crack-tip opening displacement under the biaxial loading conditions. The following conclusions may be drawn from the study.

(1). Biaxial loading may have significant influence on crack closure behaviour at low stress ratio with high maximum applied stress level. But, the biaxial loading effect at high stress ratio may be negligible.

(2). An improved Brown&Miller's theoretical model for the calculation of plastic zone size and the CTOD under biaxial loading conditions has been presented. The improved model is able to recover the Dugdale model under the uniaxial loading condition. Furthermore, under the biaxial loading conditions, the improved model can give better estimation of the plastic zone size and the CTOD assessed with the FE prediction results.

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